Overview of Underwater Acoustics

Reference used in this lecture: Lurton, X. 2002. An introduction to underwater acoustics. New York: Springer. Slides also developed by Dr. Ethem Sozer of MIT Sea Grant.

Definitions

p: pressure, measured *relative to hydrostatic*, Pa

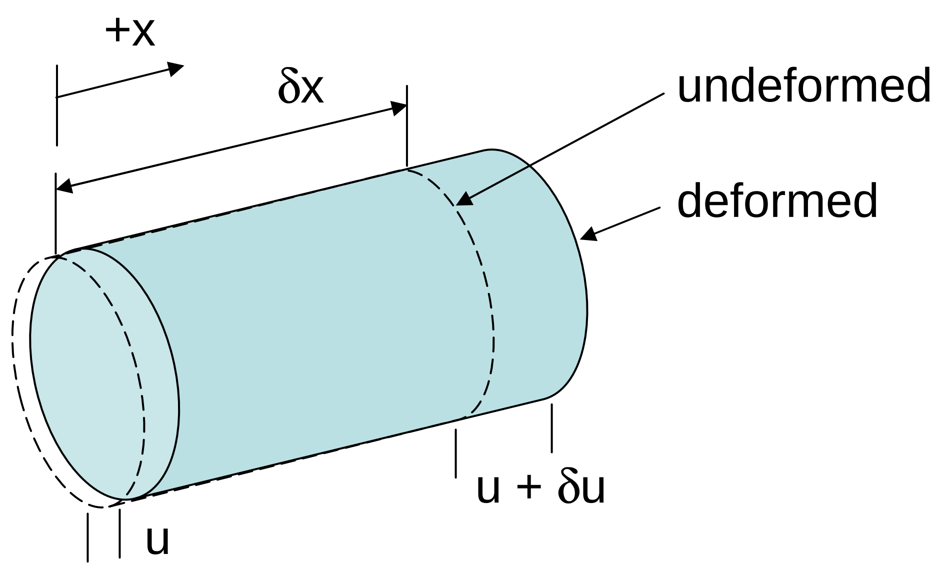
ρ: density, measured *relative to hydrostatic*, kg/m^3

E: bulk modulus of the fluid, Pa, $\delta p = E\,[\,\delta\rho / \rho\,]$

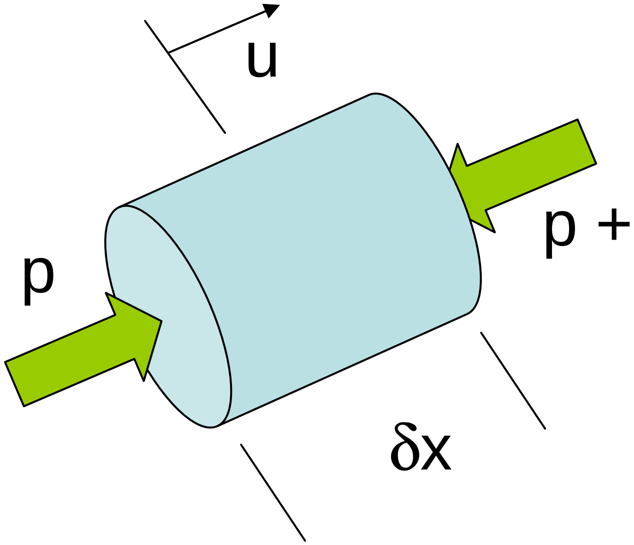
[u,v,w]: deflections in [x,y,z]-directions, relative to the hydrostatic condition, m

Then in one dimension *(pipe)*

$p = E\,[\,-\delta u / \delta x\,]$



One-dimensional Case *cont.*



<u>Newton's Law</u>:

$\delta p = - \rho \, u_{tt} \, \delta x$ OR

$p_x = - \rho \, u_{tt}$

diff wrt x

<u>Constitutive Law</u>:

$p = - E \, \delta u / \delta x$ OR

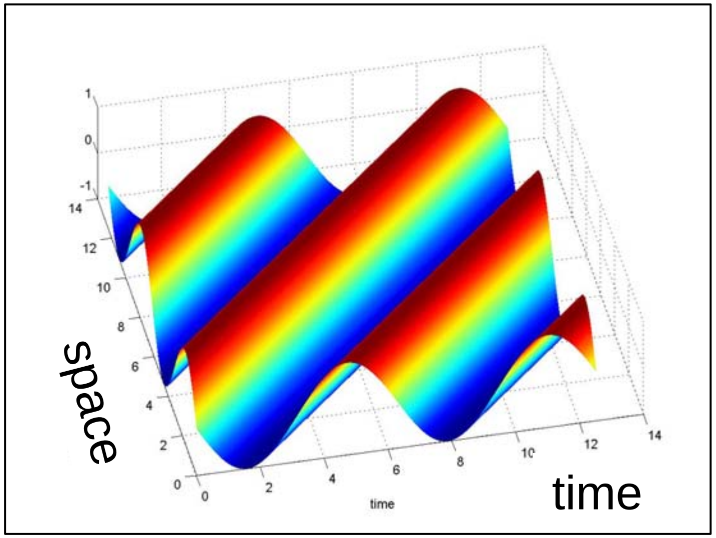
$p = - E \, u_x$

diff wrt tt

$p_{xx} = [\, \rho / E \,] \, p_{tt}$

a wave equation!

Let $p(x,t) = P_o \sin(\omega t - kx)$ ⟶



Insert this in the wave equation:

$$-P_o k^2 \sin(\,) = -[\rho / E] P_o \omega^2 \sin(\,) \rightarrow$$

$$[\omega / k]^2 = E / \rho \rightarrow$$

Wave speed $c = \omega / k = [E / \rho]^{1/2}$

This is *sound speed in water*, independent of pressure, or frequency.

$\rho \sim 1000$ kg/m^3, $E \sim 2.3e9$ N/m^2 → $c \sim 1500$ m/s

Wavelength $\lambda = 2\pi/k = 2\pi c/\omega = c/f$; **1kHz : 1.5m**

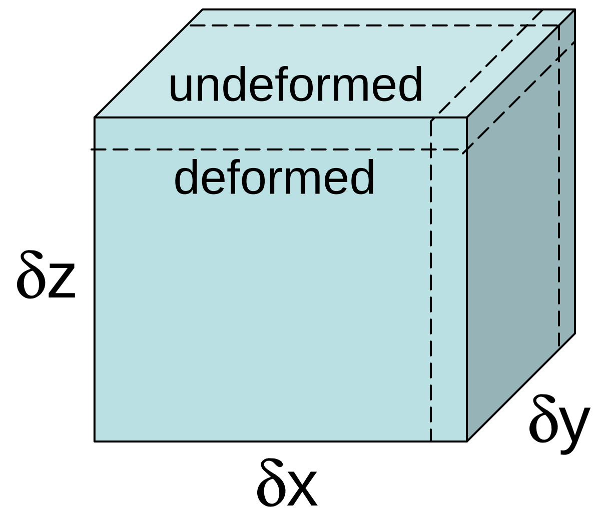
In Three Dimensions: A CUBE

Newton's Law:

$p_x = - \rho\, u_{tt} \rightarrow p_{xx} = - \rho\, u_{ttx}$

$p_y = - \rho\, v_{tt} \rightarrow p_{yy} = - \rho\, v_{tty}$

$p_z = - \rho\, w_{tt} \rightarrow p_{zz} = - \rho\, w_{ttz}$



Constitutive Law:

$- E\, u_x = p / 3 \rightarrow - E\, u_{ttx} = p_{tt} / 3$

$- E\, v_y = p / 3 \rightarrow - E\, v_{tty} = p_{tt} / 3$

$- E\, w_z = p / 3 \rightarrow - E\, w_{ttz} = p_{tt} / 3$

All directions deform uniformly

Lead to Helmholtz Equation:

$$p_{xx}+p_{yy}+p_{zz} = p_{tt} / c^2$$

or

$$\Delta p = p_{tt} / c^2$$

where Δ is the LaPlacian operator

Particle Velocity

Consider one dimension again:

$p_x = -\rho\, u_{tt}$ → $p_x = -\rho\, (u_t)_t$

If $p(x,t) = P_o \sin(\omega t - kx)$ and $u_t(x,t) = U_{to} \sin(\omega t - kx)$ →

$-kP_o \cos(\ ) = -\rho\, \omega\, U_{to} \cos(\ )$ → $U_{to} = P_o / \rho\, c$

Note velocity is in phase with pressure!

[ρ c]: ***<u>characteristic impedance</u>;***

water: $\rho c \sim 1.5e6$ Rayleighs "hard"

air: $\rho c \sim 500$ Rayleighs "soft"

In three dimensions:

$rp = -\rho\, \underline{V}_t$ where

$rp = p_x\, i + p_y\, j + p_z\, k$ and

$\underline{V} = u_t\, i + v_t\, j + w_t\, k$

Note equivalence of the following:

$\lambda = c / f$ and $\omega / k = c$

There is no dispersion relation here; this is the only relationship between ω and k!

Consider Average Power through a 1D surface:

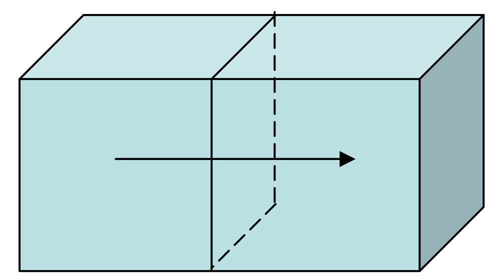
$\mathbf{P}(x) = [\, 1 / T \,]\ s^{\,T}\ p(\tau,x)\ u_t(\tau,x)\ d\tau$

$= [\, 1 / T \,]\ s^{\,T}\ P_o\ U_{to}\ \sin^2(\omega\tau - kx)\ d\tau$

$= P_o\ U_{to} / 2$

$= P_o^2 / 2\ \rho\ c = U_{to}^2\ \rho c / 2$

__Acoustic Intensity__ in W/m^2



Power per unit area is pressure times velocity

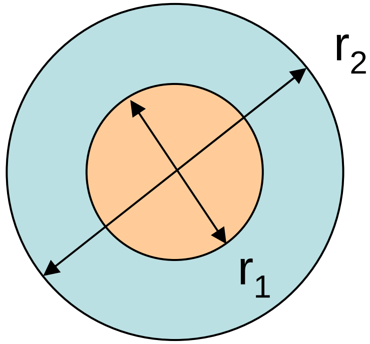
If impedance ρc is high, then it takes little power to create a given pressure level; but it takes a lot of power to create a given velocity level

Spreading in Three-Space

At time t_1, perturbation is at radius r_1; at time t_2, radius r_2 →

$\mathbf{P}(r_1) = P_o^2(r_1) / 2\rho c$

$\mathbf{P}(r_2) = P_o^2(r_2) / 2\rho c$



Assuming no losses in water; then

$\mathbf{P}(r_2) = \mathbf{P}(r_1)\, r_1^2 / r_2^2 = P_o^2\, (r_1)\, r_1^2 / 2\rho c\, r_2^2$

and

$P_o(r_2) = P_o(r_1)\, r_1 / r_2$

Let r_1 = 1 meter (standard!) →

$\mathbf{P}(r) = P_o^2(1m) / 2\rho c\, r^2$

$P_o(r) = P_o(1m) / r$

$U_{to}(r) = P_o(1m) / \rho c\, r$

Pressure level and particle velocity decrease linearly with range

Decibels (dB)

$10 * \log_{10}$ (ratio of two positive scalars):

Example:

$x_1 = 31.6$; $x_2 = 1$ → 1.5 orders of magnitude difference

$10*\log_{10}(x_1/x_2) =$ **15dB**

$10*\log_{10}(x_2/x_1) =$ **-15dB**

RECALL $\log(x_1^2/x_2^2) = \log(x_1/x_2) + \log(x_1/x_2) = 2\log(x_1/x_2)$

In acoustics, *acoustic intensity (power) is referenced to* ***1 W/m²*** *;*
pressure is referenced to ***1 μPa***

$$10*\log_{10}[\,\mathbf{P}(r) / 1\ \mathrm{W/m^2}\,] = 10*\log_{10}\,\mathbf{[}\,[\,P_o^2(r) / 2\,\rho\,c] / 1\ \mathrm{W/m^2}\,\mathbf{]}$$

$$= 20*\log_{10}[\,P_o(r)\,] - 10*\log_{10}(2\rho c)$$

$$= 20*\log_{10}[\,P_o(r) / 1\mu\mathrm{Pa}\,] - 120 - 65$$

Spreading Losses with Range

Pressure level in dB is

$20 \log_{10} [P_o(r) / 1\mu Pa] - 185 =$

$20 \log_{10} [P_o(1m) / r / 1\mu Pa] - 185 =$

$20 \log_{10} [P_o(1m) / 1\mu Pa] -$ $\underline{\mathbf{20 \log10 [r]}}$ -185

Example: At 100m range, we have lost

40dB or *four orders of magnitude* in sound intensity

40dB or *two orders of magnitude* in pressure

(and particle velocity)

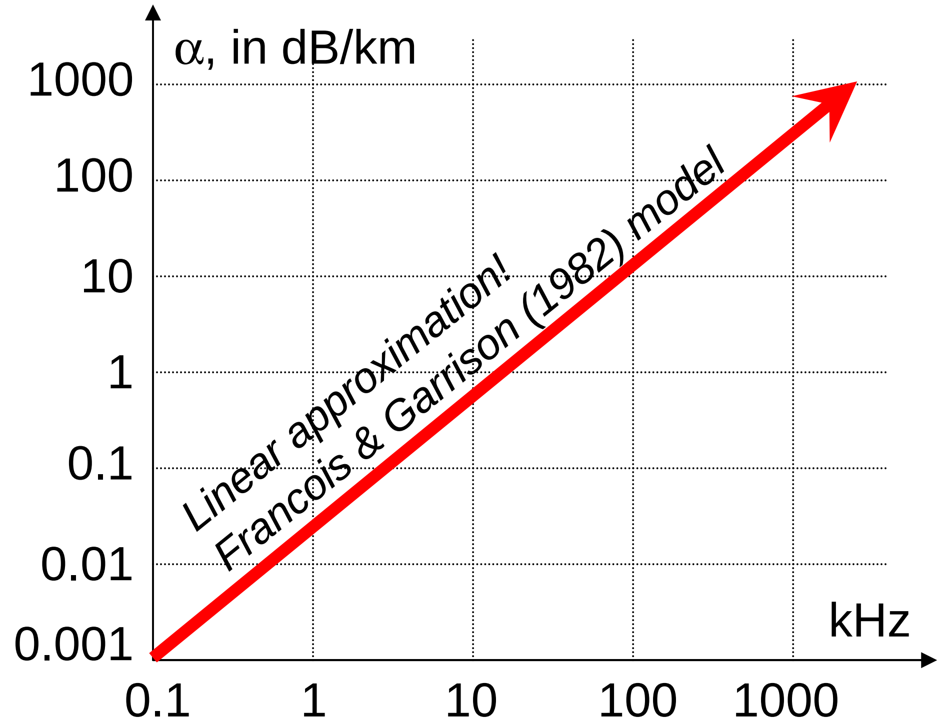
Attenuation Losses with Range

Acoustic power <u>does</u> have losses with transmission distance – primarily related to relaxation of boric acid and magnesium sulfate molecules in seawater. Also bubbles, etc.

At 100 Hz, ~1dB/1000km:
OK for thousands of km, ocean-scale seismics and communications

At 10kHz, ~1dB/km:
OK for ~1-10km, long-baseline acoustics

At 1MHz, 3dB/10m:
OK for ~10-100m, imaging sonars, Doppler velocity loggers



TL = 20log$_{10}$ r + α r (pressure transmission loss)

The Piezo-Electric Actuator

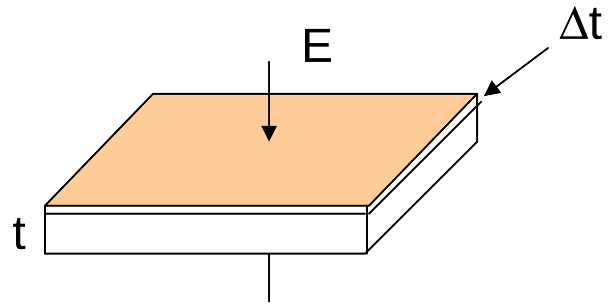
strain = constant X electric field

ε **= d** X ***E*** or

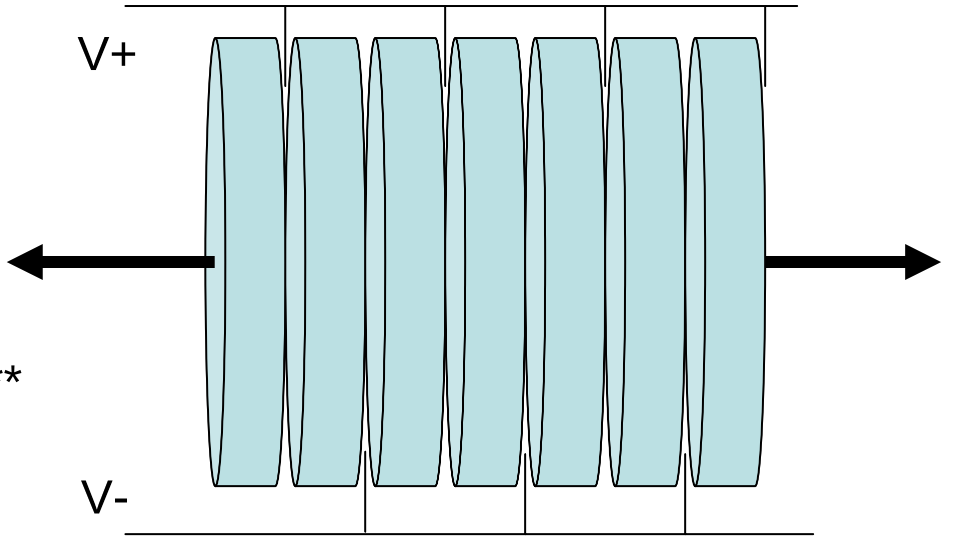
Δ***t / t = d*** X ***(V / t)***

where d = 40-750 x 10^{-12} m / V →

Drive at 100V, we get only 4-75 nm thickness change!

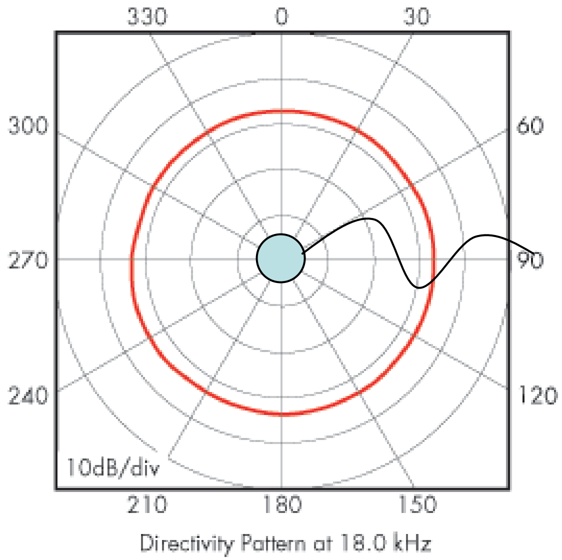


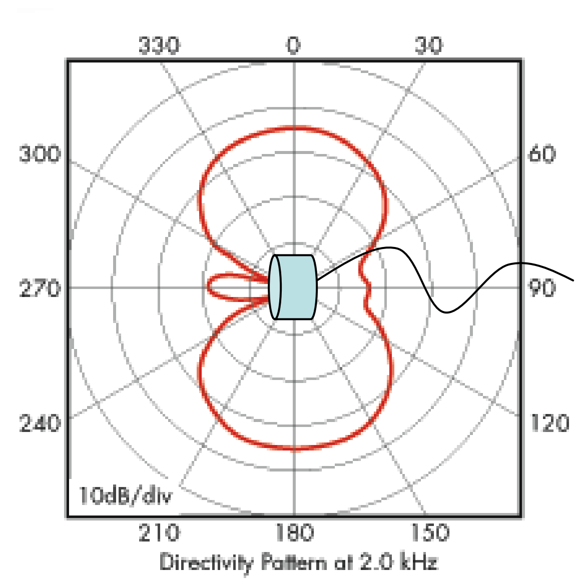
Series connection amplifies displacement



still capable of MHz performance

Directionality Pattern





- ITC 1001 spherical transducer
- Uniform response over all angles (0 to 2π) on both horizontal and vertical plane

- ITC 2010 toroidal transducer
- More gain over the sides (horizontal plane) than the over the top and bottom (vertical plane)

The Piezo-Electric Sensor

electric field = constant X stress

$$\boldsymbol{E = g} \; X \; \sigma \qquad \text{or}$$

$$\boldsymbol{V = t\, g} \; \sigma$$

where g = 15-30 x 10^{-3} V/mN

Ideal Actuator: Assume the water does not impede the driven motion of the material

Ideal Sensor: Assume the sensor does not deform in response to the water pressure waves

Typical Transducer:

120 to 150 dB re 1μPa, 1m, 1V means

$10^6 - 10^{7.5}$ μPa at 1m for each Volt applied or

1-30 Pa at 1m for each Volt applied

Typical Hydrophone:

-220 to -190 dB re 1μPa, 1V means

10^{-11} to $10^{-9.5}$ V for each μPa incident or

10^{-5} to $10^{-3.5}$ V for each Pa incident

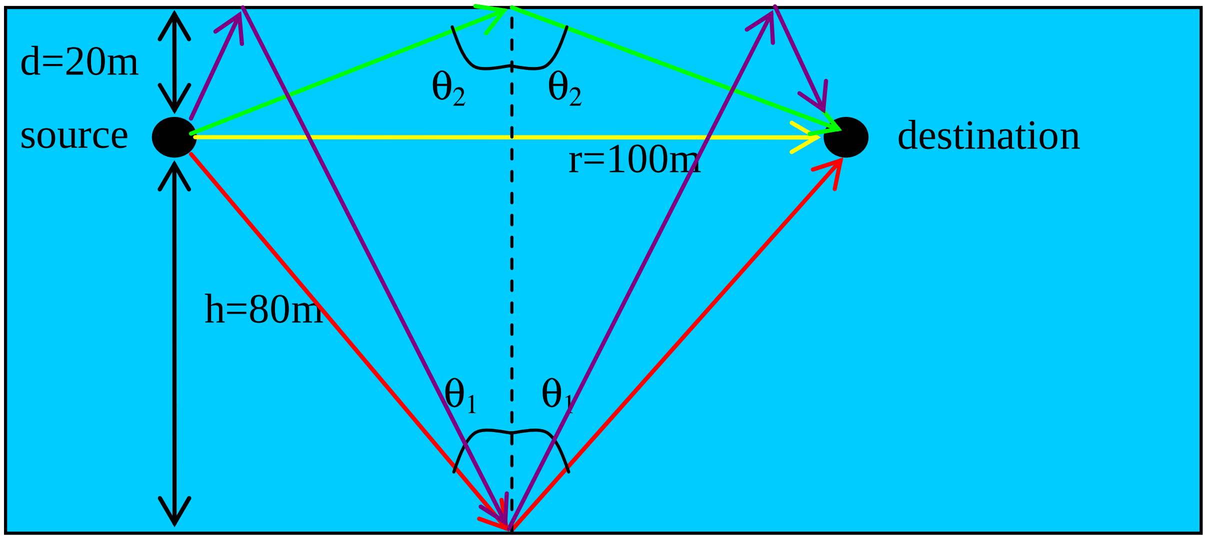
So considering a transducer with 16Pa at 1m per Volt, and a hydrophone with 10^{-4} V per Pa:

If V = 200V, we generate 3200Pa at 1m, or 3.2Pa at 1km, assuming spreading losses only;

The hydrophone signal at this pressure level will be 0.00032V or 320μV !

Shallow Water Propagation

- Assumptions:
  - Constant sound speed (c = 1500 m/s)
  - Surface and bottom are smooth



Surface reflection loss (RLs) = 1 dB
Bottom reflection loss (RLb) = 3 dB

Length of propagation paths

Direct path => d0 = 100m

Bottom reflection => d1 = 2h/cos(q1) = 107.7 m

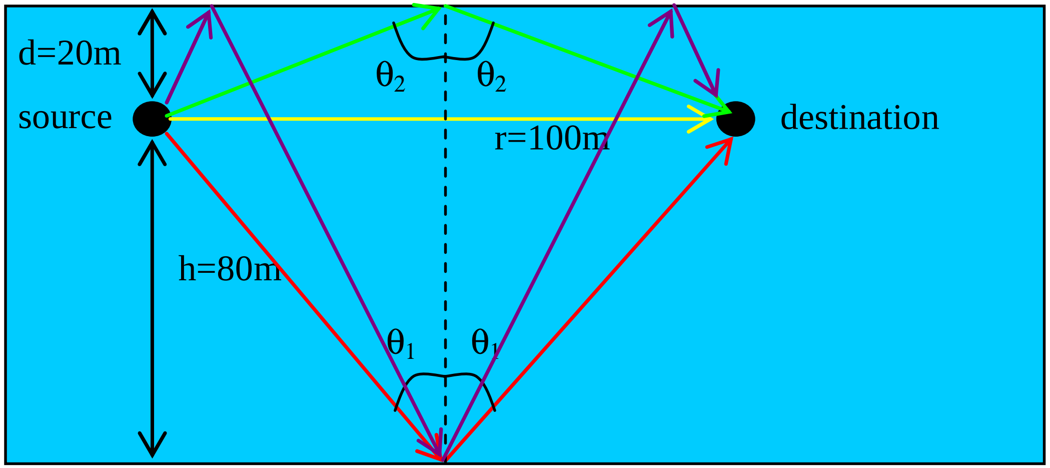
$\theta 1$ = atan(r/2h)

Surface reflection => d2 = 2d/cos($\theta 2$) = 188.7 m

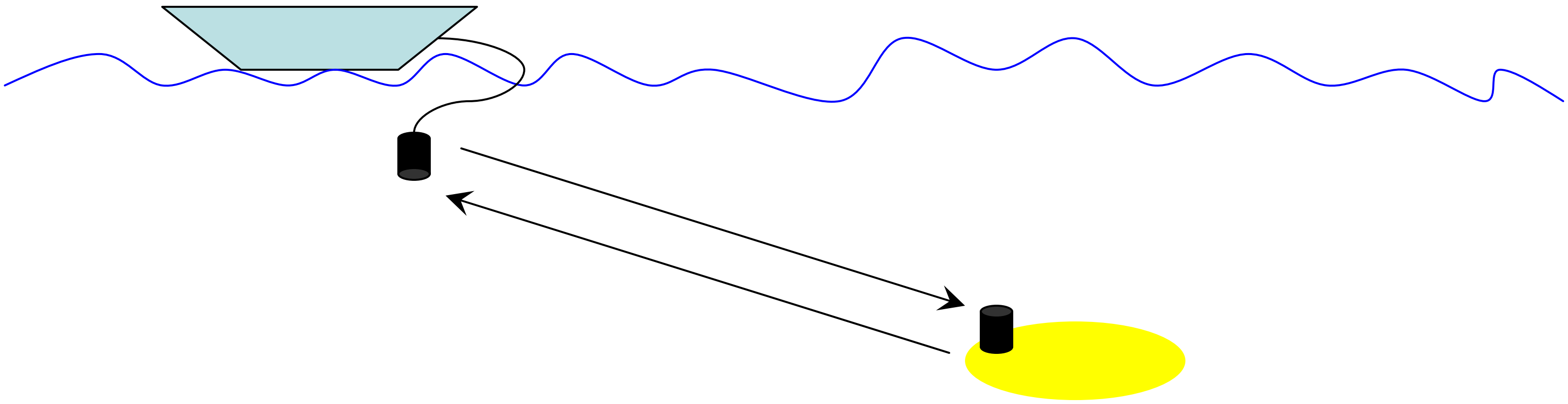
$\theta 2$ = atan(r/2d)

SBS reflection => d3 = 2(2d/cos($\theta 3$)+ h/cos($\theta 3$)) = 260 m

BSB reflection => d4 = (2d/cos($\theta 4$)+ 2(h/cos($\theta 4$))) = 399.5 m

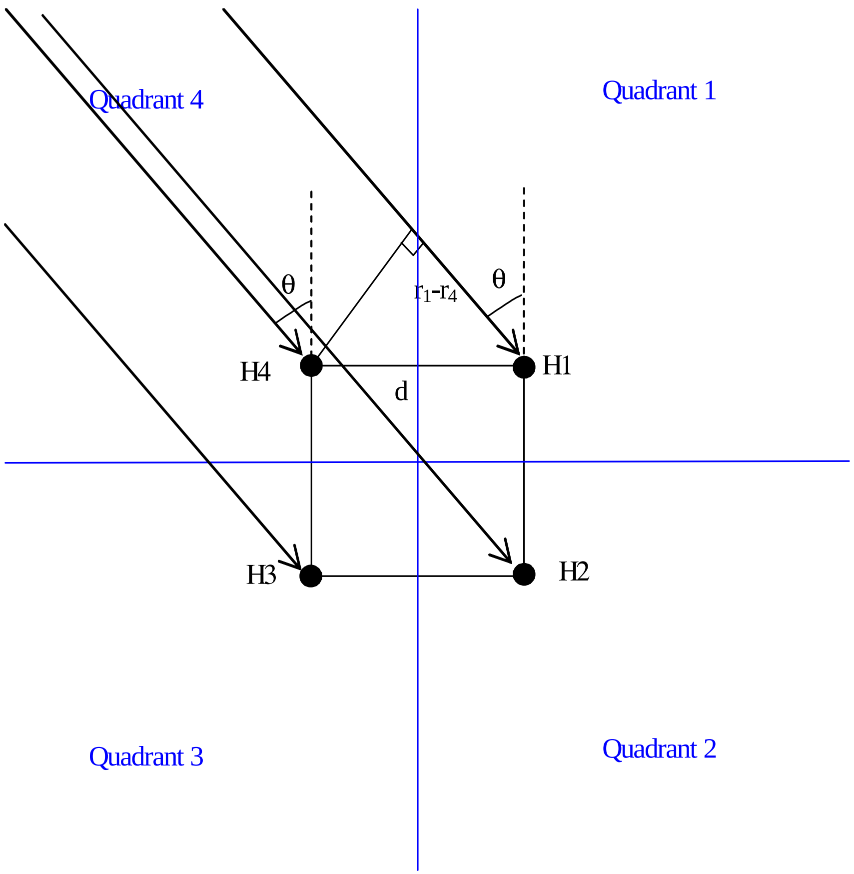


Determining the Range of a Source



- Tracker sends a pulse, $p(t) = A \sin(2\pi f_c t)$, $0<t<Ts$
- Target replies, $p1(t) = A \sin(2\pi f_c(t-\tau_p-\tau_t))$
- Tracker receives, $p2(t) = A \sin(2\pi f_c(t-\tau_p-\tau_t-\tau_p))$
- How can we measure $\tau_p+\tau_t+\tau_p$?

Determining the Direction of the Target



- Four hydrophones
- Measure delay at each hydrophone
- Compare delay pairs (τ_1, τ_2), (τ_2, τ_3),(τ_3, τ_4), (τ_4, τ_1) to find which quadrant
- Estimate the angle

$$\theta = \text{sign}(r_1 - r_4)\text{acos}(|r_1 - r_4| / d)$$