

12.215 Modern Navigation

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Review of Monday Class

Latitude and Longitude

- Simple spherical definitions
- Geodetic definition: For an ellipsoid
- Astronomical definition: Based on direction of gravity
- Relationships between the types
- Coordinate systems to which systems are referred
- Temporal variations in systems

Today's class: Heights

- Definition of heights
 - Ellipsoidal height (geometric)
 - Orthometric height (potential field based)
- Shape of equipotential surface: Geoid for Earth
- Methods for determining heights

Ellipsoidal heights

- Calculation of ellipsoid heights from Cartesian XYZ was covered in Lecture 2.
- The ellipsoid height is the distance along the normal to the reference ellipsoid from the surface of the ellipsoid to the point whose height is being calculated.
- While the geometric quantities, geodetic latitude and longitude are used for map mapping and terrestrial coordinates in general; ellipsoidal height is almost never used (although this is changing with the advent of GPS)
- Why is ellipsoidal height not used?

Orthometric heights

- The problem with ellipsoidal heights are:
 - They are new: Ellipsoidal heights could only be easily determined when GPS developed (1980's)
 - Geometric latitude and longitude have been around since Snell (optical refraction) developed triangulation in the 1500's.
 - Primary reason is that fluids flow based on the shape of the equipotential surfaces. If you want water to flow down hill, you need to use potential based heights.

Orthometric heights

- Orthometric heights are heights above an equipotential surface
- The equipotential surface is called the geoid and corresponds approximately to mean sea level (MSL).
- The correspondence is approximately because MSL is not an equipotential surface because of forces from dynamic ocean currents (e.g., there is about 1m drop over the Gulf stream which is permanently there but change magnitude depending on the strength of the current)

Mean Sea Level (MSL)

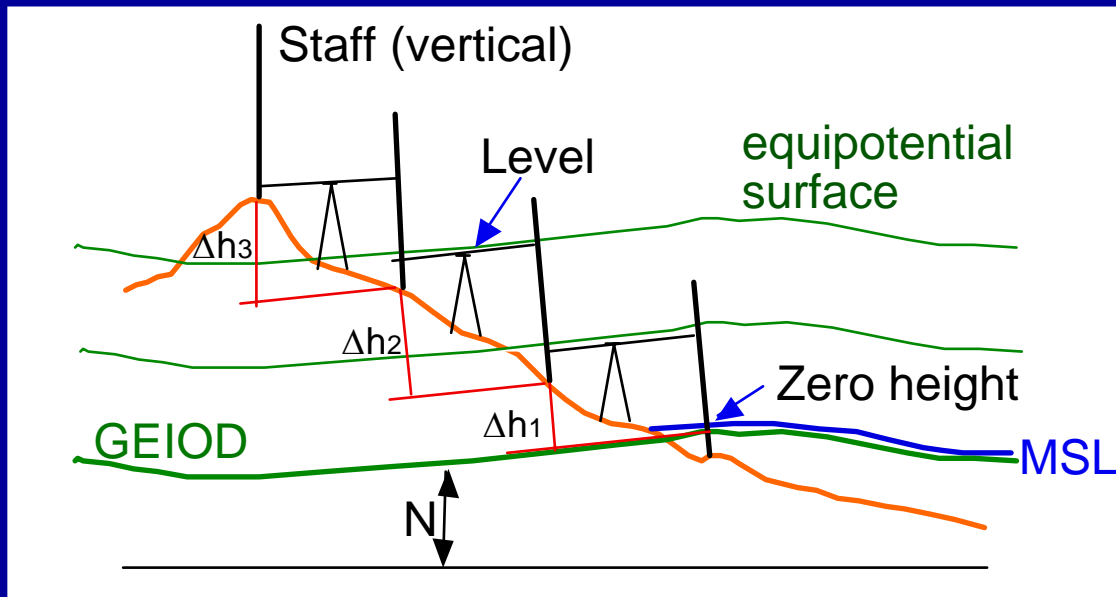
- Ocean tides also need to be considered but this can be averaged over time (signal is periodic with semi-diurnal, diurnal and long period tides. Longest period tide is 18.6 years)
- Another major advantage of MSL is that it has been monitored at harbors for many centuries in support of ocean going vessels
- Also poses a problem because dredging of harbors can change the tides.
- Land-locked countries had to rely on other countries to tell them the heights at the border.
- MSL is reasonably consistent around the world and so height datums differ by only a few meters (compared to hundreds of meters for geodetic latitude and longitude).

Height determination

- Height measurements historically are very labor intensive
- The figure on the next page shows how the technique called leveling is used to determine heights.
- In a country there is a primary leveling network, and other heights are determined relative to this network.
- The primary needs to have a monument spacing of about 50 km.

Leveling

- The process of leveling is to measure height differences and to sum these to get the heights of other points.



Orthometric height of hill is $\Delta h_1 + \Delta h_2 + \Delta h_3$

N is Geoid Height. Line at bottom is ellipsoid

Leveling

- Using the instrument called a level, the heights on the staffs are read and the difference in the values is the height differences.
- The height differences are summed to get the height of the final point.
- For the primary control network: the separation of the staffs is between 25-50 meters.
- This type of chain of measurements must be stepped across the whole country (i.e., move across the country in 50 meter steps: Takes decades and was done).

Leveling problems

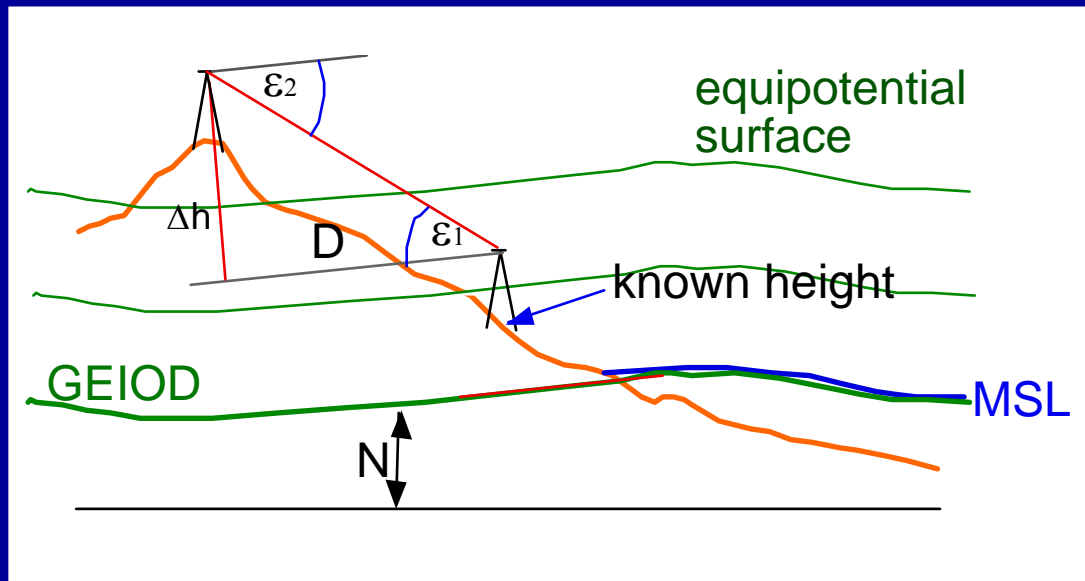
- Because heights are determined by summing differences, system very prone to systematic errors; small biases in the height differences due to atmospheric bending, shadows on the graduations and many other types of problem
- Instrument accuracy is very good for first-order leveling: Height differences can be measured to tens of microns.
- Accuracy is thought to about 1 mm-per-square-root-km for first order leveling.
- Changes in the shapes of the equipotential surface with height above MSL also cause problems.
- The difference between ellipsoidal height and Orthometric height is the Geoid height

Trigonometric Leveling

- When trying to go the tops of mountains, standard leveling does not work well. (Image trying to do this to the summit of Mt. Everest).
- For high peaks: A triangulation method is used call trigonometric leveling.
- Schematic is shown on the next slide
- This is not as accurate as spirit leveling because of atmospheric bending.

Trigonometric Leveling schematic

- Method for trigonometric leveling. Method requires that distance D is known and the elevation angles are measured. Trigonometry is used to compute Δh



Trigonometric Leveling

- In ideal cases, elevation angles at both ends are measured at the same time. This helps cancel atmospheric refraction errors.
- The distance D can be many tens of kilometers. In the case of Mt. Everest, D was over 100 km (the survey team was not even in the same country; they were in India and mountain is in Nepal).
- D is determined either by triangulation or after 1950 by electronic distance measurement (EDM) discussed later
- The heights of the instruments, called theodolites, above the ground point must be measured. Note: this instrument height measurement was not needed for leveling.

Geoid height

- Although the difference between ellipsoidal and orthometric height allows the geoid height to be determined, this method has only been used since GPS became available.
- Determining the geoid has been historically done using surface gravity measurements and satellite orbits.
- Satellite orbit perturbations reveal the forces acting on the satellite which if gravity is the only effect is the first derivative of the potential (atmospheric drag and other forces can greatly effect this assumption)

Geoid height

- The long wavelength part of geoid (greater than 1000km) is now determined from satellite orbit perturbations.
- The <1000 km wavelength use surface gravity and solve a boundary value problem where the derivative of the function which satisfies Laplace's equation is given on the boundary, and the value of the function is needed.
- Most of the great mathematicians worked on field theory trying to solve the Earth boundary value problem (Laplace, Legendre, Green, Stokes)
- The standard method of converting gravity measurements to geoid height estimates is called Stokes method.
- This field is called physical geodesy

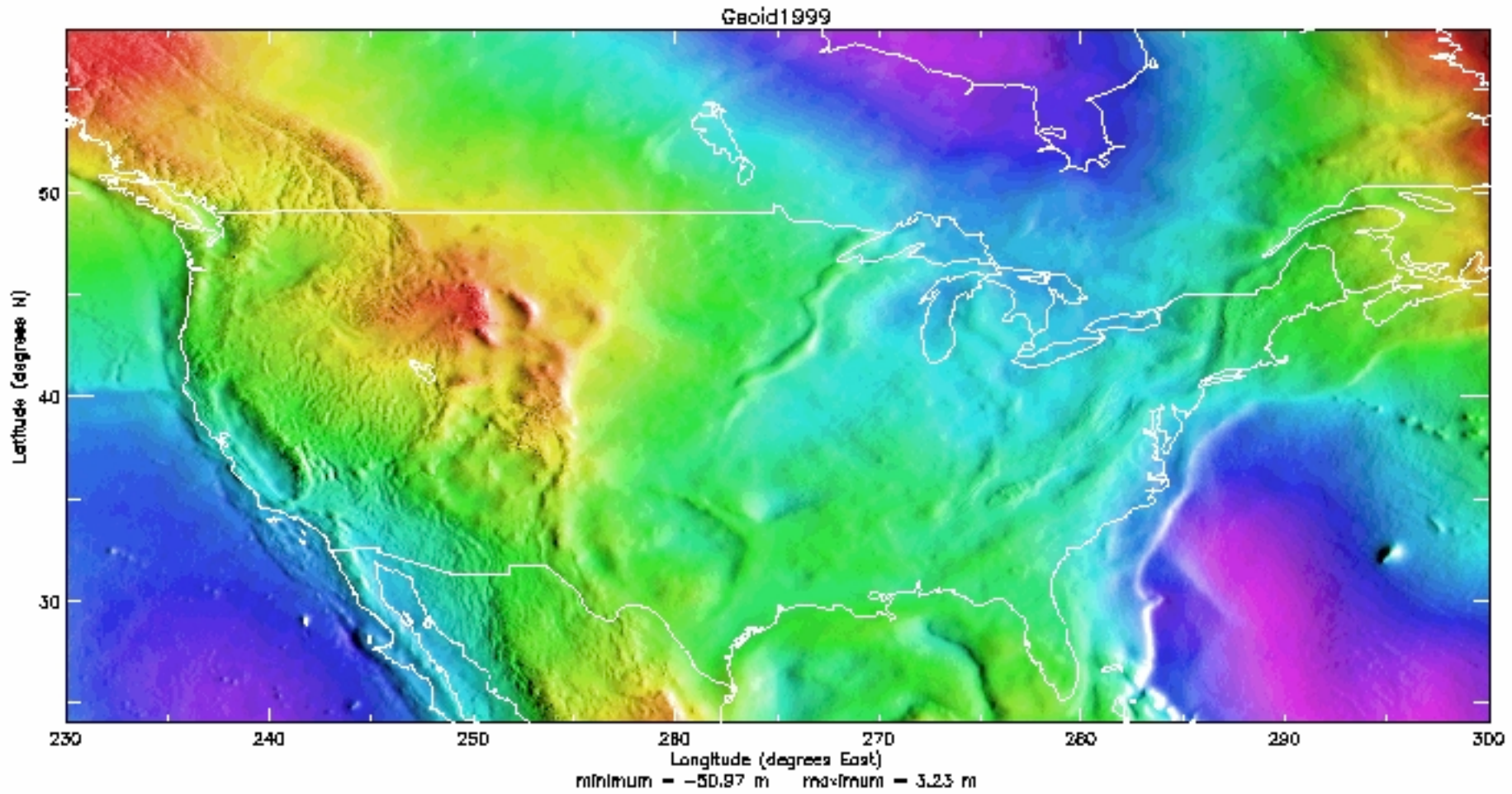
US Geoid

- National geodetic survey maintains a web site that allows geoid heights to be computed (based on US grid)

http://www.ngs.noaa.gov/cgi-bin/GEOID_STUFF/geoid99_prompt1.prl

- Near Boston geoid height is -27.688 m
- The lowest geoid height in the US is -51m and the highest is 3.2m
- Lowest value in the world is south of India and is about -100 m

US Geoid



Summary

- In Today's class we have discussed:
 - Definition of heights
 - Ellipsoidal height (geometric)
 - Orthometric height (potential field based)
 - Shape of equipotential surface: Geoid for Earth
 - Methods for determining heights
- Check the links given in this lecture for more information
- Also try web searching on physical+geodesy, geoid, radar+altimetry.