12.307 Weather and Climate Laboratory
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12.307
Project 1
Mass and wind - geostrophic/ageostrophic flow

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Abstract

The purpose of this project is to study, using meteorological observations and laboratory experiments, the relation between the wind field and the mass field in a rotating system. In part I we explore the relationship in laboratory experiments; in part II atmospheric observations of intense cyclones and hurricanes are used.

1 Part I: Laboratory Experiment

Here we will use the so called "radial inflow" (also called "balanced vortex") experiment to think about frames of reference, centrifugal and Coriolis forces and the connection between the velocity field and the pressure (mass) field. A more detailed description of the experiment and associated theory is attached.

2 Part II: Atmospheric Data

2.1 Hurricane flow: surface scatterometer winds

Begin by looking at flow in a hurricane from a list of interesting cases from the past few summers. Here are few examples:

- in the summer of 2005 the Atlantic hurricane season was particularly active with several strong hurricanes badly affecting the Southern US coastal regions - see attached satellite images of hurricanes Katrina and Rita,

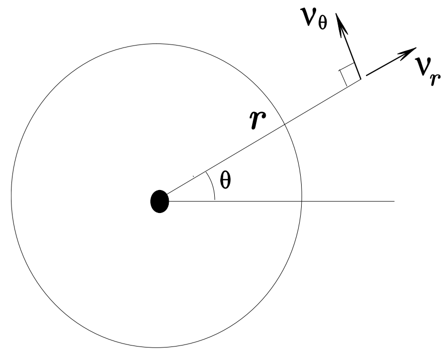

Figure 1: The velocity of a fluid parcel viewed in the rotating frame of reference: $v_{rot} = (v_\theta, v_r)$.

- in n the summer of 2006 few hurricanes formed in the Atlantic, but several intense ones formed in the Pacific - see attached satellite image of Typhoon Ioke on September 4, 2006,
- in the Summer of 2008, Hurricane Bertha was the first hurricane to form in the Atlantic - see attached image. It broke the record for longevity lasting 17 days during the month of July!

A more comprehensive list of hurricanes occurances is available in the project website. Choose one event and plot the surface winds from the Scatterometer imagery. From the observed velocities compute the Rossby number,

$$R_o = \frac{|v_\theta|}{fr} \tag{1}$$

where r is the distance from the center of the hurricane - see Fig.1. How does R_o depend on radius? How does it compare to what you observed in the radial inflow experiment? Discuss the balance of forces typical of the hurricane flow.

2.2 Hurricane flow: upper air analyzed data

For the same event plot the geostrophic wind at various pressure levels and compare it to the analyzed wind. What do you find?

Let's consider parcels of air that are trapped in closed, axi-symmetric motion, moving with constant tangential velocity v_θ along a path of curvature r - see Fig.1.

If the R_o is of order unity, then the balance of forces in the radial direction is a three-way balance between centrifugal, Coriolis and pressure gradient forces:

$$\frac{v_\theta^2}{r} + f v_\theta = g\frac{\partial h}{\partial r}$$

where h is the height of a pressure surface (see Eq.(2) in Appendix). This can be re-arranged to give:

$$v_\theta = \frac{g}{\left(f + \frac{v_\theta}{r}\right)} \frac{\partial h}{\partial r}$$

Thus we expect that the observed wind will be less than the geostrophic wind in a cyclonic situation ($v_\theta > 0$). Is your data consistent?

What about anticyclonic situations ($v_\theta < 0$)?

Solving the above equation for v_θ we find that:

$$v_\theta = -\frac{1}{2} f r \pm \left(\frac{1}{4} f^2 r^2 + g r \frac{\partial h}{\partial r}\right)^{\frac{1}{2}}$$

where the positive root must be chosen; why? Is there any quantitative agreement with your observed cyclone?

Compute the ratio of geostrophic to observed wind; how does this ratio depend on the Rossby number, Eq.(1), of your system?

What would you expect in a tornado? Can you see analogues with the balanced vortex experiment?

2.3 Planetary scale and geostrophic balance from analyzed data

Plot the 500mb height and wind from recent analyses. Notice that the analyzed wind tends to follow the 500mb height field with a speed inversely proportional to the separation of the height isolines as we expect from the geostrophic approximation.

The geostrophic relationship is:

$$\text{in height coordinates: } \mathbf{v}_g = \frac{1}{\varrho f} \mathbf{k} \times \boldsymbol{\nabla} \mathbf{p}_{z=const}$$

$$\text{in pressure coordinates: } \mathbf{v}_g = \frac{g}{f} \mathbf{k} \times \boldsymbol{\nabla} \mathbf{z}_{p=const}$$

where $f = 2\Omega \sin lat$ is the Coriolis parameter, ϱ is the density, p is the pressure, z is height, g is the acceleration due to gravity and $\mathbf{k}$ is a unit vector in the vertical.

Plot the geostrophic wind and check how close it is to the observed wind. Look at regions of large curvature, such as pronounced troughs or ridges. How close is the geostrophic wind to the observed one in these regions? Discuss the balance of forces typical of strong cyclones or anti-cyclones.

2.4 Ekman Layer and surface analyzed data

How close to geostrophic is the wind at the surface? - plot mean sea-level pressure, deduce the geostrophic wind and compare it with the observed wind. Think about the balance of forces in the surface boundary layer

The surface wind stress produces important frictional forces in the surface boundary layer of the atmosphere, extending up to an elevation of about 1 km, which retard the boundary layer winds and induces cross isobaric flow.

The horizontal momentum equation in the surface boundary layer can be written as (see 12.003 notes):

$$\frac{D\mathbf{v}}{Dt} + f\mathbf{k} \times \mathbf{v} + \frac{1}{\rho}\nabla p = \mathbf{F}$$

where the frictional force is $\mathbf{F} = \frac{1}{\rho}\frac{\partial \boldsymbol{\tau}}{\partial z}$ and τ is the stress.

Supposing that $\mathbf{v}_0$ is the wind at the surface and that the frictional force over the boundary layer acts against it, sketch the balance of forces - frictional, pressure gradient and Coriolis - in the boundary layer. Mark on your diagram $\mathbf{v}_0$ and $\mathbf{v}_g$, where $\mathbf{v}_g$ is the geostrophic wind in balance with pressure gradient forces. Deduce that $\mathbf{v}_0$ must have a component directed down the pressure gradient.

If $\boldsymbol{\tau}_0$, the frictional stress at the ground, is given by

$$\boldsymbol{\tau}_0 = -\varrho c_d v_0 \mathbf{v}_0$$

where c_d is the drag coefficient, show that γ, the angle between $\mathbf{v}_0$ and $\mathbf{v}_g$, is given by

$$\gamma = \tan^{-1}\left(\frac{c_d v_0}{\Delta z f}\right)$$

and

$$\frac{v_0}{v_g} = \frac{1}{\sqrt{1 + \left(\frac{c_d v_0}{\Delta z f}\right)^2}}$$

where Δz is the depth of the boundary layer and f is the Coriolis parameter.

If c_d has a value of, on the average, 2×10^{-3}, $\Delta z = 10^3 m$ and $f = 10^{-4} s^{-1}$, calculate γ and $\frac{v_0}{v_g}$. Hence complete and discuss the table below (c_d varies from 2×10^{-3} to about half this value over the oceans up to a few times this value over rugged mountains).

$\left(\frac{c_d v_0}{\Delta z f}\right)$	γ	$\frac{v_0}{v_g}$
0	0	1
0.4		
0.8		
1.2		
1.6		
2.0		

What happens in the limit that the frictional force overwhelms the Coriolis force?

3 Appendix: Geostrophic, cyclostrophic and gradient wind balance

Consider parcels of air trapped in closed, axi-symmetric motion, moving with constant tangential velocity v_θ along a path of curvature r - see Fig.1. The radial balance of forces away from frictional boundary layers and in the steady state is:

$$\text{gradient wind:} \frac{v_\theta^2}{r} + f v_\theta = g \frac{\partial h}{\partial r} \tag{2}$$

Eq (2) is known as the 'gradient wind' relation.

The Rossby number, $R_o = \frac{v_\theta}{fr}$ measures the ratio of $\frac{v_\theta^2}{r}$ to $f v_\theta$. There are two limit cases: If $R_o << 1$, then we have 'geostrophic balance':

$$\text{geostrophic balance: } f v_\theta = g \frac{\partial h}{\partial r} \tag{3}$$

If $R_o >> 1$ then the 'cyclostrophic' relationship can be used:

$$\text{cyclostrophic balance:} \frac{v_\theta^2}{r} = g \frac{\partial h}{\partial r} \tag{4}$$