Chapter 6

Rossby waves and planetary scale motions

6.1 Observed planetary scale waves in the atmosphere

Fig. 6.1 shows (solid contour; interval 4hPa) a typical northern hemisphere surface pressure map. It shows a rich structure, mostly of "synoptic scale" systems, especially small low-pressure storm systems. These have a range of sizes and intensities; there is a particularly large and vigorous storm over Iceland.

If we look, at the same time, at upper air charts, we see the influence of these storm systems weakening in the analysis. Fig 6.2 shows the height of the 500hPa pressure surface (solid contours; interval 60m) at the same time. The intense surface features are much less obvious here. Rather, the midlatitude jet is apparent[1] in the belt of tight height gradient around the hemisphere. However, there are strong wavy perturbations of the jet, usually of larger scale than the features that dominated the surface analysis (except over N America, where "synoptic" scale features are apparent at 500hPa also.) In terms of zonal wavenumber (the number of wavelengths around a latitude circle), the large-scale upper level disturbances have typical wavenumbers 1-4. These scales are referred to as *planetary*, and the wave motions on these scales as *planetary waves.* These waves migrate both east-

[1] Through geostrophic balance, the tight height gradient implies rapid flow along the height contours.

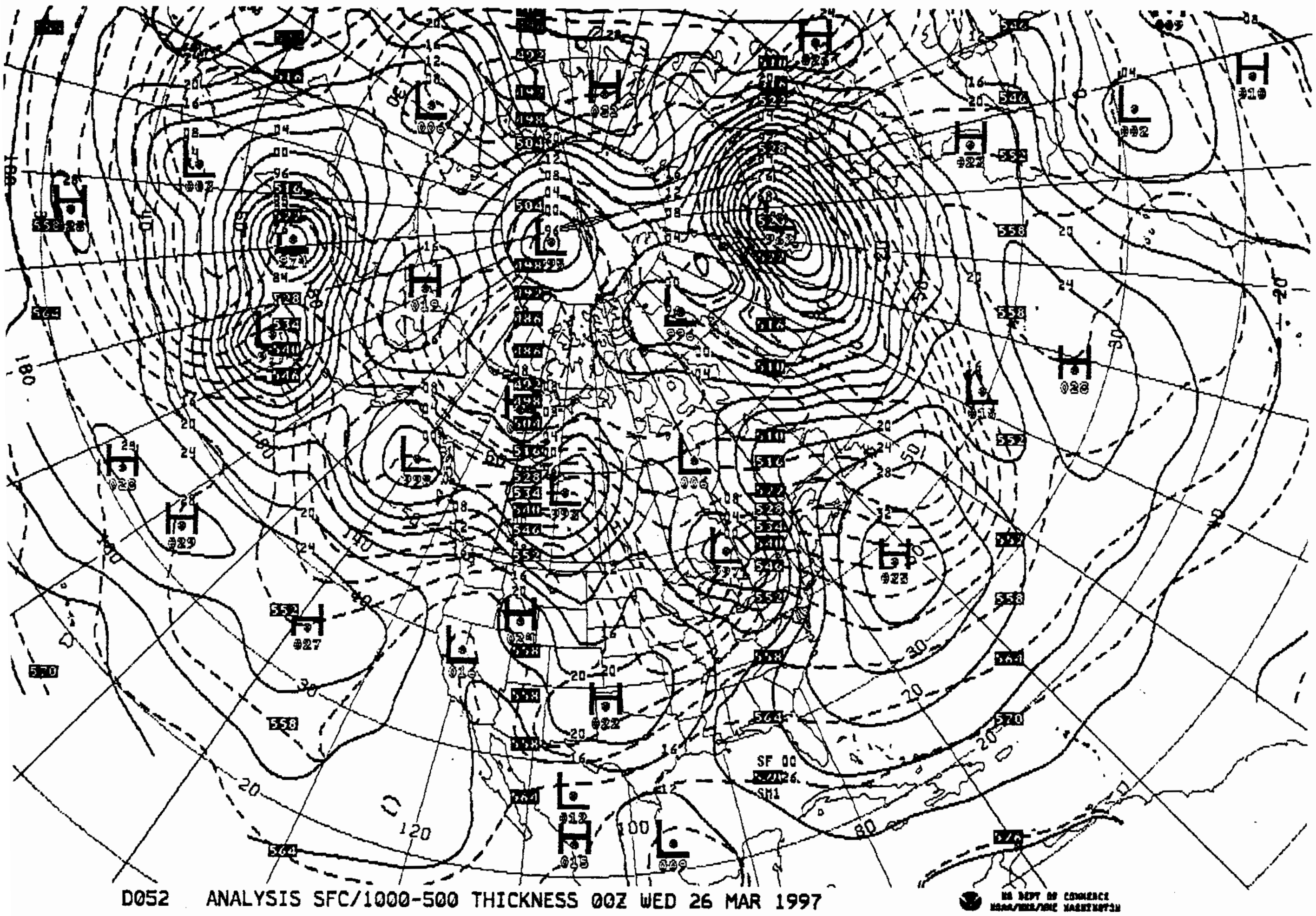

Figure 6.1: Surface pressure analysis (solid contours), 26 March 1997.

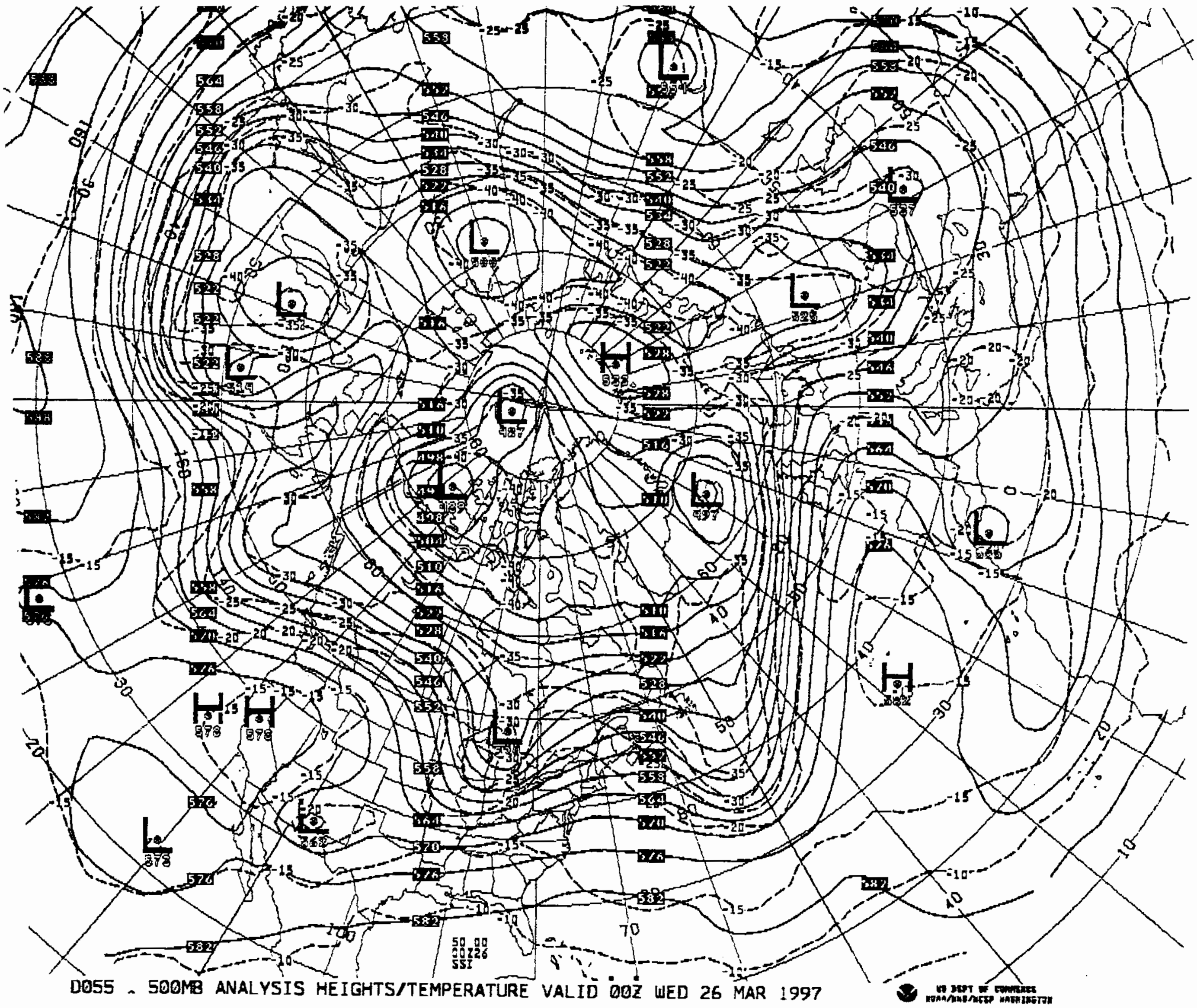

Figure 6.2: 500hPa analysis (solid contours), 26 Mar 1997.

ward and (sometimes) westward; they also include a substantial *stationary* component. This latter fact is evident from Fig. 6.3, which shows the N

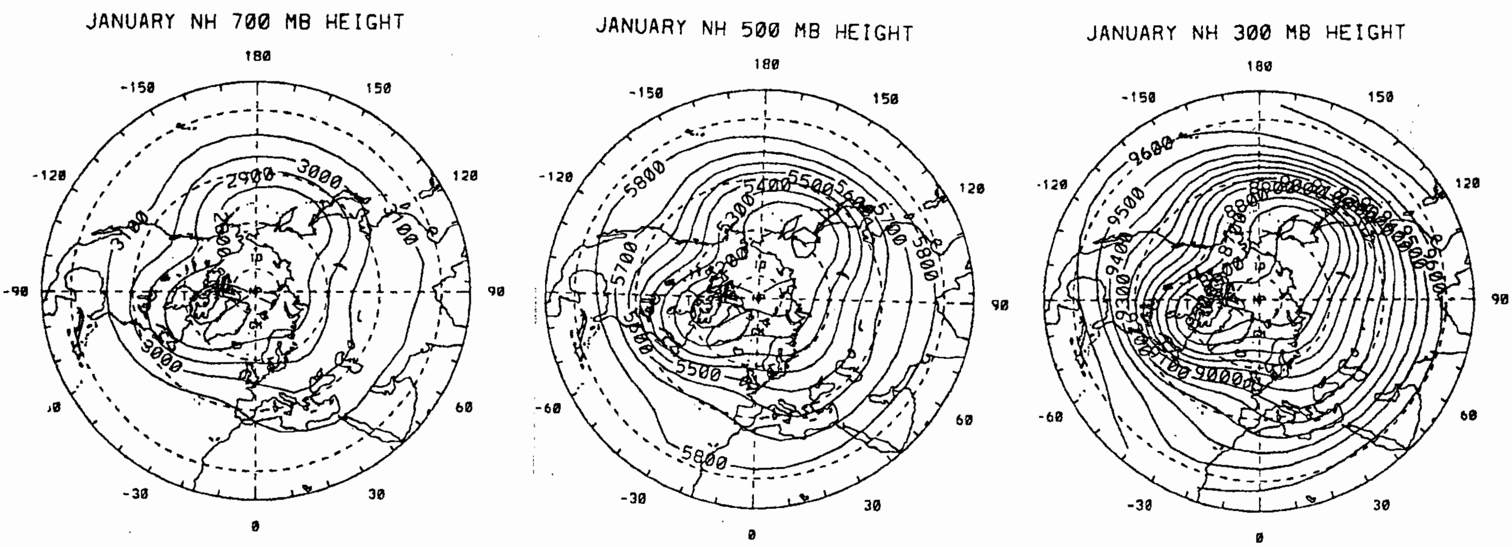

Figure 6.3: Long-term January mean heights, 300 - 700hPa.

hemisphere geopotential height in the lower (700hPa), middle (500hPa), and upper (300hPa) troposphere, averaged over 12 Januarys. The averaging has suppressed any signal from the mobile, synoptic scale storm systems, as well as from mobile planetary waves. What remains is the stationary component. As can be seen from the figure, this component is substantial. Note:

1. The time-averaged flow (along the height contours) departs significantly

from zonality;

2. in some regions, the mean flow departs greatly from being eastward, e.g. near the east coast of N America, where storm systems will tend to be steered by the mean wind to move up the coast, and to the west of N America and Europe, where a southwesterly fetch in the prevailing winds is an ameliorating influence on wither climate;

3. the wave phase is stationary, despite the mean almost-westerly flow: why are the waves not "blown away" by the wind?

4. these waves are vertically coherent, illustrating the Taylor-Proudmann effect, and giving us some hope that a barotropic analysis will be adequate to reveal the underlying dynamics.

6.2 Theory of Rossby waves

6.2.1 The β-plane

We saw in the derivation of the barotropic vorticity equation the potential importance of the fact that the Coriolis parameter varies with latitude, a consequence of spherical geometry. However, dealing with spherical geometry is (a little) more complicated than with planar geometry, so it is common to represent a strip of the sphere—limited in latitude but going all the way around the world in longitude—as a plane, as in Fig. 6.4. We consider a strip centered on longitude ϕ_0, and define a y coordinate $y = a(\phi - \phi_0)$, and an x coordinate $x = a\lambda$, where λ is longitude. Since $f = f(\phi) = 2\Omega \sin\phi$, in the (x, y) system it becomes $f = f(y)$. Assuming that the width of the strip is small enough, we can approximate $f(\phi)$ as a Taylor series about the central latitude:

$$f(\phi) \simeq f(\phi_0) + (\phi - \phi_0)\left(\frac{df}{d\phi}\right)(\phi_0) + \ldots$$

where

$$\begin{aligned} f(\phi_0) &= 2\Omega \sin\phi_0 \; ; \\ df.d\phi_0 &= 2\Omega \cos\phi_0 \; . \end{aligned}$$

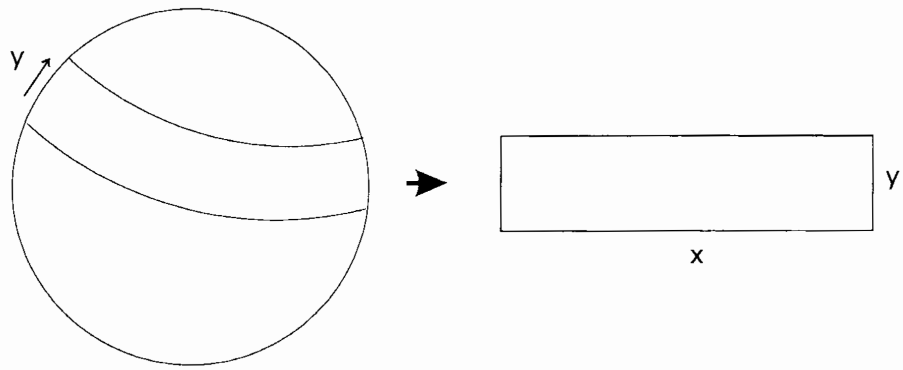

Figure 6.4: The β-plane.

Substituting for y, we get

$$f(y) = f_0 + \beta y \ , \tag{6.1}$$

where $f_0 = 2\Omega \sin\phi_0$ and

$$\beta = \frac{2\Omega}{a} \cos\phi_0 \ .$$

Note that, for a latitude of $\pi/4$, $\beta = 1.617 \times 10^{-11}\mathrm{m}^{-1}\mathrm{s}^{-1}$. Note that, though the sign of f changes from N to S hemisphere, β is always positive (since f always increases northward).

6.2.2 Small amplitude barotropic waves on a motionless basic state

Neglecting viscous effects, the barotropic vorticity equation (??) becomes

$$\frac{d\zeta_a}{dt} = \frac{\partial\zeta}{\partial t} + \mathbf{u}\cdot\nabla\zeta_a = 0 \ ;$$

absolute vorticity is conserved following the flow. Suppose now the motions of interest are small amplitude disturbances to a motionless basic state on a

β-plane, for which $\zeta_a = f$ (relative motion is zero, hence relative vorticity is zero) where f is given by (6.1). Since there is no perturbation to f, we have

$$\begin{aligned} (u, v) &= (u', v') , \\ \zeta_a &= f + \zeta' , \end{aligned}$$

where the primes denote the perturbations. Neglecting terms quadratic in the primed quantities, we have (since f is a function of y only)

$$\frac{\partial \zeta'}{\partial t} + \mathbf{u}' \cdot \nabla f = \frac{\partial \zeta'}{\partial t} + \beta v' = 0 .$$

Since $\zeta = \nabla^2 \psi$, $\zeta' = \nabla^2 \psi'$, and, with $v' = \partial \psi' / \partial x$ (from the definition of streamfunction), we can easily get a single equation for ψ':

$$\frac{\partial}{\partial t} \left(\frac{\partial^2 \psi'}{\partial x^2} + \frac{\partial^2 \psi'}{\partial y^2} \right) + \beta \frac{\partial \psi'}{\partial x} = 0 .$$

If we look for solutions of the form

$$\psi' = Re \left[\Psi \exp i \left(kx + ly - \omega t \right) \right] ,$$

we get the dispersion relation for Rossby waves:

$$\omega = - \frac{\beta k}{k^2 + l^2} . \tag{6.2}$$

This function is plotted in Fig. 6.5. [Note that $\omega l / \beta = -x/(x^2 + 1)$, where $x = k/l$.] Note that:

1. $\omega / k = -\beta / \left(k^2 + l^2\right) < 0$: the phase speed is negative. So the phase of Rossby waves (on a motionless state) always propagates *westward*;

2. since ω is a nonlinear function of $\mathbf{k}$, Rossby waves are *dispersive.*

3. From Fig. 6.5 it is clear that $\partial \omega / \partial k > 0$ for $k/l > 1$, and $\partial \omega / \partial k < 0$ for $k/l < 1$: the group velocity of Rossby waves is eastward for zonally short waves, westward for zonally long waves.

4. The magnitude of the group velocity (judge by the slope of Fig. 6.5) is, typically, greater for the westward-propagating long waves than for the eastward-propagating short waves.

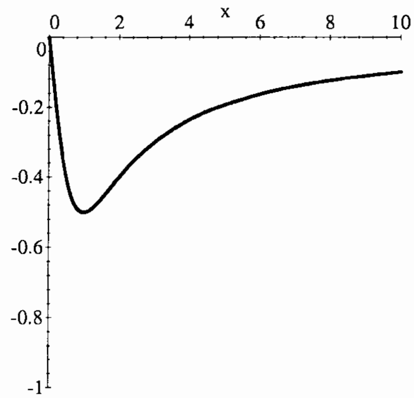

Figure 6.5: The function $-x/(x^2+1)$.

6.2.3 Typical values

At 45 degN,

$$\beta = \frac{2\Omega}{a}\cos 45\,\mathrm{deg} = \frac{2\pi\sqrt{2}}{86400 \times 6.37 \times 10^6} = 1.6145 \times 10^{-11}\mathrm{m}^{-1}\mathrm{s}^{-1}\ .$$

A typical midlatitude disturbance might have a half-wavelength of 5000km in both directions, so

$$k = l \simeq \frac{\pi}{5 \times 10^6} = 6.28 \times 10^{-7}\mathrm{m}^{-1}$$

and then

$$\omega = -\frac{1.6145 \times 10^{-11}}{2 \times 6.28 \times 10^{-7}} = -1.29 \times 10^{-5}\mathrm{s}^{-1}\ .$$

The period is

$$\begin{aligned} \frac{2\pi}{|\omega|} &= \frac{2\pi}{1.29 \times 10^{-5}} = 4.87 \times 10^5\mathrm{s} \\ &\simeq \frac{4.87 \times 10^5}{86400} = 5.6\mathrm{d}\ . \end{aligned}$$

The westward phase speed is

$$c = -\frac{\omega}{k} = \frac{1.29 \times 10^{-5}}{6.28 \times 10^{-7}} = 20.5\mathrm{ms}^{-1}\ .$$

So the typical periods and phase speeds (relative to a stationary atmosphere) for these *planetary scale* Rossby waves are of order (days) and comparable with wind velocities, and so are meteorologically significant.

6.2.4 Mechanism of Rossby wave propagation

From (6.2), it is clear that the propagation of Rossby waves (indeed, the existence of the waves themselves) is dependent on the existence of the planetary vorticity gradient, β. In fact, had we allowed the basic state to have relative vorticity, it would have been the gradient of the mean absolute vorticity, rather than just β, that appeared in (6.2). How does a basic state vorticity gradient lead to waves? Consider Fig. 6.6. We assume that there

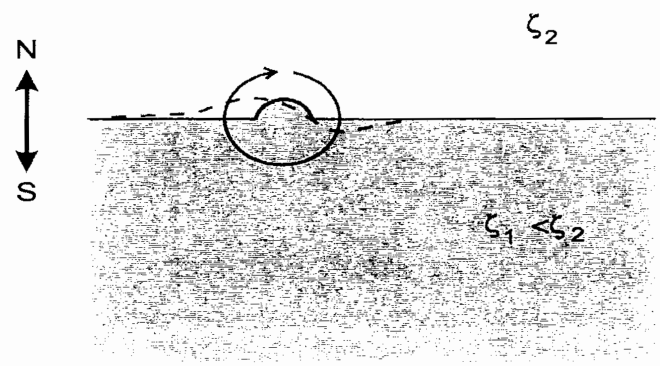

Figure 6.6:

are two regions of uniform vorticity, separated initially by a straight E-W boundary. North of the boundary, the absolute vorticity is ζ_2; to the south, it is ζ_1. Since the Coriolis parameter increases northward, we specify that $\zeta_1 < \zeta_2$. Now let's perturb the interface as shown on the figure, locally and northward. There is now a perturbation in the vorticity field, which is zero everywhere except in the bulge in the interface, where the vorticity perturbation is $\zeta_1 - \zeta_2 < 0$: the anomaly is negative (and therefore clockwise). Just as perturbing an array of electric charges would induce an anomalous electric field, this vorticity perturbation will induce an anomalous circulation. In fact, the streamfunction of the perturbed circulation is ψ' where $\nabla^2\psi' = \zeta'$.

So the problem of determining the circulation is essentially the same as that for the circulation around a point vortex in Section ??: hence the induced circulation will be clockwise, decaying as $1/r$ from the vorticity anomaly, much as depicted schematically in the figure.

Now, because absolute vorticity is conserved following the flow, it is simply advected by the circulation. The effect of the induced circulation on the vorticity distribution will be to advect the interface as shown: northward to the west, southward to the east. As the initial perturbation was northward, the perturbation itself tends to move toward the west—this is the westward phase propagation we noted from (6.2). The spreading, and changing of shape of the perturbation—manifested, amongst other things, by the developing southward perturbation to the east—is a manifestation of the dispersion we also noted.

6.3 Rossby waves in westerly flow

6.3.1 Dispersion relation: stationary waves and dispersion

The planetary scale waves observed in the atmosphere do not always show phase propagation westward, even though they are indeed Rossby waves. Some propagate to the east, some to the west, and as we saw earlier, there is substantial part of the planetary wave field that is stationary. The reason of course is that, unlike the simple preceding theory, the midlatitude atmosphere has mean westerly flow. In uniform flow, the preceding results for phase and group velocity should be interpreted as applying *relative to the background flow*, so the short waves (slow phase velocity relative to the flow) actually propagate to the east; only for sufficiently long waves is the westward Rossby wave propagation strong enough to overcome advection.

In a uniform background eastward flow U, the dispersion relation becomes[2]

$$\omega = Uk - \frac{\beta k}{k^2 + l^2} \ . \tag{6.3}$$

Just as for the "rock in the river" problem, it is possible to have *stationary*

[2]Relative to the moving flow, the phase velocity, from (6.2), is $\tilde{c} = -\beta / \left(k^2 + l^2\right)$; so relative to the ground, $c = \tilde{c} + U$, whence $\omega = ck = (\tilde{c} + U)\, k = Uk - \beta k / \left(k^2 + l^2\right)$.

waves for which the frequency is zero, provided $U > 0$. This happens when

$$k^2 + l^2 = \kappa_s^2 \,, \tag{6.4}$$

where $\kappa_s = \sqrt{\beta/U}$ is known as the *stationary wavenumber.* For typical midlatitude values $U = 30\text{ms}^{-1}$, $\beta = 1.5 \times 10^{-11}\text{m}^{-1}\text{s}^{-1}$, $\kappa_s^{-1} \simeq 1400\text{km}$, so such waves have typical wavelength $2\pi/\kappa_s \simeq 9000\text{km}$, which at 45° latitude corresponds approximately to zonal wavenumber 3. From (6.3), the zonal component of group velocity is

$$c_{gx} = U + \beta \frac{(k^2 - l^2)}{(k^2 + l^2)^2} \,;$$

given (6.4) and some manipulation, it follows that, for stationary waves with $k^2 + l^2 = \kappa_s^2 = \beta/U$,

$$c_{gx}(\omega = 0) = 2k^2 \frac{U}{\beta} \;:$$

the zonal group velocity is eastward.

6.3.2 Forced stationary waves

We are now equipped to understand a simple representation of atmospheric stationary waves. The fact that these waves have a rather special value of phase velocity—zero—tells us that there is something special about forcing them: the forcing itself must be stationary. In fact, there are many ways such waves could be forced: by flow over very large-scale mountain ranges (the Himalaya, the Rockies, Antarctica, primarily), by geographically fixed regions of heating (which affect vorticity by ways we will discuss later), and by other, more subtle, means. The details of the waves produced by localized, stationary forcing depend on the nature of the forcing; however, in light of the above, there are some general things we can say, specifically:

1. The stationary wave will be located to the east of the forcing (since the group velocity has an eastward component), and

2. the length scale of the response will be determined by the inverse of stationary wavenumber.

These features are apparent in explicit solutions such as illustrated in Fig. 6.7.

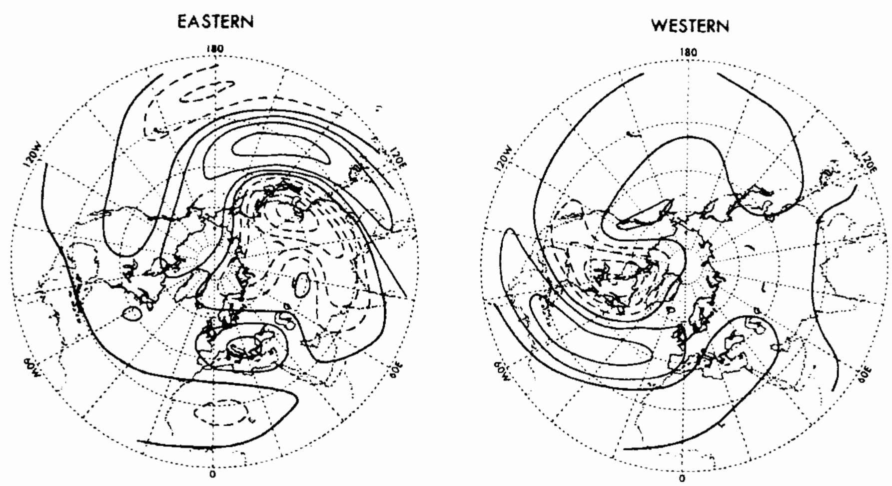

Figure 6.7: Flow over a localized mountain. Numerical solutions for the perturbation streamfunction ψ' for flow over (left) mountains in the eastern hemisphere (Tibet, mostly, with a small contribution from the Alps) and (right) the western hemisphere (mostly the Rockies). ote the Rossby waves propagating "downstream" (eastward) of the mountains.

6.3.3 Vertical structure

The theory developed thus far has been based on the assumption that the flow is barotropic. In reality, there are density variations in the atmosphere, which allow the existence of *baroclinic* (*i.e.*, non-barotropic) motions. The vertical structure of the Rossby wave train produced by a localized mountain is shown in Fig. 6.8. In this figure, we can see two components of

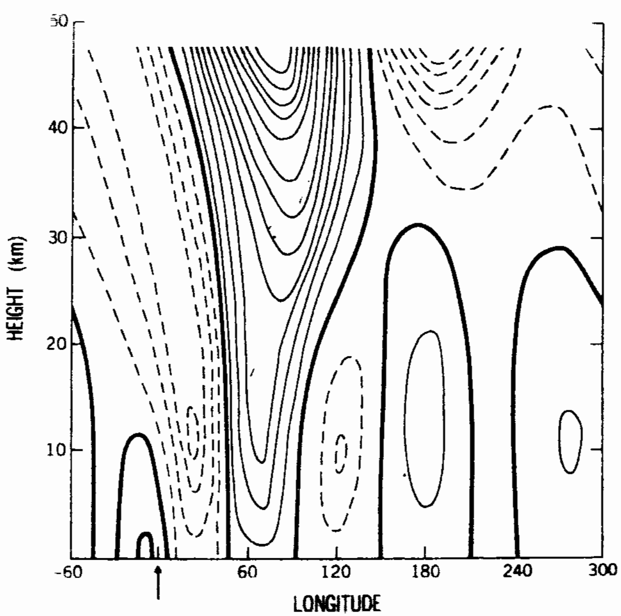

Figure 6.8: Perturbation streamfunction as a function of longitude and height for a 3D calculation of the response to flow over an isolated mountain (location marked by an arrow).

the response: a *surface wave*, which is trapped near the surface, just like an ocean surface wave is trapped at the ocean surface; and a *vertically-propagating component*. The former behaves very much like the barotropic waves we have been discussing. The latter is a wave that propagates in all three directions, including upward [cf. internal gravity waves, Chapter 3]. (In fact, the winds must be westerly aloft for vertical propagation, which restricts this behavior to the winter half-year.) Waves that propagate to great heights reach large amplitude: because the atmospheric density decreases with height, an upward propagating wave becomes "focussed" into less and less mass the higher it goes, and thus must increase its velocity perturbations to compensate. These planetary Rossby waves dominate the meteorology of the winter stratosphere.

6.4 Rossby waves in the ocean

The ocean supports Rossby waves, just as the atmosphere does, obeying the same dispersion relation, and for the same reasons. (In fact, for barotropic motions, the theory does not discriminate between atmosphere and ocean.) The coastal boundaries of the ocean prevent a sustained east-west circulation (except in the Southern Ocean) and sustained east-west propagation of the waves themselves, so in practice there are many differences. The presence of coasts means that ocean basins can support trapped modes, for one thing. However, much of the large-scale variability of the ocean can be described as Rossby waves, albeit in a less organized way than for the atmosphere. However, there is one central aspect of ocean dynamics that may not appear to involve Rossby waves, but in fact does: the existence of western boundary currents.

6.4.1 Western intensification

It is evident from (6.2) and the ensuing discussion that Rossby wave behavior is zonally asymmetric. In particular, we saw that the group propagation of long Rossby waves is fast and westward, while that of short waves is slow and eastward. As illustrated in Fig. 6.9, this has dramatic consequences for ocean dynamics. Any large-scale disturbance in mid-ocean will generate Rossby waves; the larger scale of these will propagate rapidly westward. Before long, they will reach the western boundary of the ocean where they will be reflected. Unlike gravity (or light) waves, the reflected waves will not simply be a mirror image of the incident waves: the reflected waves must have an eastward component of group velocity and so must be of short zonal wavelength. Moreover, they will propagate relatively slowly, more so than the incoming waves. Thus, there will be a kind of "traffic jam" at the western boundary—information can get in more readily that it gets out. The information that accumulates there will involve motions of small zonal scale.

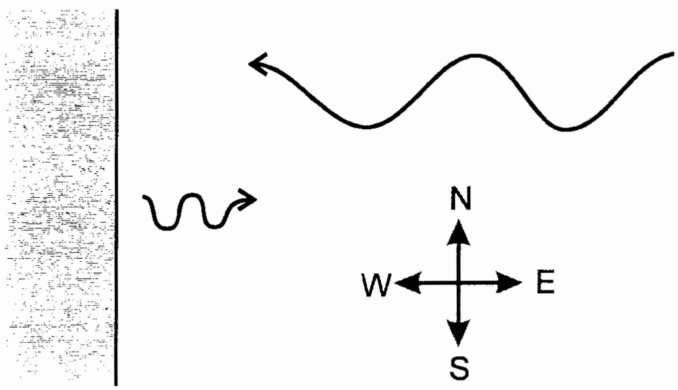

Figure 6.9:

This is the underlying dynamical reason for the existence of strong boundary currents on the western, rather than eastern, sides of the ocean. The underlying reason for the east-west asymmetry is β, the northward gradient of planetary vorticity.

6.5 Vorticity and potential vorticity in a fluid of varying depth

Now consider pseudo-barotropic motion in a fluid of varying depth. By "pseudo"-barotropic we mean that the horizontal flow is independent of the vertical coordinate (thus satisfying the Taylor-Proudmann theorem) but, because of depth variations, cannot be exactly nondivergent. So the system we will consider is an inviscid shallow water system, with a base that is not necessarily flat, as shown in Fig. 6.10. The system is assumed to be rotating with *uniform* Coriolis parameter f.

Our rotating shallow water equations are

$$\begin{aligned} \frac{du}{dt} - fv &= -g\frac{\partial h}{\partial x} \\ \frac{dv}{dt} + fu &= -g\frac{\partial h}{\partial y} \\ \frac{dH}{dt} &= -H\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right) . \end{aligned} \tag{6.5}$$

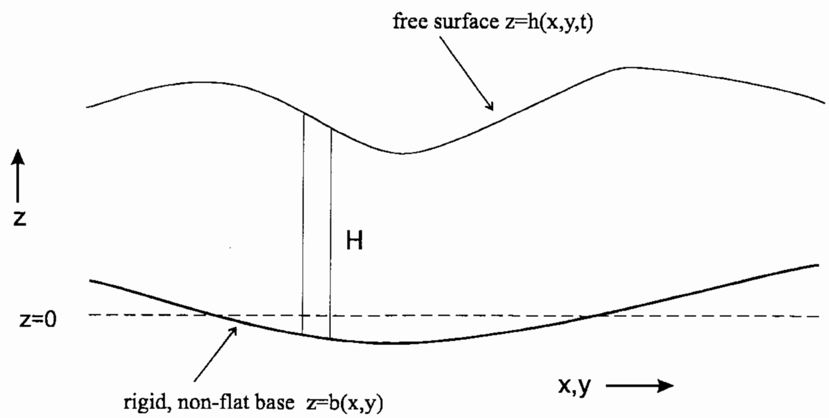

Figure 6.10: Shallow water model with varying depth.

Here, $H(x, y, t) = h - b$ is the *total* depth, where $h(x, y, t)$ is the height of the free surface and $b(x, y)$ the height of the bottom boundary. Note that the continuity equation involves H rather than h, because the total mass convergence into the column is $\rho H \nabla \cdot u$, and the rate of change of column mass (following the flow) is dH/dt, rather than dh/dt.

Now, let's form our vorticity equation in the usual way, by taking $\partial/\partial x$ of the 2nd eq. $-\partial/\partial y$ of the 1st. As before,

$$\begin{aligned}
\frac{\partial}{\partial x}\left(\frac{dv}{dt}\right) - \frac{\partial}{\partial y}\left(\frac{du}{dt}\right) &= \frac{\partial}{\partial x}\left(\frac{\partial v}{\partial t}\right) - \frac{\partial}{\partial y}\left(\frac{\partial u}{\partial t}\right) \\
&\quad + \frac{\partial}{\partial x}\left(u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y}\right) - \frac{\partial}{\partial y}\left(u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right) \\
&= \frac{d}{dt}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right) + \frac{\partial u}{\partial x}\frac{\partial v}{\partial x} + \frac{\partial v}{\partial x}\frac{\partial v}{\partial y} - \frac{\partial u}{\partial y}\frac{\partial u}{\partial x} - \frac{\partial v}{\partial y}\frac{\partial u}{\partial y} \\
&= \frac{d}{dt}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right) + \left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right)\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right) \\
&= \frac{d\zeta}{dt} + \left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right)\zeta \, .
\end{aligned}$$

Then we get

$$\frac{d\zeta_a}{dt} + \zeta_a\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right) = 0 \, ,$$

where $\zeta_a = f + \zeta$ is absolute vorticity, as before. So absolute vorticity is not conserved in this system: it can change whenever the divergence is nonzero (we'll see why). But using the 3rd equation of (6.5), the divergence is just

$$\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right) = -\frac{1}{H}\frac{dH}{dt} .$$

Substituting,

$$\frac{d\zeta_a}{dt} - \frac{\zeta_a}{H}\frac{dH}{dt} = H\frac{d}{dt}\left(\frac{\zeta_a}{H}\right) = 0 ,$$

and so

$$\frac{d}{dt}\left(\frac{\zeta_a}{H}\right) = 0 . \tag{6.6}$$

What (6.6) tells us is that, although absolute vorticity is not conserved, there is a quantity that is conserved following the flow: this quantity is

$$P = \frac{\zeta_a}{H}$$

and is known as the *potential vorticity.* What it means can be seen in the following. Suppose, as shown in Fig. 6.11, that a cylindrical column, initially with absolute vorticity ζ_a and length H, is stretched along its length. Mass continuity demands that the column must contract laterally as it is stretched; angular momentum conservation then dictates that the fluid must spin faster. Eq. (6.6) tells us that ζ_a increases in proportion to H: this process is known as *vortex stretching.*

6.6 Rossby waves in a fluid of varying depth

Consider now perturbations to a otherwise motionless fluid (so $\zeta = 0$ in the absence of perturbations) contained between sloping surfaces, as in Fig. 6.12. The column depth, $H(y)$ is a linear function of y, and we assume the perturbation velocities to be small, so that we can linearize. The potential vorticity equation (6.6) is

$$\left(\frac{\partial}{\partial t} + u\frac{\partial}{\partial x} + v\frac{\partial}{\partial y}\right)\left(\frac{\zeta_a}{H}\right) = 0 ,$$

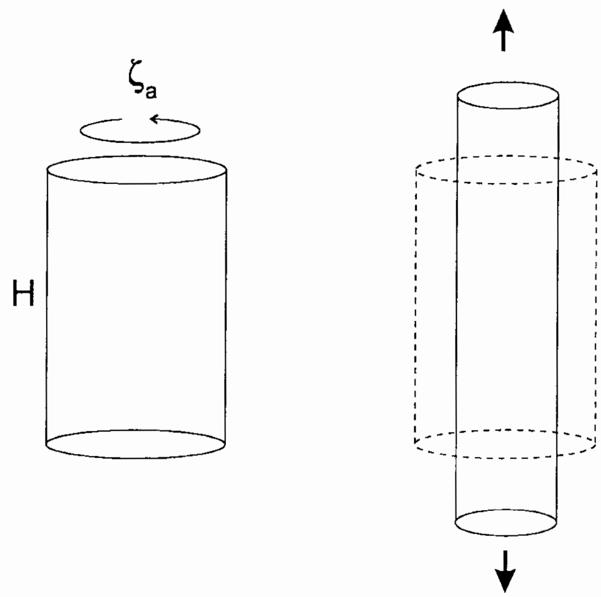

Figure 6.11: Illustrating vortex stretching.

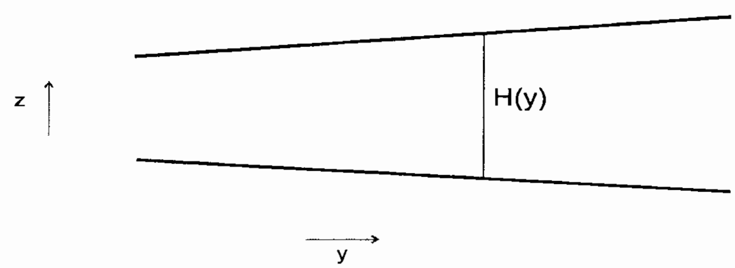

Figure 6.12:

whence

$$\frac{1}{H}\left(\frac{\partial}{\partial t}+u\frac{\partial}{\partial x}+v\frac{\partial}{\partial y}\right)\zeta_a-\frac{\zeta_a}{H^2}\left(\frac{\partial}{\partial t}+u\frac{\partial}{\partial x}+v\frac{\partial}{\partial y}\right)H=0\ .$$

Since $H = H(y)$ and $\zeta_a = f + \zeta'(x, y, t)$, this linearizes to give

$$\frac{\partial \zeta'}{\partial t}+v'\tilde{\beta}=0\ ,$$

where

$$\tilde{\beta}=-\frac{f}{H}\frac{dH}{dy}\ . \tag{6.7}$$

Thus, the vorticity equation becomes precisely equivalent to that in the exactly barotropic case on a β-plane, with in this case $\tilde{\beta}$—a measure of the gradient of fluid depth—replacing the gradient of f. Thus, *e.g.*, a sloping ocean bottom can give rise to Rossby waves, called "topographic Rossby waves", just as can the curvature of the Earth.

In fact, in the case of the Earth's curvature, the two effects are just another way of saying the same thing. Each is illustrated in Fig. 6.13. On the left, we take a traditional view of the atmosphere (or ocean), which is assumed to be contained within a spherical shell of depth D. The "vertical" is defined to be the local upward normal to the surface, and the component of planetary vorticity in this direction is $2\Omega \sin\phi = f$, the Coriolis parameter. Since the thickness of the fluid in the vertical direction is D, the potential vorticity is

$$P=\frac{f}{D}=\frac{2\Omega\sin\phi}{D}\ ,$$

and its gradient is

$$\frac{1}{a}\frac{dP}{d\phi}=\frac{2\Omega}{aD}\cos\phi=\frac{1}{D}\frac{df}{dy}=\frac{\beta}{D}\ .$$

In this view, the depth of the fluid column, D, never changes, so conservation of potential vorticity P implies conservation of absolute vorticity ζ_a. If a fluid column is moved northward to where f is greater, $\zeta_a = f + \zeta$ is conserved by ζ decreasing as f increases—so a northward displacement induces anticyclonic (negative) relative vorticity.

In the second view, we define the direction of the Earth's rotation vector to be the "vertical". The component of planetary vorticity in this direction

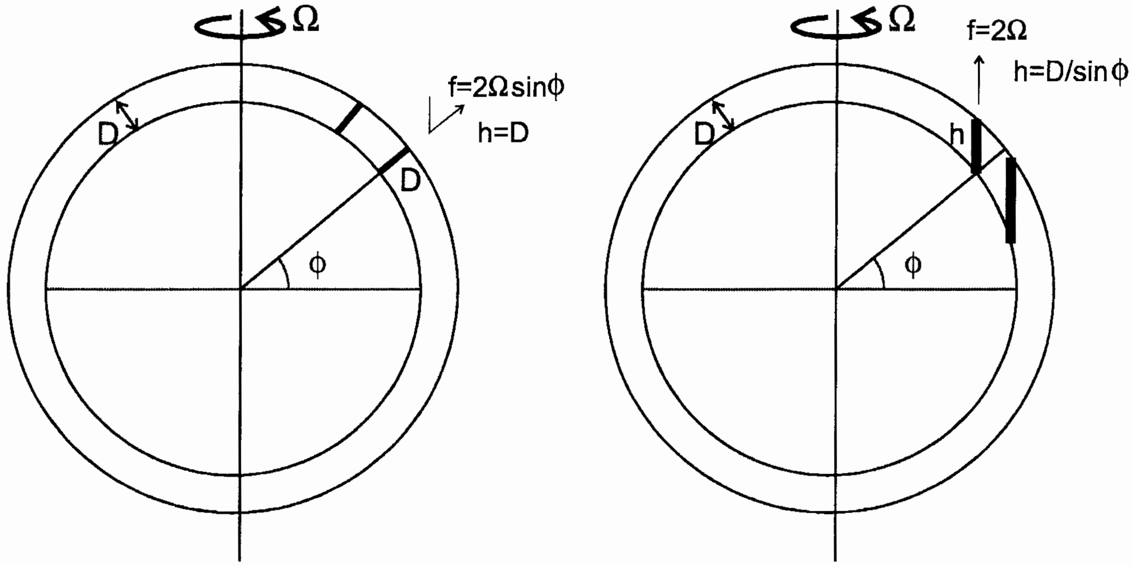

Figure 6.13: Illustrating the equivalence between the two forms of beta in spherical geometry.

is just 2Ω, which is of course constant. But the thickness of the atmospheric shell in this is not constant, but is $h = D/\sin\phi$. So the potential vorticity is

$$P = \frac{\zeta_a}{h} = \frac{2\Omega\sin\phi}{D},$$

just the same! And its gradient is

$$\frac{1}{a}\frac{dP}{d\phi} = \frac{2\Omega}{a}\frac{d}{d\phi}\left(\frac{1}{h}\right) = \frac{2\Omega}{a}\frac{d}{d\phi}\left(\frac{\sin\phi}{D}\right) = \frac{\beta}{D}.$$

So the PV gradient is (of course) exactly the same as in the first case, but we see it differently. In this viewpoint, the planetary vorticity is everywhere 2Ω, but as fluid columns move north or south, their length changes. A northward displacement produces a contraction of the column: in response (in order to conserve P) the absolute vorticity $2\Omega + \zeta$ must decrease, so ζ must become anticyclonic (negative).

6.7 GFD experiment: topographic Rossby waves in the lee of a ridge

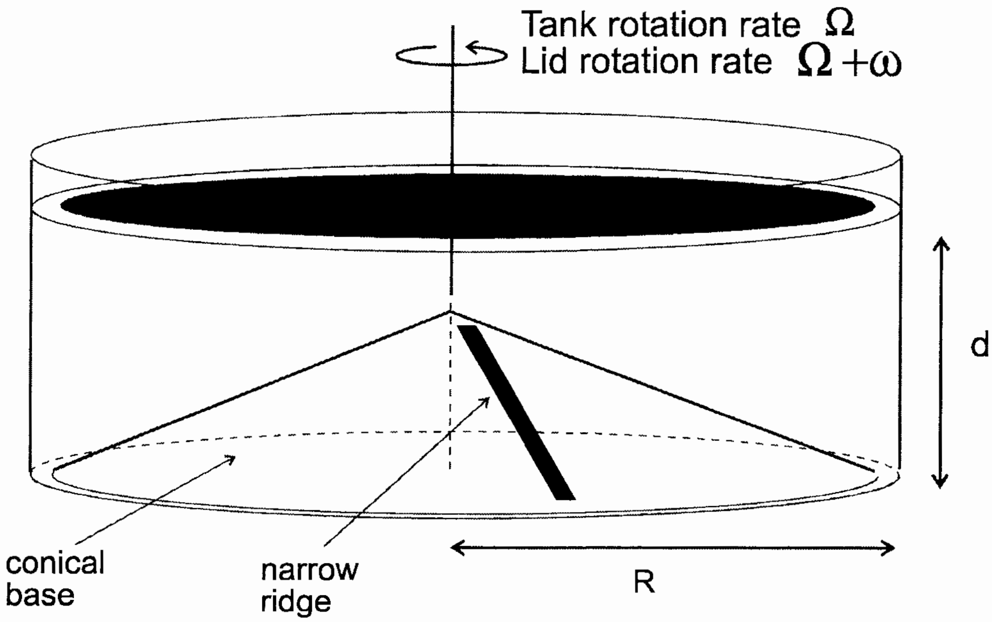

Figure 6.14: Schematic of the tank experiment.

Fig 6.14 show the set-up. A cylindrical tank, on a turntable rotating at rate Ω, is fitted with a conical base; since the deepest water is at the outer rim, that corresponds to the equator. The effective β in this setup is

$$\tilde{\beta} = \frac{2\Omega}{H}\frac{dH}{dr} = \frac{2\Omega}{R}\frac{\delta H}{H} .$$

A lid rotates cyclonically relative to the tank at rate ω. This drives flow (of angular velocity $\sim \omega/2$) in the tank, over a small, straight ridge on the conical base. We expect this to produce a train of stationary Rossby waves of total wavenumber

$$\kappa \simeq \sqrt{\frac{\tilde{\beta}}{U}}$$

where $U = (R/2)(\omega/2)$ is the flow at radius $R/2$. So we expect the magnitude of the wavelength to be

$$\frac{2\pi}{\kappa} = 2\pi\sqrt{\frac{U}{\tilde{\beta}}} = \pi\sqrt{\frac{R\omega R}{2\Omega}\left(\frac{H}{\delta H}\right)} = \pi R \times \sqrt{\left(\frac{\omega}{2\Omega}\right) / \left(\frac{\delta H}{H}\right)} \, .$$

We will have $\omega \approx 0.1\Omega$, and $\delta H \approx H/2$, so we expect

$$\frac{2\pi}{\kappa} \approx \pi R \times 0.3 \, .$$

Since, at mid-channel, a wave of zonal wavenumber one has wavelength πR, this will give us something like zonal wavenumber 3. (We will update these numbers when we do the experiment.)

6.8 Further reading

Observational and theoretical aspects of Rossby waves are covered in several geophysical fluid dynamics texts, including

"An Introduction to Dynamic Meteorology", J.R. Holton, Academic Press, 1979 (2nd edition).