Laboratory I: Image Analysis

Due: September 28, 2005

Exercise I

The goal of this exercise is to become familiar with point counting and measuring mean intercept lengths for a two-phase sample. Using the attached image of hot pressed calcite with melt and the transparency, you will first conduct a point count analysis of the calcite grains and then measure the mean intercept length for the calcite grains following the procedures below.

Point Counting

Position the grid over the calcite and melt image, and count number of points that fall on the calcite grains. Points that lie on a boundary should be counted as 1/2 (Hilliard, 1968). Next, reposition the grid and the count the number of points falling on the calcite phase again. For error analysis, it is necessary to keep the tallies from each position separate. Repeat this process approximately 15 times. Sum the number of points counted for calcite grains P_c, and then divide this total by the total number of grid points P_T to give the ratio P_c/P_T, generally denoted by simply P_P.

Error Analysis

After repositioning the grid 15 times, calculate the variance of your measurements. Variance σ^2 is the standard deviation squared, and provides some idea of the heterogeneity of the microstructure. It is defined as

$$\sigma^2(x) = \frac{1}{n-1}\sum_1^n \left(x_i - \overline{x}\right)^2 \tag{1}$$

for n independent measurements. In (1) n represents the total number of positions of the grid, $\overline{x}$ is the mean number of hits, and x_i is the number of hits at grid position i. Find $\overline{x}$, and then determine the variance of your measurements.

Accuracy

For a randomly oriented sample the total number of points you have to count to achieve accuracy with 95% confidence limits is given by DeHoff (1968),

$$P_T = \left[\frac{200}{(\%acc)} \frac{\sigma(P_P)}{P_P} \right]^2.$$ (2)

In (2) $\sigma(P_P)$ represents the square root of the variance calculated from (1), and *%acc* is a measure of the accuracy of you measurements. Calculate the accuracy of your measurements. How many points would you need to count to achieve 5% accuracy? 1% accuracy? How long would it take you to achieve these accuracies? (95% confidence limit equals to +/-5% accuracy).

Mean Intercept Length

Lengths of intersected phases (l_C, l_M for calcite and melt, respectively) are measured along a randomly applied straight line across the microstructure, summed, and then divided by the total length of the line to get the lineal fraction of a certain phase (L_L). To calculate the variance of the linear fraction it is necessary to keep track of the length of each calcite grain and melt phase along the linear transect.

Using the line provided on the transparency, position the line over the sample image and measure the intercept lengths of both the calcite (l_C) and melt phases (l_M). To do this, measure the length of each of the calcite grains and melt phase along the line. Since the mean intercept length technique has units of length, it will be necessary to scale your measurements to the sample using the scale bar at the bottom of the image. Reposition the line and repeat this process for 15 random directions across the image. For error analysis and consistency, keep the entire length of the line within the image boundaries. Calculate the value of the linear fraction by first summing the intercept lengths of the calcite and dividing by total transect length to determine $(L_L)_C$. Repeat for the melt phase to determine $(L_L)_M$. For a two phase system how is $(L_L)_C$ related to $(L_L)_M$?

Error Analysis

To determine the variance of $(L_L)_C$ and $(L_L)_M$, it is first necessary to calculate the standard deviation of the intercept lengths of the calcite $\sigma(l_C)$ and melt phase $\sigma(l_M)$ using (1). The calculation of the variance in the lineal fraction L_L for the calcite and melt phases is then given by

$$\left[\frac{\sigma(L_L)}{L_L} \right]^2 = \left(\frac{1}{N_L} \right)(1 - L_L)^2 \left(\left[\frac{\sigma(l_C)}{\bar{l}_C} \right]^2 + \left[\frac{\sigma(l_M)}{\bar{l}_M} \right]^2 \right),$$ (3)

where L_L is the linear fraction of the phase of interest, and N_L is the total number of intercepts of the phase of interest. How do the variance of for the calcite and melt phase compare?

Exercise II

The purpose of this exercise is twofold: first, to accustom you to using image analysis software; and, second, to familiarize you with statistical methods used in image analysis. Using computer-imaging software (e.g. NIH (freeware available for Macs--http://rsb.info.nih.gov/nih-image/Default.html), ImageJ (Similar to NIH, available for Windows, same website), Scion (freeware available for Windows, http://www.scioncorp.com), you will determine the grain size distribution of the image squaregrains.tif.

ImageJ and Scion will be available on the computer (the new one) at the microscope room. A useful website to familiar yourself with the image analysis software: http://www.nist.gov/lispix/imlab/labs.html

Computer Image Analysis

Load figure squaregrains.tif into the image analysis program of your choice, and calculate the area of the square grains. Be sure to calibrate the image analysis program with the correct scale in order to obtain correct values for the grain sizes. Make a table of grain sizes, and plot a histogram of the number of grains versus grain size. Vary the size of the grain size bins to get a histogram with a maximum number of hits of ~10.

Statistical Analysis

Two important distribution functions in geology are the normal (Gaussian) and log normal distributions. Many distributions of random variables in nature fall into one of these two distributions. For example, distributions in the earth sciences that follow a log normal distribution are,

1) The ϕ-fraction in sedimentology, which is defined from the lognormal distribution.
2) Sedimentary bed thicknesses
3) Permeability and pore size
4) Concentration of trace elements in rocks
5) The magnitude of earthquakes

To begin statistical analysis, first determine the mean $\overline{A}$ and standard deviation $\sigma(A)$ of your area measurements. The probability density function of the normal distribution is

$$P\left(A \mid \overline{A}, \sigma\right) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left[-\left(\frac{A-\overline{A}}{\sigma}\right)^2 \Big/ 2\right], \tag{4}$$

where, the probability of measuring an area A is $P(A|\overline{A},\sigma)$ for a mean area value $\overline{A}$ and standard deviation σ. The probability density function for the lognormal function is written

$$P\left(A \mid \overline{A}, \sigma\right) = \frac{1}{\sigma A \sqrt{2\pi}} \exp\left[-\frac{\left(\ln A - \overline{A}/\sigma\right)^2}{2}\right],$$ (5)

where, like (4) probability of measuring an area A is P(A|$\overline{A}$,σ) for a mean area value $\overline{A}$ and standard deviation σ. Plot (4) and (5) as a function of area and compare the morphology of the curves to histogram generated in the computer image analysis section. Which distribution is more appropriate for the squaregrains.tif? Why? There are many types of probability distribution functions (e.g. Poisson, Raleigh, etc.). Do you think other types of distributions may be better/worse?

Exercise III

Measuring grain sizes and distributions may sound objective and unambiguous; however, once you begin to analyze real rocks, analysis techniques and subjective decisions can yield to a range in estimates for the same feature. In this exercise you will determine the size of grains using three methods, the mean intersect length, the projected area diameter, and Feret's diameter.

Analysis Technique

The software programs mentioned above can quickly calculate mean intercept length, projected area diameter, and Feret's diameter for sample with clear grain boundaries or distinct phases. However, many of the images we deal with in geology, look more like the quatz.tif than squaregrains.tif. Therefore, some type of image processing is necessary before attempting to quantifying grain size statistics. In the software programs this is done though such processes as thresholding and detecting grain edges. These image-processing techniques vary in their success rates. For this exercise, attempt to threshold and detect the edges of quartz. How well does each of these techniques reflect the actual quartz.tif image? Include your best thresholding image with your lab report.

When software techniques such as thresholding, etc., do not work, sometimes it become necessary to trace the image by hand and then scan the traced image into a computer. (One could likewise trace the image in Adobe Photoshop, for example, but generally is it easier and less frustrating to do by hand.) Trace the image, scan it into a computer, and use the image analysis programs described above to calculate the mean intercept length, the projected area diameter and the Feret's diameter.

Mean Intercept Length

Using the technique described in exercise I, determine the mean intercept length of the quartz grains in the horizontal and vertical directions separately. Calculate the standard deviation of your measurements using (1).

Projected Area Diameter

Determine the area of each of the grains. The projected area diameter is equal to

$$d^2 = \frac{4A}{\pi}.$$ (6)

Calculate the standard deviation using (1).

Feret's Diameter

Along the same lines used in the mean intercept method, calculate the Feret's diameter of the grains. The Feret's Diameter is distance between two parallel lines touching opposite sides of the grain. For a vertically oriented transect the Feret's diameter for a grain is shown below
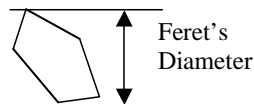
Generally, Feret's diameter is the greatest distance possible between any two points along the boundary of a region of interest. For this measurement, you can use ImageJ, which is on the computer in the microscope room. Determine the standard deviation using (1).

Discussion

Compare the application of each of these methods to quartz.tif. Is there a large difference between the measurements? If so, why? How do you think processing/tracing the image could have affected your measurements?