Strain production and preferred orientation

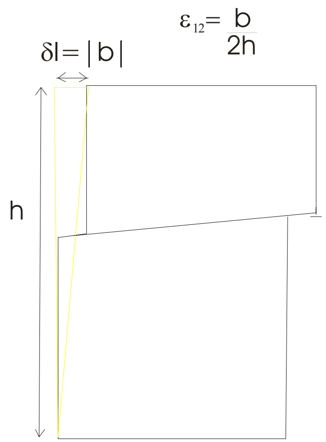
Groves and Kelly,
Crystallography And Crystal Defects,
1970. Chapter 6.

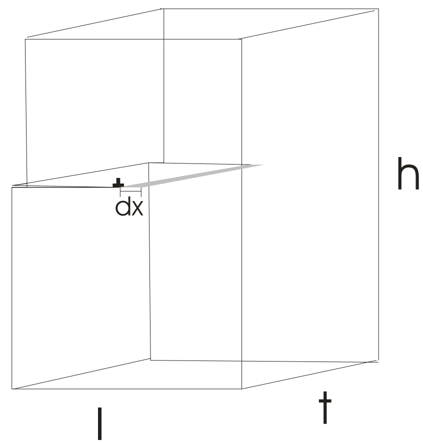
Wenk, H.-R. Chapter 10,

in

Karato and Wenk, *Plastic deformation of minerals and rocks*, *Rev. Mineral. Geochem. Vol. 51*, 2002

Strain during glide





- for n dislocations slipping:

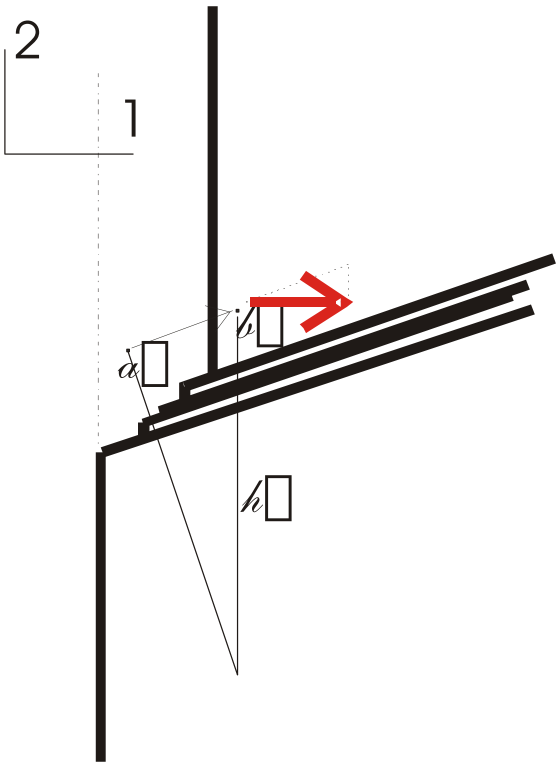
$$\varepsilon_{ij}^{total} = n\frac{b}{2h}$$

- For an increment:

$$d\gamma = \frac{b}{h}\frac{dx}{l}\frac{t}{t}$$

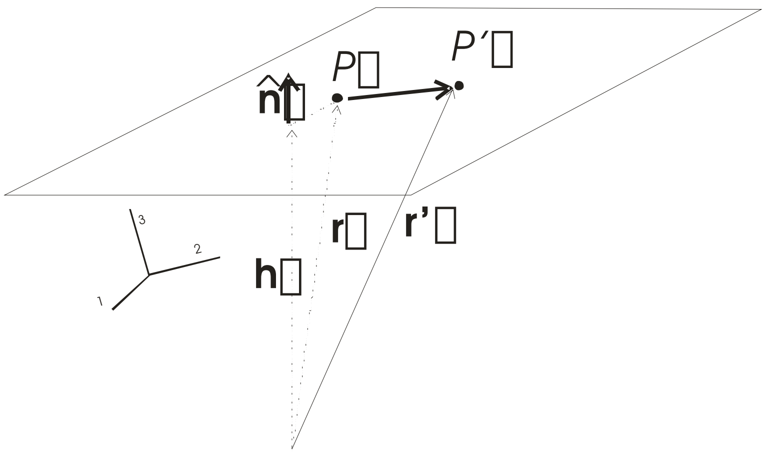
$$d\varepsilon = \frac{bda}{2V}$$

Inclined Slip Plane



- Strain
  - Burgers Vector
  - Normal to glide plane
  - Number of dislocations

Strain elements from glide



$$e_{ij} \triangleq \frac{du_i}{dx_j} \equiv \varepsilon_{ij} + \omega_{ij}$$

$$\left|\mathbf{PP'}\right| = \alpha h \boldsymbol{\beta} = \alpha\left(\mathbf{r}\bullet\mathbf{n}\right)\boldsymbol{\beta}$$

where   is a unit Burgers vector

and $\alpha = \dfrac{s}{h}$

$$e_{11} = \frac{\partial}{\partial x_1}\left(\alpha\left(\mathbf{r}\bullet\mathbf{n}\right)\boldsymbol{\beta}\right) \equiv \alpha\frac{\partial}{\partial x_1}\left(x_k n_k\right)\beta_1 = \boxed{\alpha n_1 \beta_1 = e_{11}}$$

Strain and Rotation

$$\varepsilon_{ij} = \frac{\alpha}{2}\begin{pmatrix} 2n_1\beta_1 & n_1\beta_2 + n_2\beta_1 & n_1\beta_3 + n_3\beta_1 \\ \bullet & 2n_2\beta_2 & n_2\beta_3 + n_3\beta_2 \\ \bullet & \bullet & 2n_3\beta_3 \end{pmatrix}$$

$$\omega_{ij} = \frac{\alpha}{2}\begin{pmatrix} 0_1 & n_2\beta_1 - n_1\beta_2 & n_3\beta_1 - n_1\beta_3 \\ n_1\beta_2 - n_2\beta_1 & 0 & n_3\beta_2 - n_2\beta_3 \\ n_1\beta_3 - n_3\beta_1 & n_2\beta_3 - n_3\beta_2 & 0_3 \end{pmatrix}$$

- No component of β or n in k direction→$\varepsilon_{ik}=0$
- No climb or diffusion

  $$-n_1\beta_1 = n_2\beta_2 + n_3\beta_3$$

- Rotation (and strain) depend on activity (α)

Independent Slip Systems

- Distinct systems can give rise to same strain. (e.g. interchange **n** and β**)**
- If strain element unique, then *independent.*
- No more than two β's on the same plane can be independent.
- Crystallographic symmetry can increase number of strain elements for a particular slip systems.

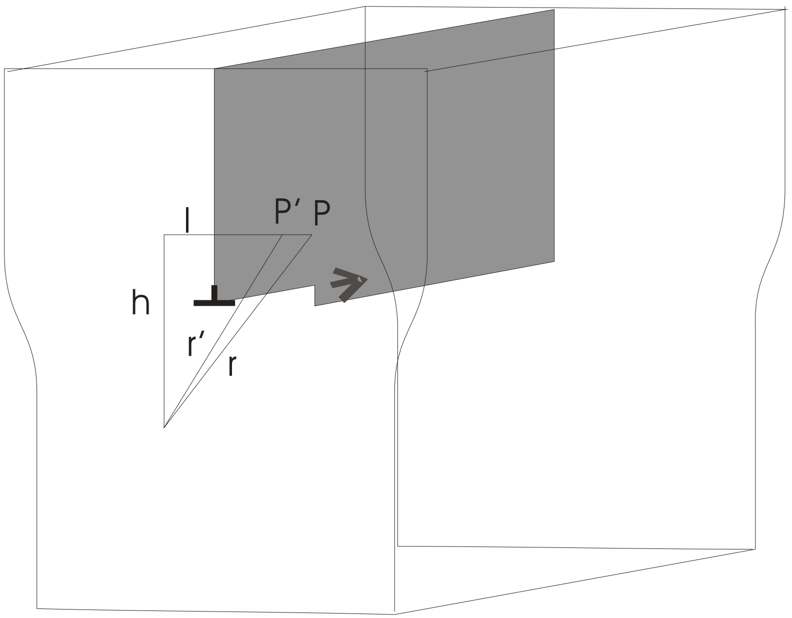
Strain from climb

- For climb only

$$\gamma = s/l \qquad \mathbf{u} = \gamma(\mathbf{r}\cdot\boldsymbol{\beta})\boldsymbol{\beta}$$

$$e_{ij} = \frac{\partial}{\partial x_j}\left(\gamma(\mathbf{r}\cdot\boldsymbol{\beta})\boldsymbol{\beta}\right) \equiv \gamma\frac{\partial}{\partial x_j}(x_k\beta_k)\beta_j = \gamma(\beta_i)\beta_j$$

$$\boxed{\varepsilon_{ij} = \frac{\gamma}{2}\left(\beta_i\beta_j + \beta_j\beta_i\right) = \gamma\beta_i\beta_j}$$



- Strain is irrotational
- Depends only on β not **n.**
- Open system, so 6 ind. s.s.
- Three β's climbing and gliding give 6 systems.

Taylor-von Mises Criterion

- Low T, glide easier than climb. Dilatancy may result.
- For homogeneous, non-dilatant, creep,
  5 independent slip systems must be present.

  von Mises Criterion

- If dilatant, 6 independent slip systems necessary.
- If condition not fulfilled
  - twinning
  - climb or diffusion
  - void production
  - inhomogeneous flow

Independence of Slip Systems

- Convert vectors to Cartesian system. Choose 5 easiest systems.
- If no climb allowed, express strain as 5-dimensional vector. $\left[\dot{\varepsilon}_{11}-\dot{\varepsilon}_{22},\, \dot{\varepsilon}_{33}-\dot{\varepsilon}_{22},\, \dot{\varepsilon}_{12},\, \dot{\varepsilon}_{23},\, \dot{\varepsilon}_{13}\right]$
- Form 5x5 matrix, take determinant

$$\begin{vmatrix} (\varepsilon_{11}-\varepsilon_{33})^{I} & (\varepsilon_{11}-\varepsilon_{33})^{II} & (\varepsilon_{11}-\varepsilon_{33})^{III} & (\varepsilon_{11}-\varepsilon_{33})^{IV} & (\varepsilon_{11}-\varepsilon_{33})^{V} \\ (\varepsilon_{22}-\varepsilon_{33})^{I} & (\varepsilon_{22}-\varepsilon_{33})^{II} & (\varepsilon_{22}-\varepsilon_{33})^{III} & (\varepsilon_{22}-\varepsilon_{33})^{IV} & (\varepsilon_{22}-\varepsilon_{33})^{V} \\ \dot{\varepsilon}_{12}{}^{I} & \dot{\varepsilon}_{12}{}^{II} & \dot{\varepsilon}_{12}{}^{III} & \dot{\varepsilon}_{12}{}^{IV} & \dot{\varepsilon}_{12}{}^{V} \\ \dot{\varepsilon}_{23}{}^{I} & \dot{\varepsilon}_{23}{}^{II} & \dot{\varepsilon}_{23}{}^{III} & \dot{\varepsilon}_{23}{}^{IV} & \dot{\varepsilon}_{23}{}^{V} \\ \dot{\varepsilon}_{13}{}^{I} & \dot{\varepsilon}_{13}{}^{II} & \dot{\varepsilon}_{13}{}^{III} & \dot{\varepsilon}_{13}{}^{IV} & \dot{\varepsilon}_{13}{}^{V} \end{vmatrix} = 0$$

Deformation of Polycrystals

- If 5 independent ss's available, homogenous, non-dilatant flow possible.
- If inhomogeneous flow possible, then 4 ss's sufficient.
- If dilatancy required, flow is pressure dependent.
- With only two ss's, impossible to get pressure independent flow.
  - Basal slip, e.g. mica.

Texture, Fabric, and Preferred Orientation

- Texture: Geometrical aspects of component particles of a rock, including size, shape, and arrangement.
- Fabric: Orientation in space of elements of which rock is composed.
  That factor of the texture which depends on the relative sizes and shapes, and the arrangement of the component crystals.
- Preferred orientation: A rock in which the grains are more or less systematically oriented by shape or [by crystallographic orientation].
  - *Dictionary of geological terms*, Am. Geolog. Inst., Dolphin Books, 1962.

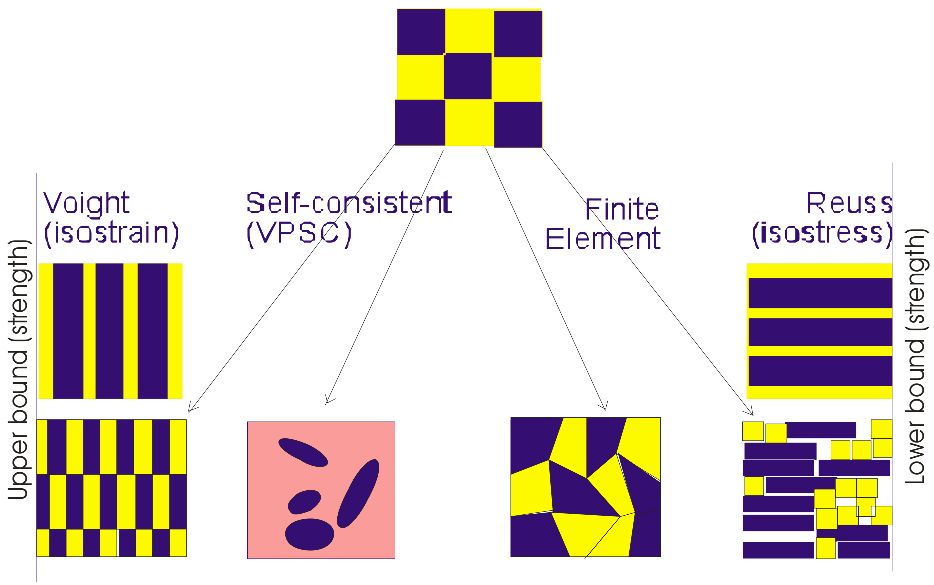
Methods of measuring

- Optical
- X-ray pole figure goniometer
- Synchrotron X-rays
- Neutron diffraction
- TEM
- EBSD (EBSP)
    - Wenk, H-R. in Plastic Deformation of minerals and rocks, Rev. Min. and Geochem. Vol.51, 2002.

Data representation

- Pole figures: Density distribution of a single pole plotted in a stereographic plot relative to the sample coordinates.
- ODF: An orientation probability distribution function of three Euler angles

Simulations



- Taylor- equi-strain
  - ≅Voight elastic bound
  - fcc bcc metals (hi symm.)
  - upper bound in strength
- Equi-stress-Sachs
  - ≅Voight elastic bound
  - lower bound
  - heterogeneous strain
- Self-consistent (VPSC)
- Finite element

Processes

- Constitutive law
- Grain growth/Recrystallization
- Metamorphic reactions
- Dilatancy