

1. Acoustic waves are small adiabatic perturbations of a medium of otherwise uniform density and pressure. Assuming the specific entropy s can be written as $s = s(p, \rho)$, write the governing equations of inviscid motion for disturbances propagating in (x, z) . Find the equation for the perturbation pressure p remembering the definition of adiabatic compressibility $\left(\frac{\partial p}{\partial \rho}\right)_s$.

Require wave-like solutions for p and find the dispersion relation, the phase speed and the group velocity. Draw the contours of constant ω in the (k, m) space. Are the waves dispersive? What significance does this have for communication by speech?

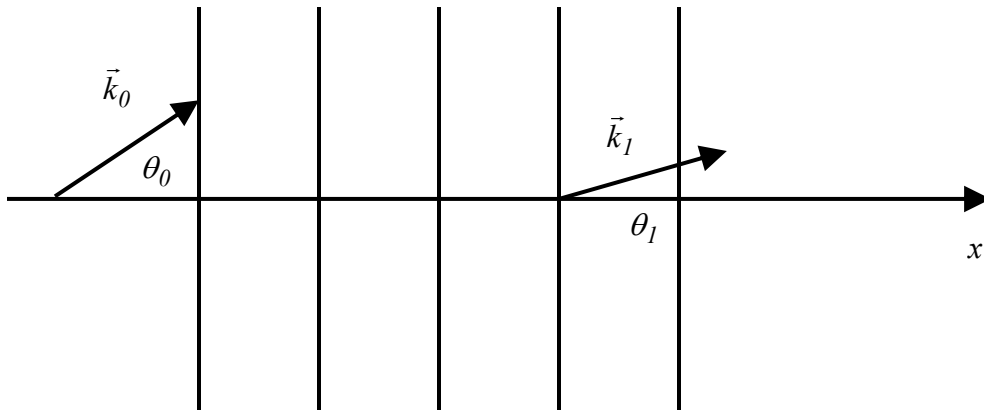
2. A surface gravity wave moves in x – direction in a fluid layer of depth D . At a certain distance L the wave impinges on a current $U(x)$, moving also in x – direction. Therefore, no motion in the y – direction is created even though the dispersion relation is changed.

Consider the two cases in which $U(x) > 0$, and $U(x) < 0$, i.e. the wave impinges on a current moving in its same direction or on a contrary direction.

Find the wavenumbers that allow the wave to ride on the current and discuss how its wavelength is modified for $U \gtrless 0$.

(Hint: Use the graphic method applied in class to the dispersion relationship of different waves.)

3. Consider shallow water surface gravity waves impinging on a topography slowly varying in x :



The lines are lines of constant h . At the initial point the wavenumber \vec{k}_0 and angle of incidence θ_0 are shown. At the final point the wavenumber \vec{k}_1 and angle of incidence θ_1 are also shown.

Find the ratio $\frac{\sin \theta_1}{\sin \theta_0}$ in terms of the phase speeds (c_1, c_0) , i.e. find Snell's Law for the waves.

4. In Homework Set 3, Problem 2, you found the internal wave solution in the unbounded domain using the hydrostatic approximation in the equations of motion. Consider the same equations in the oceanic wave guide with boundaries at the free surface η and constant depth D .

Find the normal modes following the procedure used in class for the non-hydrostatic case and compare the two sets of solutions. What is the major difference between them?

5. The potential vorticity conservation equation in a homogeneous layer of fluid on the β -plane and total thickness $h = \eta + D$ is:

$$\frac{d}{dt} \left(\frac{\zeta + f}{h} \right) = 0 \quad (1)$$

where $\zeta = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}$ is the vertical component of relative vorticity, $f = f_0 + \beta y$ the planetary vorticity, η the free surface and D the flat bottom.

Consider now the homogeneous layer to be bounded again by a free surface $\eta(x, y, t)$ but also by a variable topography $h_b(x, y)$ so that the total depth of the layer is $h = D + \eta - h_b$ where D is the mean depth.

In the QG limit $\varepsilon = \frac{U}{f_0 L} \ll 1$ and $\frac{\eta}{D} = \frac{h_b}{D} = 0(\varepsilon) \ll 1$, derive the $0(\varepsilon)$ vorticity equation.

Using the order zero geostrophic and hydrostatic relationships, express the zero-order geostrophic flow in terms of the free surface η and define the related geostrophic streamfunction. Discuss the physical meaning of the different terms in the vorticity equation.

6. Consider linear waves on the interface between two nonrotating immiscible fluids with densities ρ_1 and ρ_2 . Each layer is infinitely deep and moving with a constant mean velocity $U_n, n = 1, 2$.

- a) Show that the linearized dynamic boundary condition at $z = 0$ can be written as

$$\rho_1 \left(\frac{\partial \phi_1}{\partial t} + U_1 \frac{\partial \phi_1}{\partial x} + g\eta \right) = \rho_2 \left(\frac{\partial \phi_2}{\partial t} + U_2 \frac{\partial \phi_2}{\partial x} + g\eta \right)$$

where $u'_n = \nabla \phi_n$ is the wave induced velocity. The total velocity $u_n = U_n + u'_n$.

- b) Show that the linear kinematic boundary conditions at the interface ($z = 0$) can be written as

$$\phi_{nz} = \eta_t + U_n \eta_x \quad (n = 1, 2)$$

- c) Derive the dispersion relation for waves in x -direction ($\phi \sim e^{i(kx - \omega t)}$). Discuss the behavior of the waves as ρ_n , U_n and k are varied. Can ω be complex? If so, what is the physical meaning?

7. Reconsider the normal mode problem in the nonrotating case with $N^2 = \text{constant} > 0$ and the fluid layer of depth D contained between two rigid surfaces at $z = 0, -D$. Then consider the case where $N^2 < 0$, i.e. when heavy fluid is initially in top of lighter fluid, so $\frac{d\rho_0}{dz} > 0$.

What is the condition for the normal modes to be again sinusoidal functions of z ? What are the frequencies of the normal modes? What is the vertical wavelength corresponding to the maximum frequency?

Discuss the time evolution of the perturbation fields and its physical implications.

8. Find the reflected group velocity \bar{c}_{gr} for internal wave packets impinging on the bottom slopes sketched below, $z = ax$, and discuss the type of reflection.

(n indicates the direction perpendicular to the bottom slope.)

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