Problems with radiative equilibrium solution:

- Too hot at and near surface
- Too cold at a near tropopause
- Lapse rate of temperature too large in the troposphere
- (But stratosphere temperature close to observed)

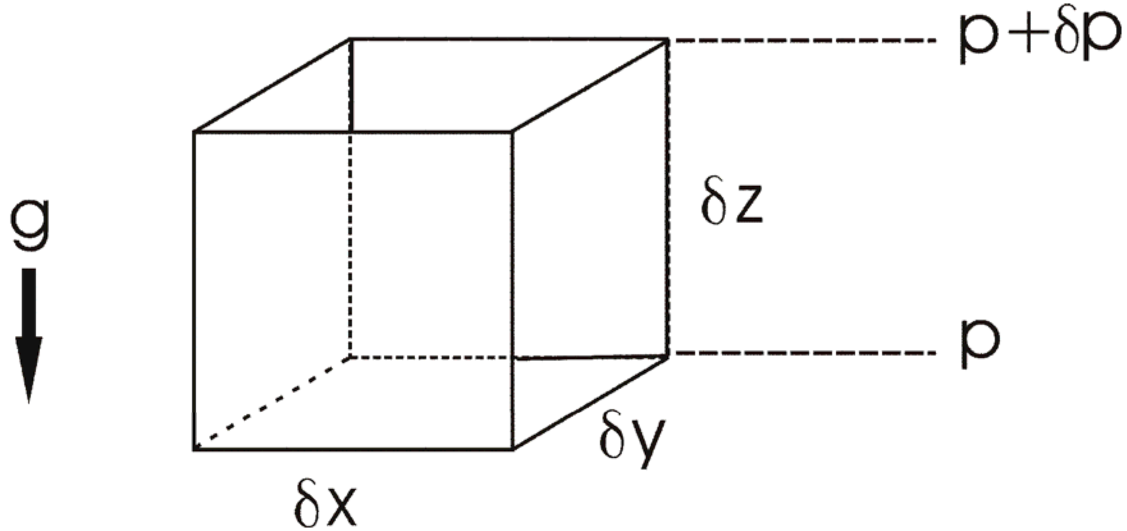
Missing ingredient: Convection

- As important as radiation in transporting enthalpy in the vertical
- Also controls distribution of water vapor and clouds, the two most important constituents in radiative transfer

When is a fluid unstable to convection?

- Pressure and hydrostatic equilibrium
- Buoyancy
- Stability

Hydrostatic equilibrium:



$$Weight: \quad -g\rho\delta x\delta y\delta z$$

$$Pressure: \quad p\delta x\delta y-\left(p+\delta p\right)\delta x\delta y$$

$$F=MA: \quad \rho\delta x\delta y\delta z\frac{dw}{dt}=-g\rho\delta x\delta y\delta z-\delta p\delta x\delta y$$

$$\frac{dw}{dt}=-g-\alpha\frac{\partial p}{\partial z}, \qquad \alpha={}^{1}\!/_{\rho}=specific\ volume$$

Pressure distribution in atmosphere at rest:

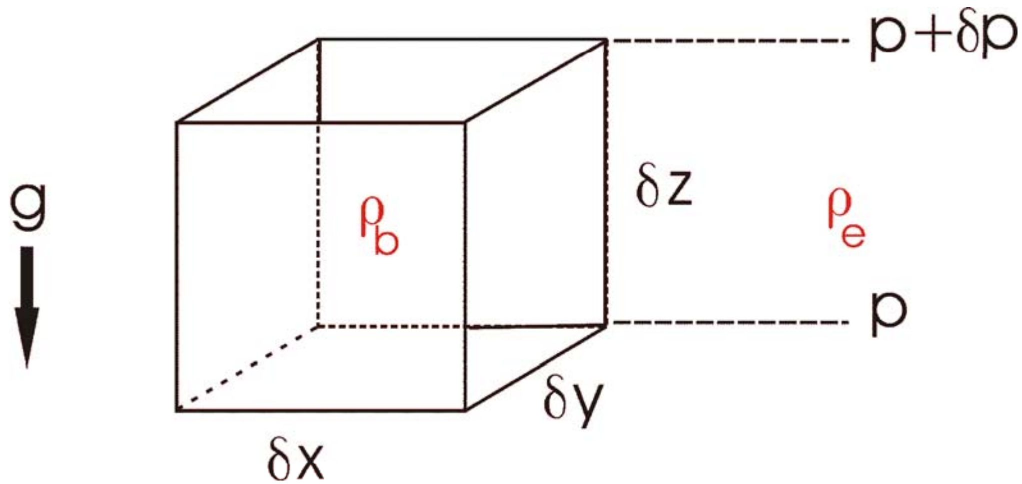
$$Ideal\ gas: \quad \alpha = \frac{RT}{p}, \qquad R \equiv \frac{R^*}{\bar{m}}$$

$$Hydrostatic: \quad \frac{1}{p}\frac{\partial p}{\partial z} = \frac{\partial \ln(p)}{\partial z} = -\frac{g}{RT}$$

$$Isothermal\ case: \quad p = p_0 e^{-z/H}, \qquad H \equiv \frac{RT}{g} = "scale\ height"$$

Earth: H~ 8 Km

Buoyancy:



$$Weight: \quad -g\rho_b \delta x \delta y \delta z$$

$$Pressure: \quad p\delta x \delta y - (p + \delta p)\delta x \delta y$$

$$F = MA: \quad \rho_b \delta x \delta y \delta z \frac{dw}{dt} = -g\rho_b \delta x \delta y \delta z - \delta p \delta x \delta y$$

$$\frac{dw}{dt} = -g - \alpha_b \frac{\partial p}{\partial z} \qquad but \quad \frac{\partial p}{\partial z} = -g / \alpha_e$$

$$\rightarrow \quad \frac{dw}{dt} = g\frac{\alpha_b - \alpha_e}{\alpha_e} \equiv B$$

Buoyancy and Entropy

Specific Volume: $\alpha = {}^{1}\!/_{\rho}$

Specific Entropy: s

$$\alpha = \alpha(p, s) \qquad Maxwell: \left(\frac{\partial \alpha}{\partial s}\right)_p = \left(\frac{\partial T}{\partial p}\right)_s$$

$$\left(\delta \alpha\right)_p = \left(\frac{\partial \alpha}{\partial s}\right)_p \delta s = \left(\frac{\partial T}{\partial p}\right)_s \delta s$$

$$B = g\frac{\left(\delta \alpha\right)_p}{\alpha} = \frac{g}{\alpha}\left(\frac{\partial T}{\partial p}\right)_s \delta s = -\left(\frac{\partial T}{\partial z}\right)_s \delta s \equiv \Gamma \delta s$$

The adiabatic lapse rate:

$$First\ Law\ of\ Thermodynamics:$$

$$\dot{Q} = T\frac{ds_{rev}}{dt} = c_v\frac{dT}{dt} + p\frac{d\alpha}{dt}$$

$$= c_v\frac{dT}{dt} + \frac{d\left(\alpha p\right)}{dt} - \alpha\frac{dp}{dt}$$

$$= \left(c_v + R\right)\frac{dT}{dt} - \alpha\frac{dp}{dt}$$

$$= c_p\frac{dT}{dt} - \alpha\frac{dp}{dt}$$

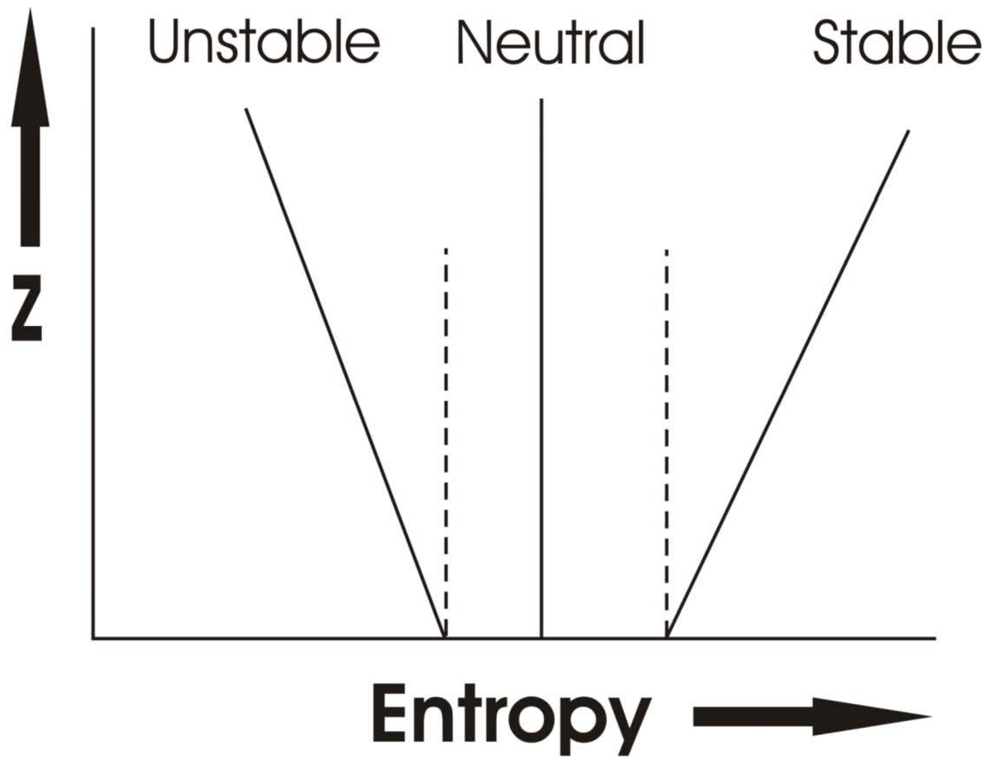
$$Adiabatic: \quad c_p dT - \alpha dp = 0$$

$$Hydrostatic: \quad c_p dT + g dz = 0$$

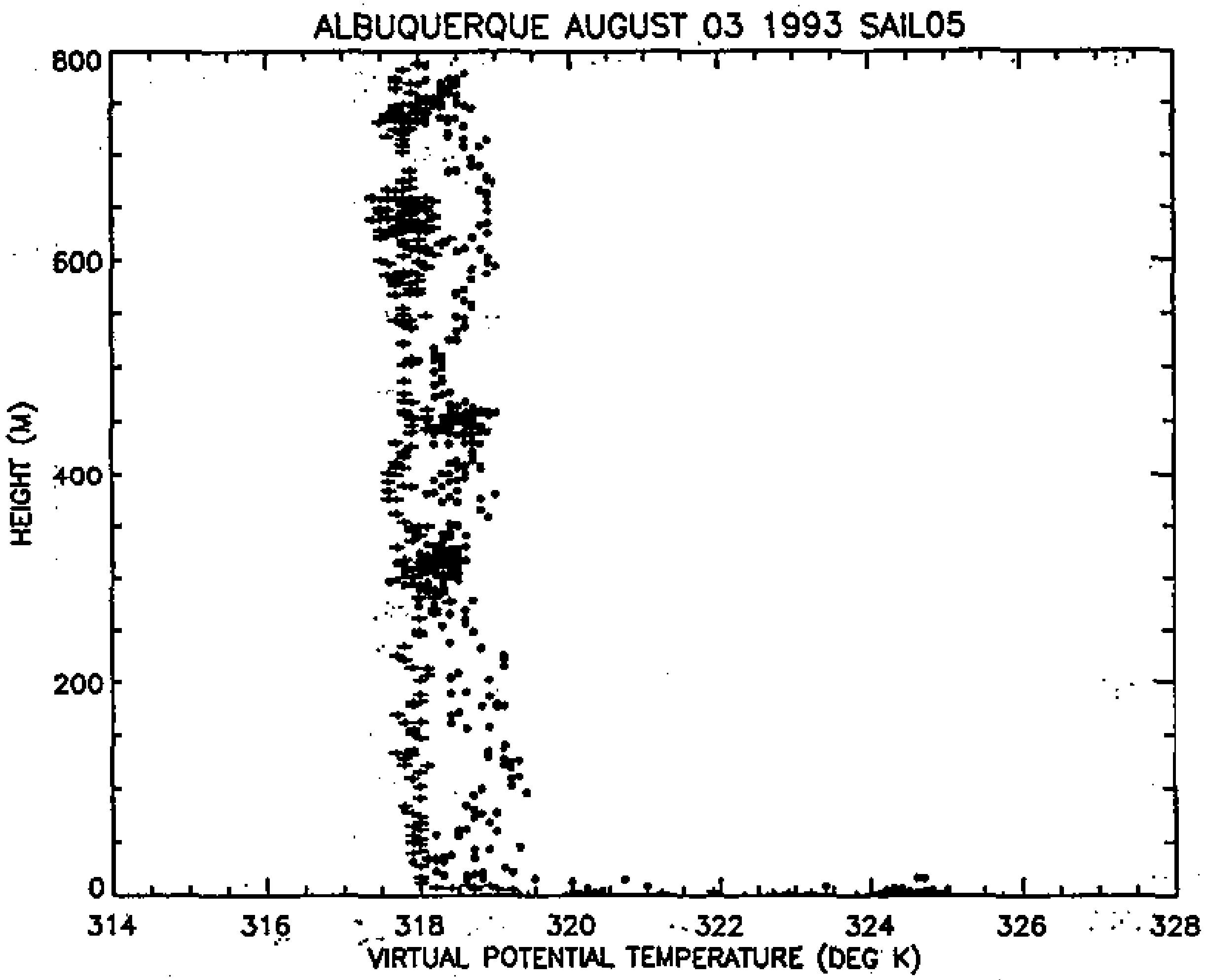
$$\rightarrow \quad \left({dT}/{dz}\right)_s = -{g}/{c_p} \equiv -\Gamma_d$$

$$\Gamma = g/c_p$$

Earth's atmosphere: $\Gamma = 1\,K/100\,m$



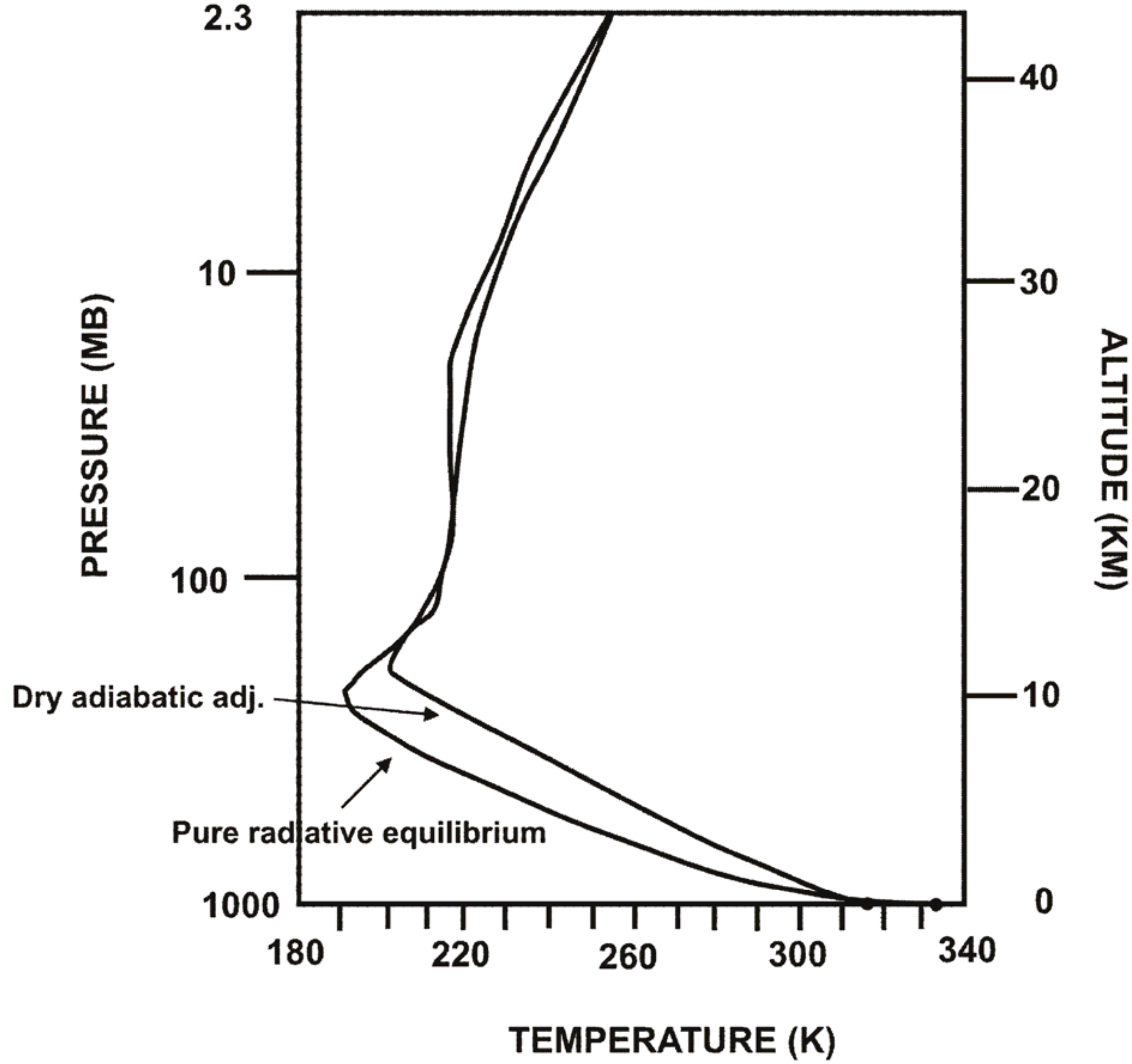
Model Aircraft Measurements (Renno and Williams, 1995)



Radiative equilibrium is unstable in the troposphere

Re-calculate equilibrium assuming that tropospheric stability is rendered neutral by convection:

Radiative-Convective Equilibrium



Better, but still too hot at surface, too cold at tropopause

Above a thin boundary layer, most atmospheric convection involves phase change of water:

Moist Convection

Moist Convection

- Significant heating owing to phase changes of water
- Redistribution of water vapor – most important greenhouse gas
- Significant contributor to stratiform cloudiness – albedo and longwave trapping

Water Variables

Mass concentration of water vapor (*specific humidity*):

$$q \equiv M_{H_2O} / M_{air}$$

Vapor pressure (partial pressure of water vapor): *e*

Saturation vapor pressure: *e**

C-C: $$e* = 6.112\, hPa\, e^{\frac{17.67(T-273)}{T-30}}$$

Relative Humidity: $$\mathcal{H} \equiv e / e*$$

The Saturation Specific Humidity

Ideal Gas Law:

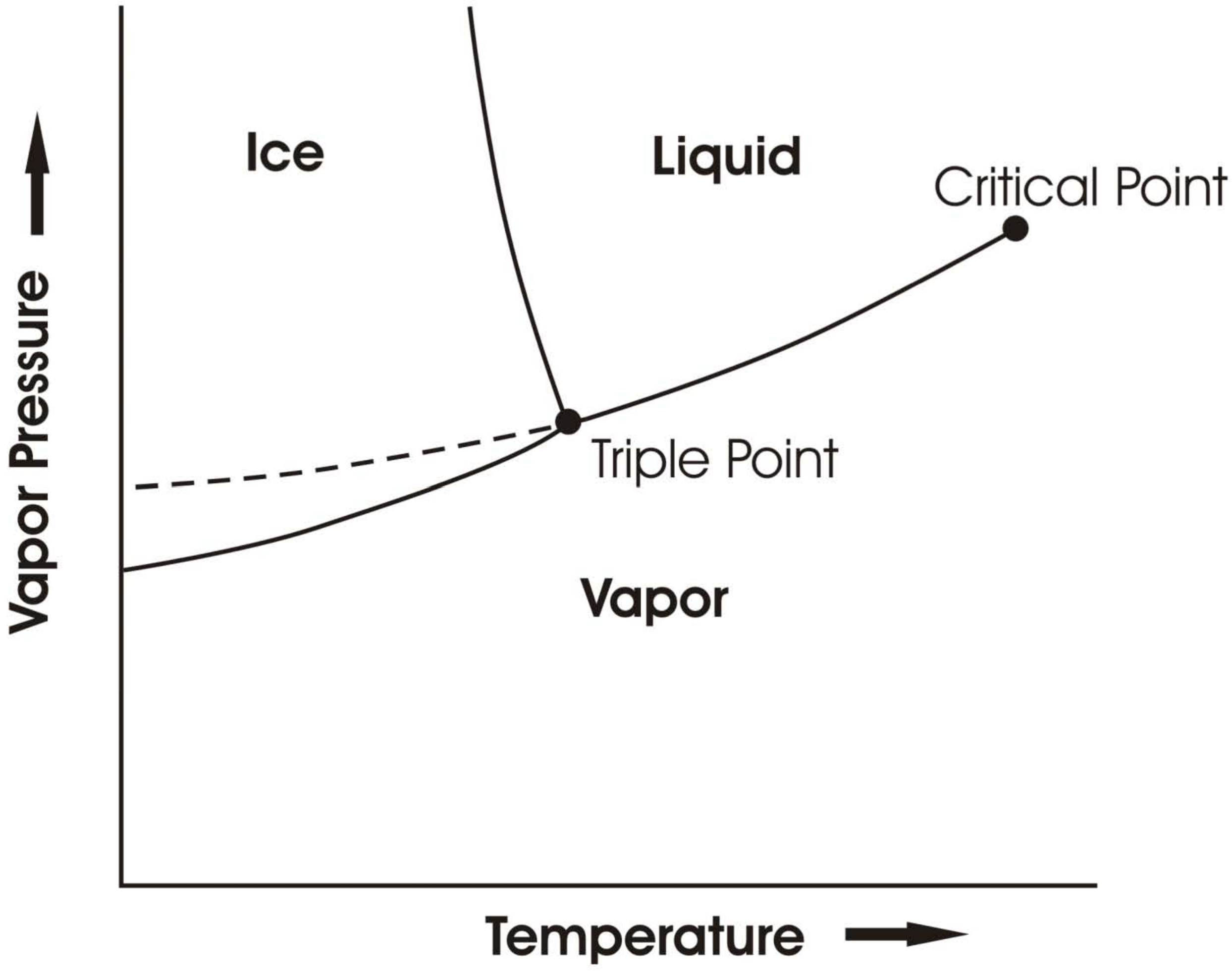
$$p_d = \rho_d \frac{R^* T}{\bar{m}}$$

$$e = \rho_v \frac{R^* T}{m_v}$$

$$q = {\rho_v}/{\rho} = {\rho_v}/{\rho_v + \rho_d} = \frac{m_v}{\bar{m}} \frac{e}{p - e\left(1 - \frac{m_v}{\bar{m}}\right)}$$

$$q^* = \frac{m_v}{\bar{m}} \frac{e^*}{p - e^*\left(1 - \frac{m_v}{\bar{m}}\right)}$$

Phase Equilibria



Bringing Air to Saturation

$$e \simeq qp\left(\bar{m}/m_v\right)$$

$$e^* = e^*(T)$$

1. Increase *q (or p)*

2. Decrease *e* (T)*

12.811 Tropical Meteorology
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