Using $$\frac{\partial T}{\partial s^*} = \frac{\partial T}{\partial M}\frac{dM}{ds^*},$$

We can re-write (18) as $$\frac{\partial T_o}{\partial M} \cong -\frac{Ri_c}{r_t^2}\left(\frac{dM}{ds^*}\right). \tag{19}$$

We can also re-write (6) as $$M_b = r_b^2\left(\frac{1}{2}f - \left(T_b - T_o\right)\frac{ds^*}{dM}\right) \tag{20}$$

Boundary layer entropy $$h\frac{ds_b}{dt} = C_k\,|\,\mathbf{V}\,|\left(s_0^* - s_b\right) + C_d\,\frac{|\,\mathbf{V}\,|^3}{T_b} \tag{21}$$

Boundary layer angular momentum $$h\frac{dM}{dt} = -r\,|\,\mathbf{V}\,|V \tag{22}$$

Combine (21) and (22):

$$\frac{ds_b}{dM} = -\frac{C_k}{C_D}\frac{\left(s_0^* - s_b\right)}{rV} - \frac{|\mathbf{V}|^2}{T_b rV}$$

Let $\quad s_b \simeq s^*, \quad |\mathbf{V}| \simeq V \simeq V_b, \quad r \simeq r_b$

$$\rightarrow \frac{ds^*}{dM} = -\frac{C_k}{C_D}\frac{\left(s_0^* - s^*\right)}{r_b V_b} - \frac{V_b}{T_b r_b} \qquad (23)$$

Balance condition (8):

$$\frac{V_b}{r_b} = -\left(T_b - T_o\right)\frac{ds^*}{dM} \qquad (24)$$

Eliminate V_b between (23) and (24):

$$\left(\frac{ds^*}{dM}\right)^2 = \frac{T_b}{T_o}\frac{C_k}{C_D}\frac{\left(s_0^* - s^*\right)}{r_b^2\left(T_b - T_o\right)} \qquad (25)$$

Eliminate r_b^2 between (20) and (25):

$$\left(\frac{ds^*}{dM}\right)^2 + 2\gamma\frac{ds^*}{dM} - \frac{\chi f}{T_b - T_o} = 0, \qquad (26)$$

where

$$\chi \equiv \frac{T_b}{T_o}\frac{C_k}{C_D}\frac{s_0^* - s^*}{2M}$$

Remember that

$$\frac{\partial T_o}{\partial M} \cong -\frac{Ri_c}{r_t^2}\left(\frac{dM}{ds^*}\right) \qquad (19)$$

Integrate (26) and (19) inward from some outer radius r_o, defined such that

$$V = 0 \quad at \quad r = r_o$$

In general, integrating this system will not yield $T_o = T_t$ at $r = r_{max}$. Iterate value of r_t until this condition is met.

If $V \gg fr$, we ignore dissipative heating, and we neglect pressure dependence of s_0^*, then we can derive an approximate closed-form solution.

Assuming that *Ri* is critical in the outflow leads to an equation for T_o that, coupled to the interior balance equation and the slab boundary layer lead (surprisingly!) to a closed form analytic solution for the gradient wind (as represented by angular momentum, *M*, at the top of the boundary layer:

$$\left(\frac{M}{M_m}\right)^{2-\frac{C_k}{C_D}} = \frac{2\left(\frac{r}{r_m}\right)^2}{2-\frac{C_k}{C_D}+\frac{C_k}{C_D}\left(\frac{r}{r_m}\right)^2}, \qquad (27)$$

$$\left(\frac{fr_o^2}{2V_m r_m}\right)^{2-\frac{C_k}{C_D}} = \frac{2\left(\frac{r_o}{r_m}\right)^2}{2-\frac{C_k}{C_D}+\frac{C_k}{C_D}\left(\frac{r_o}{r_m}\right)^2}. \qquad (28)$$

$$r_m \cong \frac{1}{2} fr_o^2 V_m^{-1}\left(\frac{1}{2}\frac{C_k}{C_D}\right)^{\frac{1}{2-\frac{C_k}{C_D}}} \qquad (29)$$

The maximum wind speed, V_m , found from maximizing the radial dependence of wind speed in the solution (27) is given by

$$V_m^{2-\frac{C_k}{C_D}} = V_p^2\left(\frac{2r_m}{fr_o^2}\right)^{\frac{C_k}{C_D}} \qquad (30)$$

$$V_p^2 \equiv \frac{C_k}{C_D}\left(T_b - T_t\right)\left(s_0 * - s_e *\right)$$

Substituting (29) into (30) gives

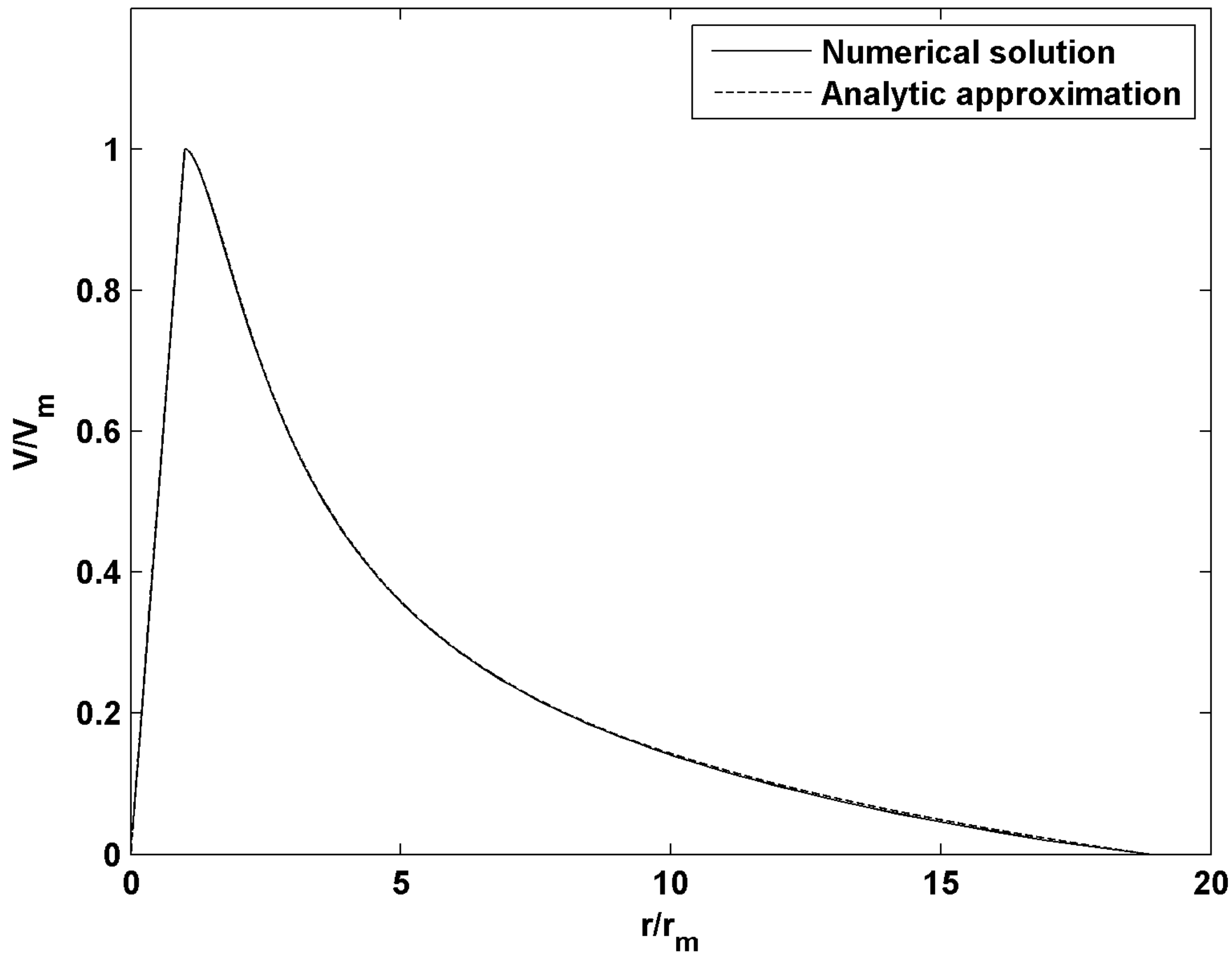
$$V_m^2 \cong V_p^2 \left(\frac{1}{2}\frac{C_k}{C_D}\right)^{\frac{\frac{C_k}{C_D}}{2-\frac{C_k}{C_D}}} \qquad (31)$$

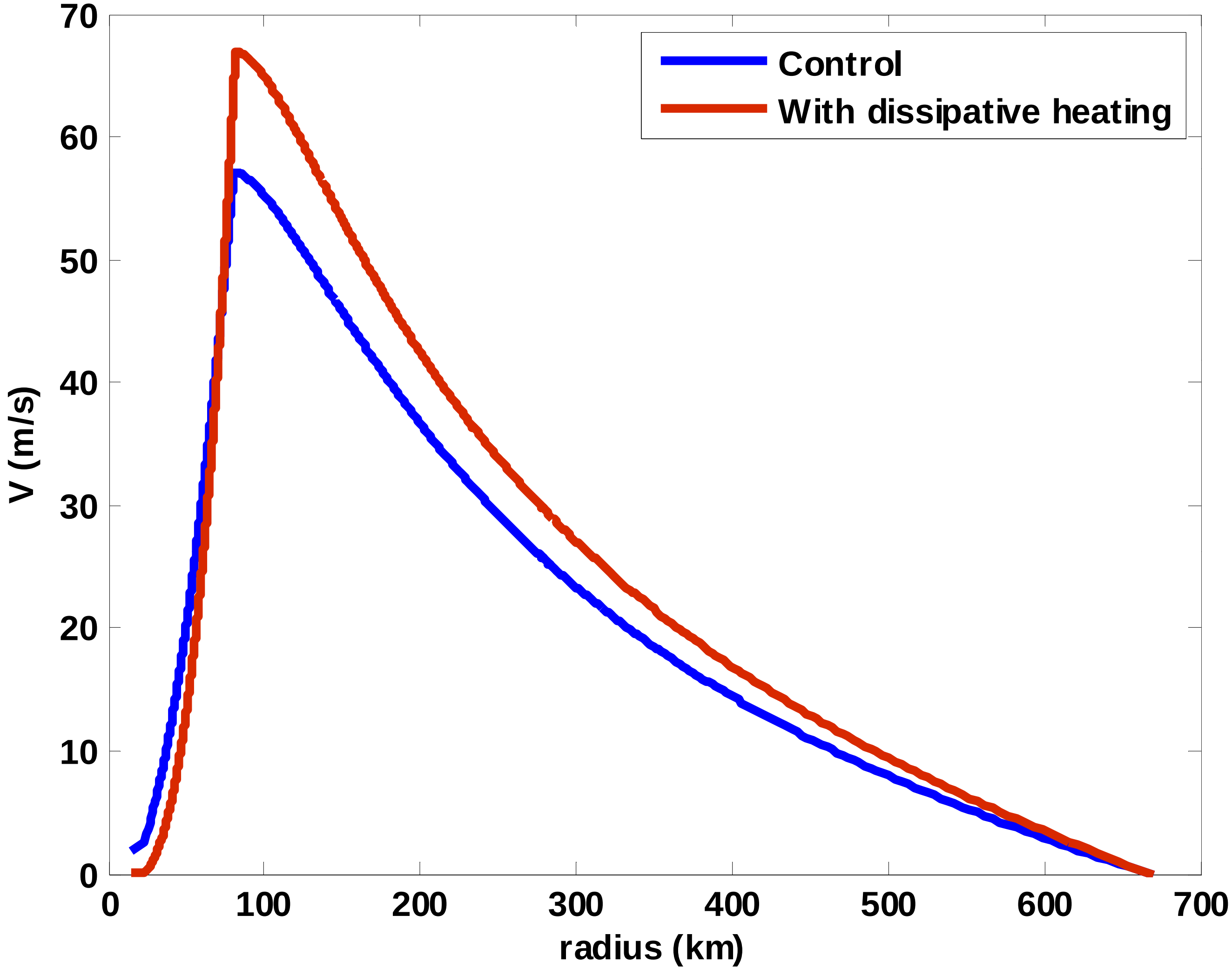
Substituting (31) into (29) gives

$$r_m \cong \left(\frac{1}{2}\right)^{\frac{3}{2}} \frac{f r_o^2}{\sqrt{\left(T_b - T_t\right)\left(s_0 * - s_e *\right)}}$$

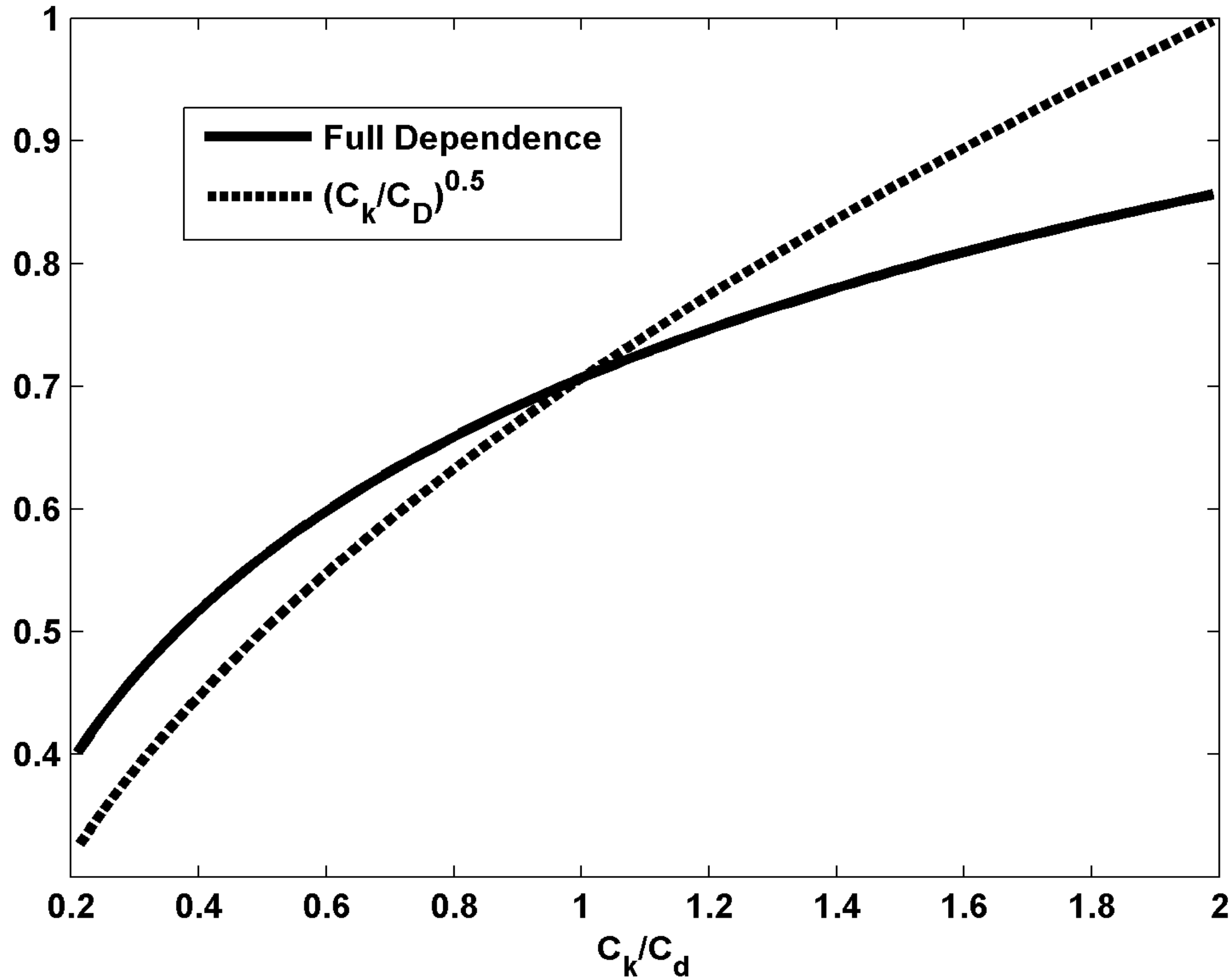
Also,

$$r_t^2 = r_m^2 \frac{C_D}{C_k} Ri_c$$

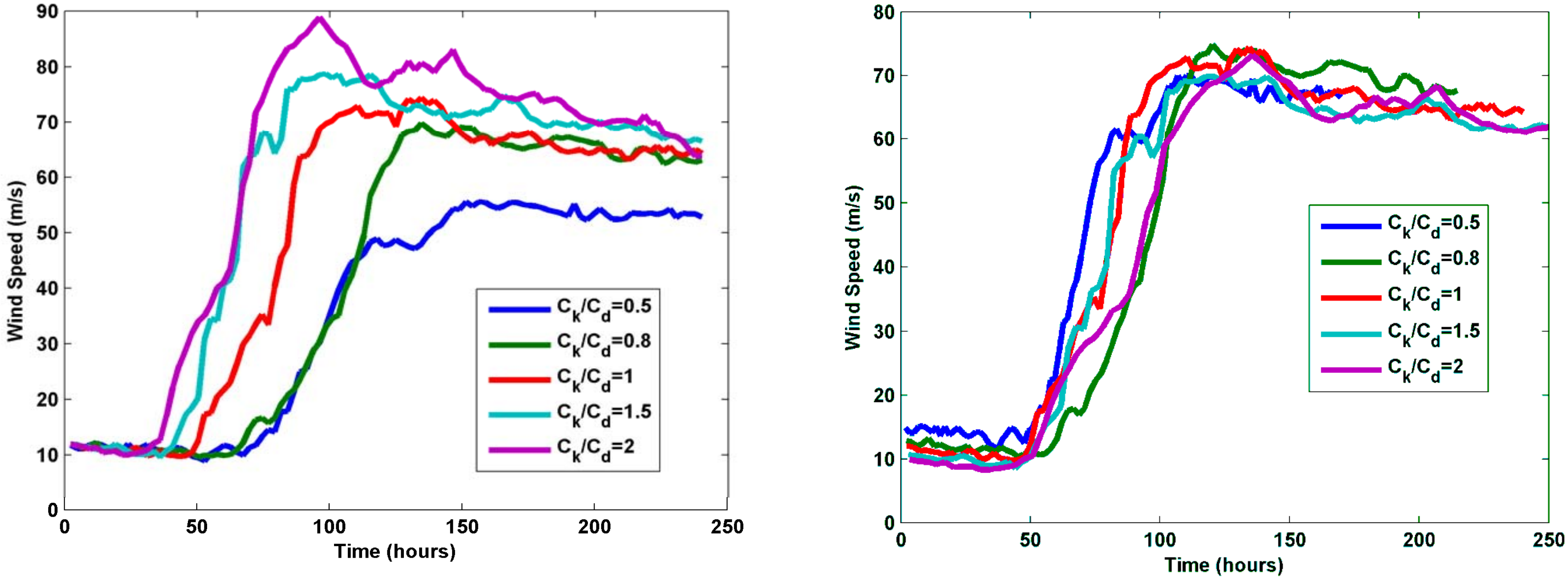

Numerical solution
Analytic approximation
V/V_m
r/r_m
0
0.2
0.4
0.6
0.8
1
0
5
10
15
20



70
60
50
40
30
20
10
0
V (m/s)
0
100
200
300
400
500
600
700
radius (km)
Control
With dissipative heating

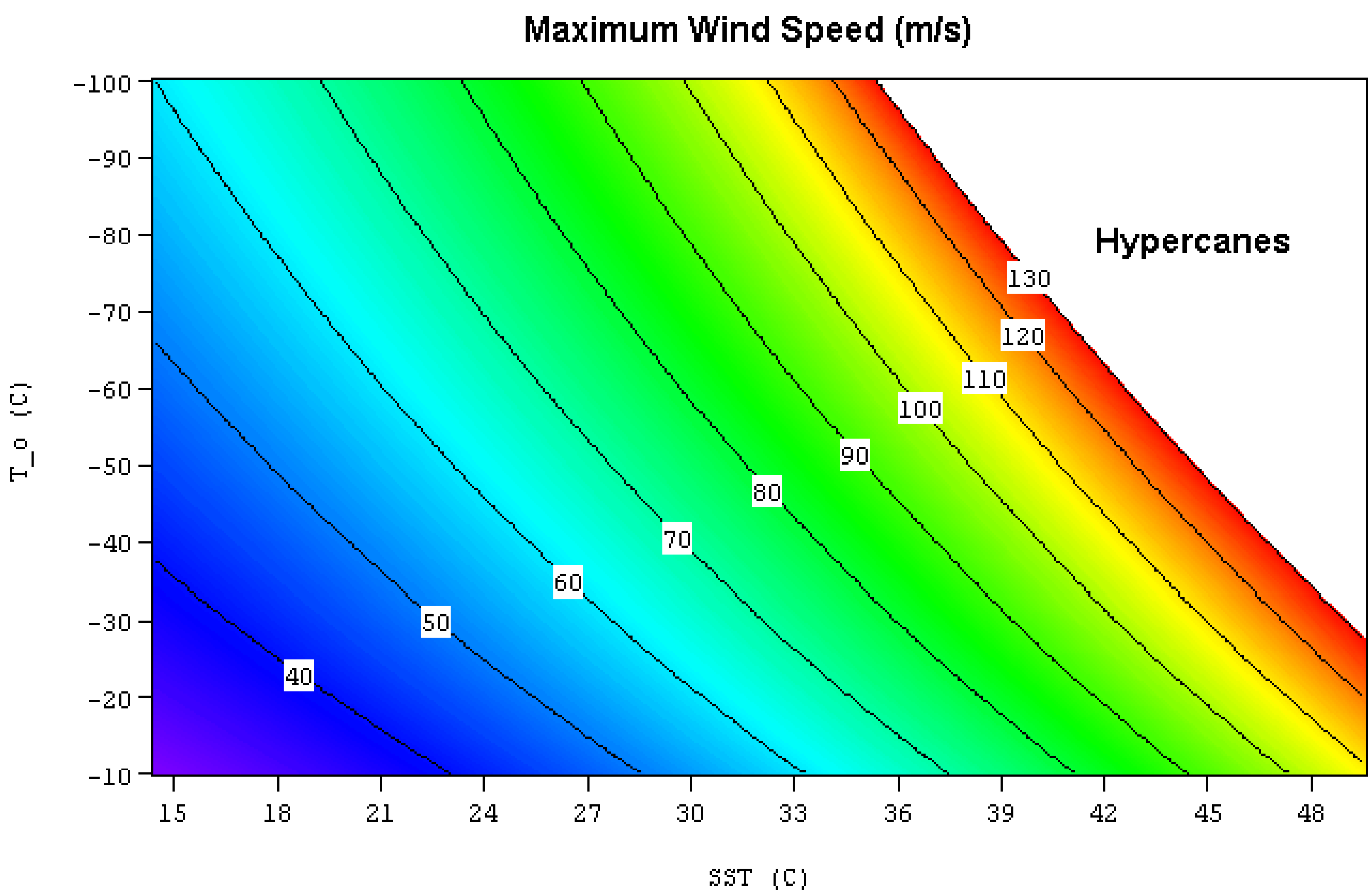


1
0.9
0.8
0.7
0.6
0.5
0.4
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
2
Full Dependence
$(C_k/C_D)^{0.5}$
C_k/C_d

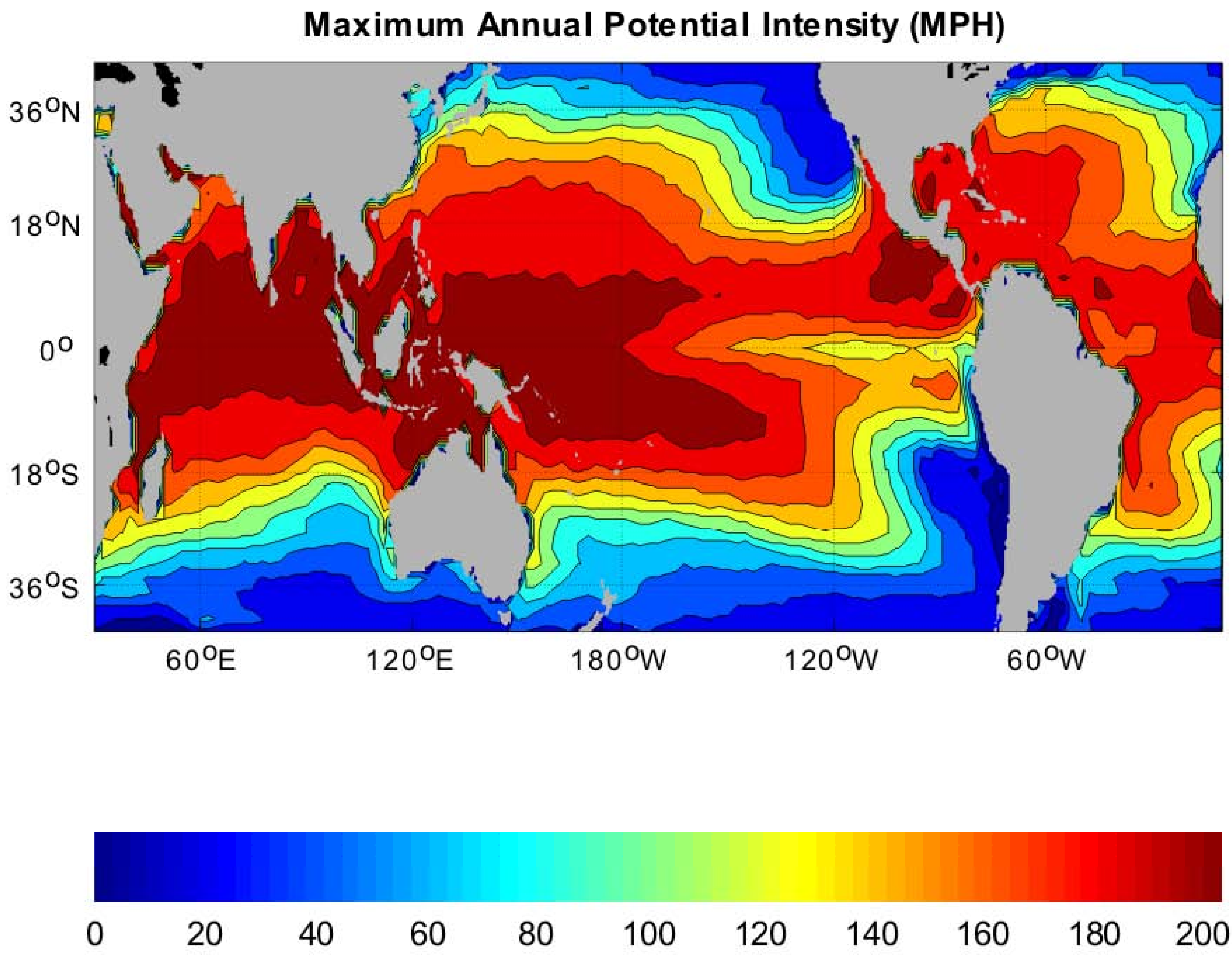


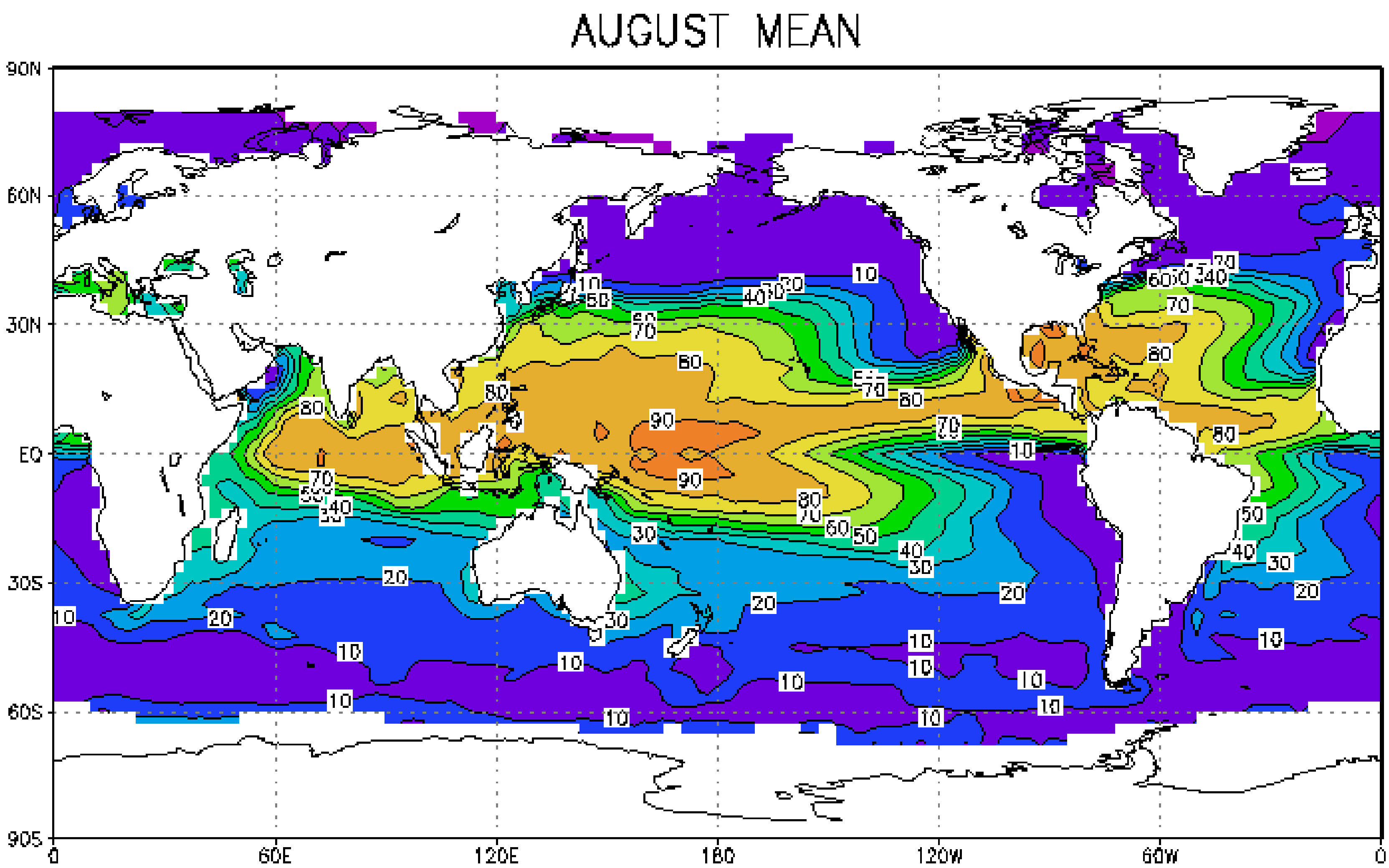
Numerical integrations with RE87 model (no dissipative heating, no pressure dependence of k_0*) : Left, regular variables; Right: Velocity scaled by (31) and time scaled by the inverse square-root of the enthalpy exchange coefficient.

Effects of Pressure-Dependence of Surface Saturation Enthalpy

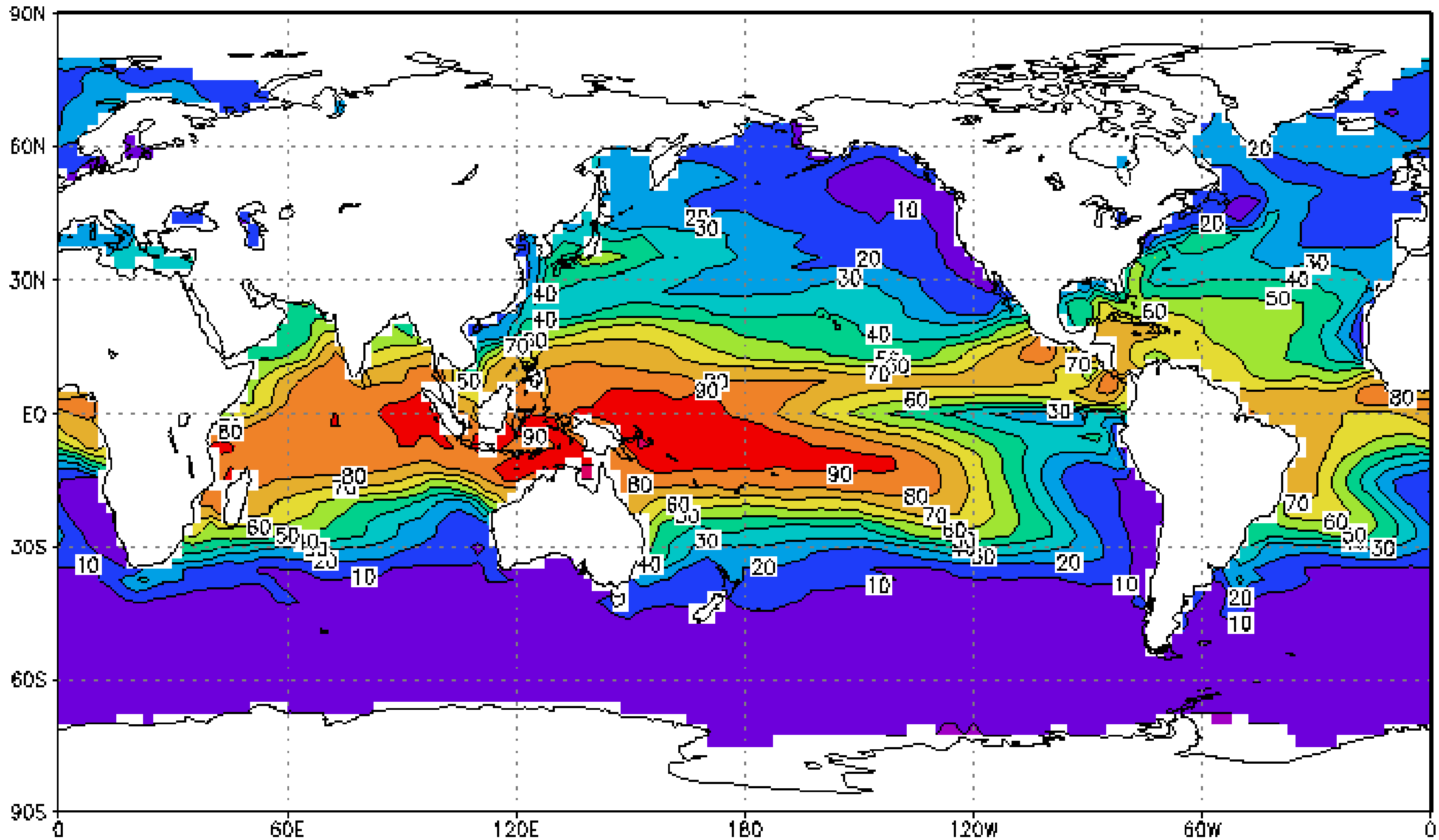


$\mathscr{H} = 0.75 \quad C_k/C_D = 1.2$

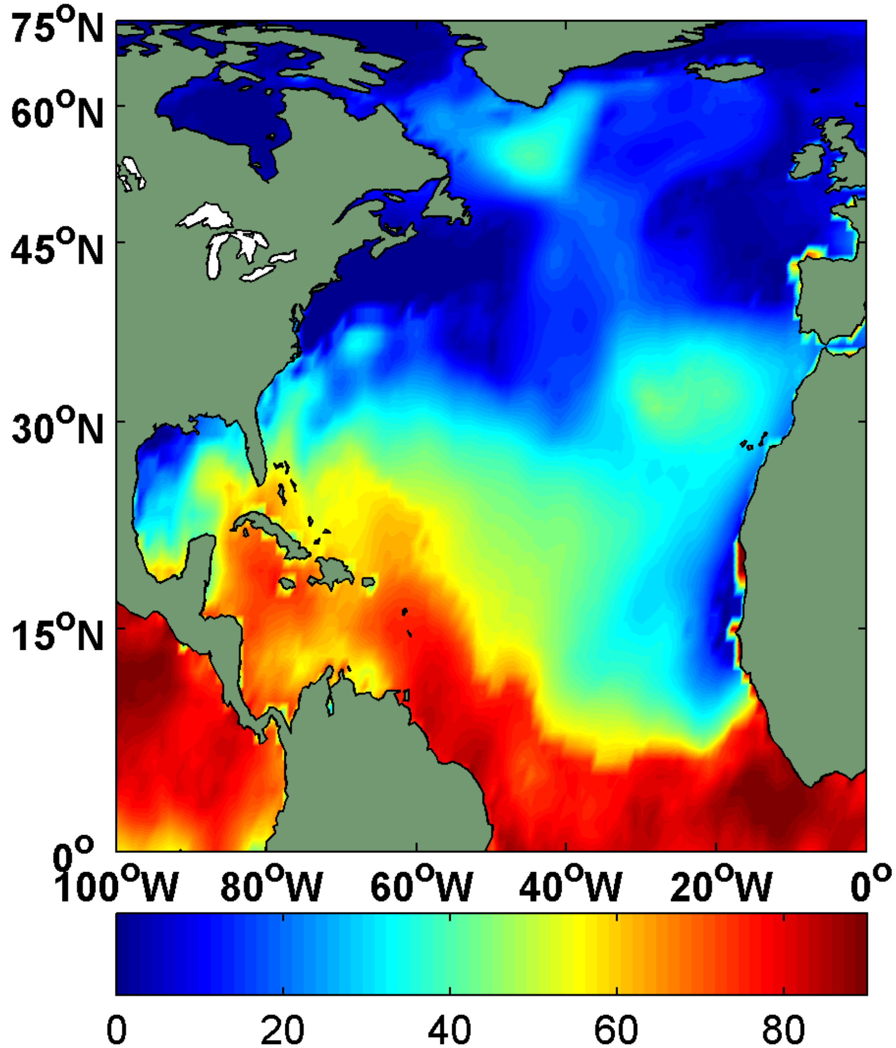

Maximum Annual Potential Intensity (MPH)
36ºN
18ºN
0º
18ºS
36ºS
60ºE
120ºE
180ºW
120ºW
60ºW
0
20
40
60
80
100
120
140
160
180
200

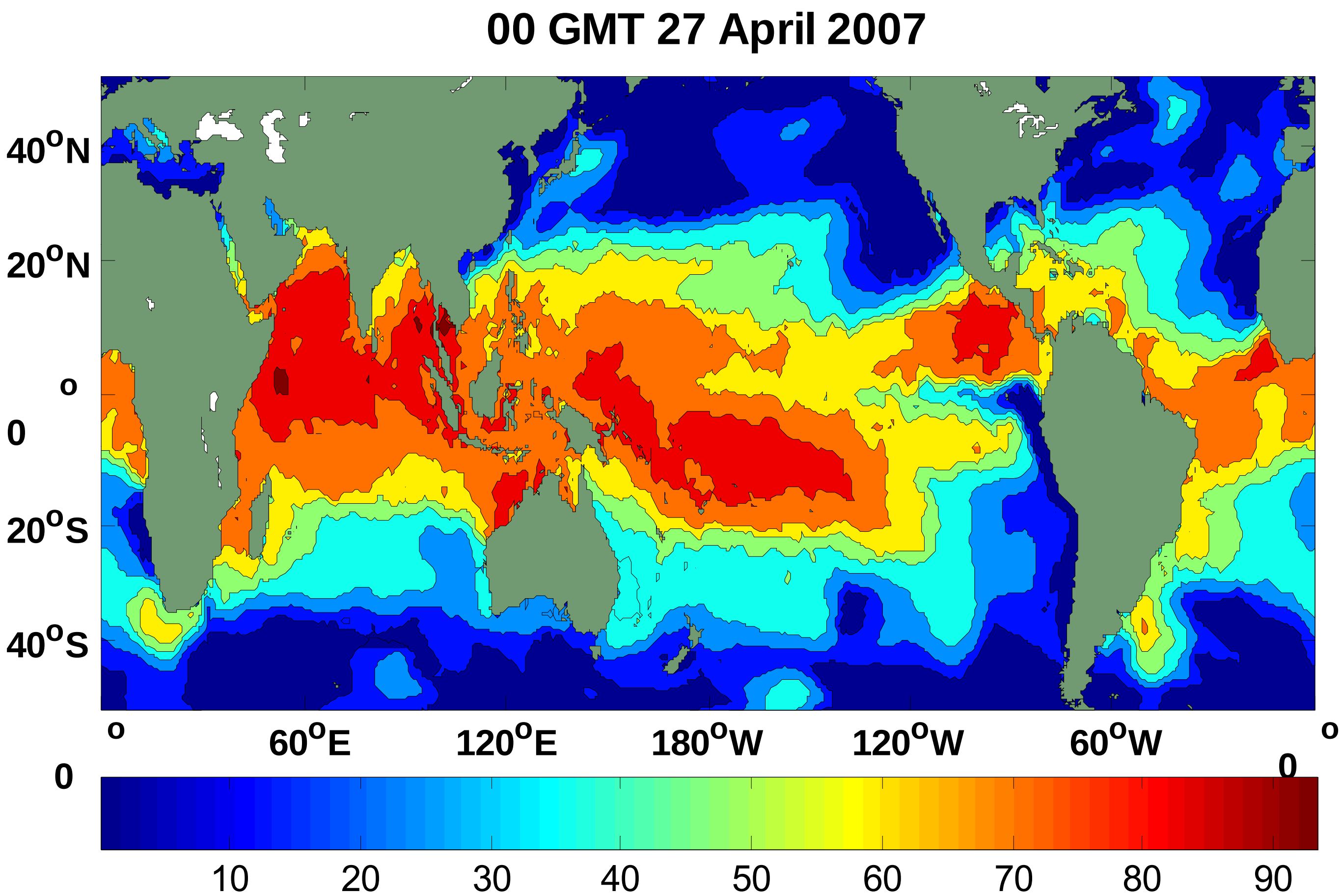

AUGUST MEAN
90N
60N
30N
EQ
30S
60S
90S
0
60E
120E
180
120W
60W
0

JANUARY MEAN

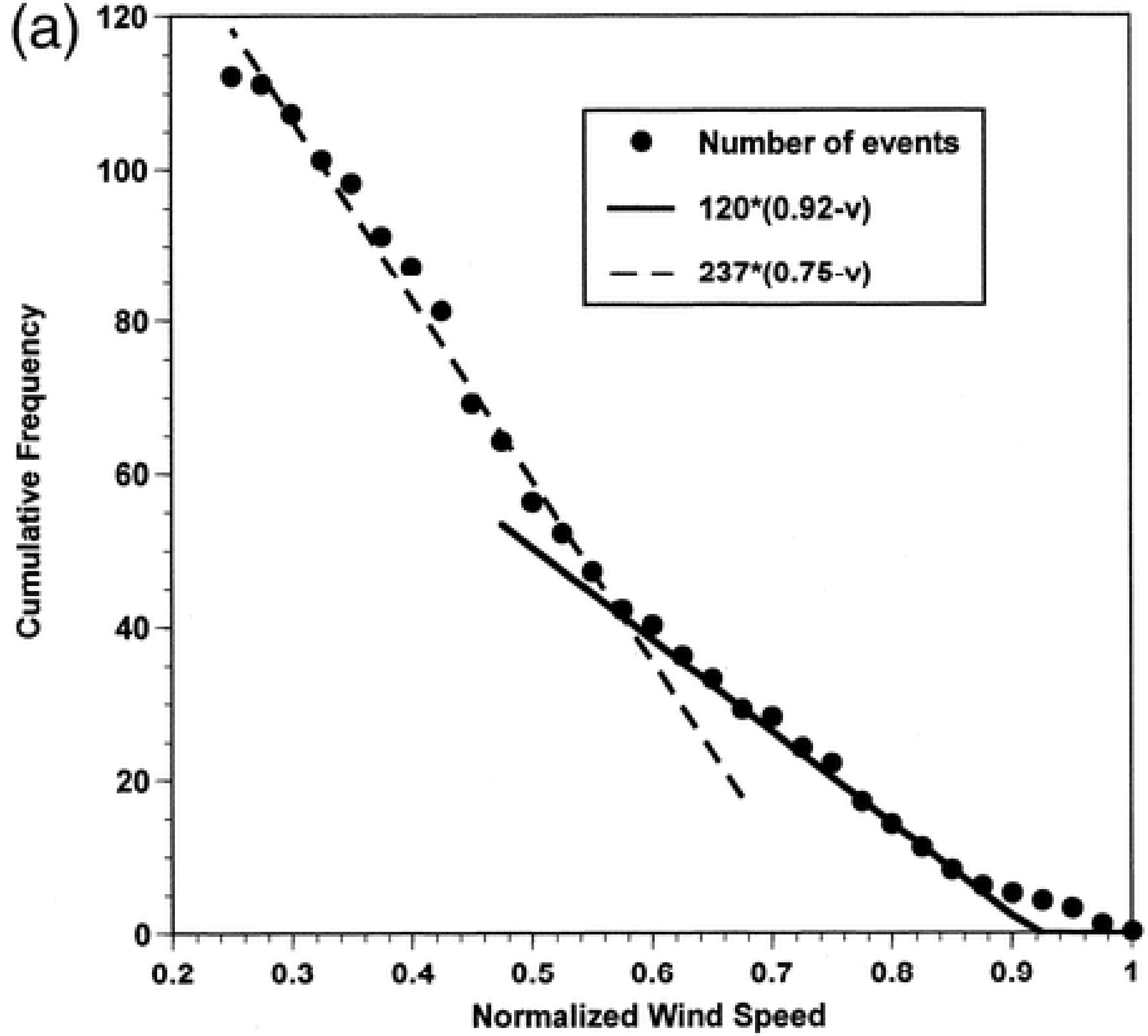


27 April 2011 12 GMT

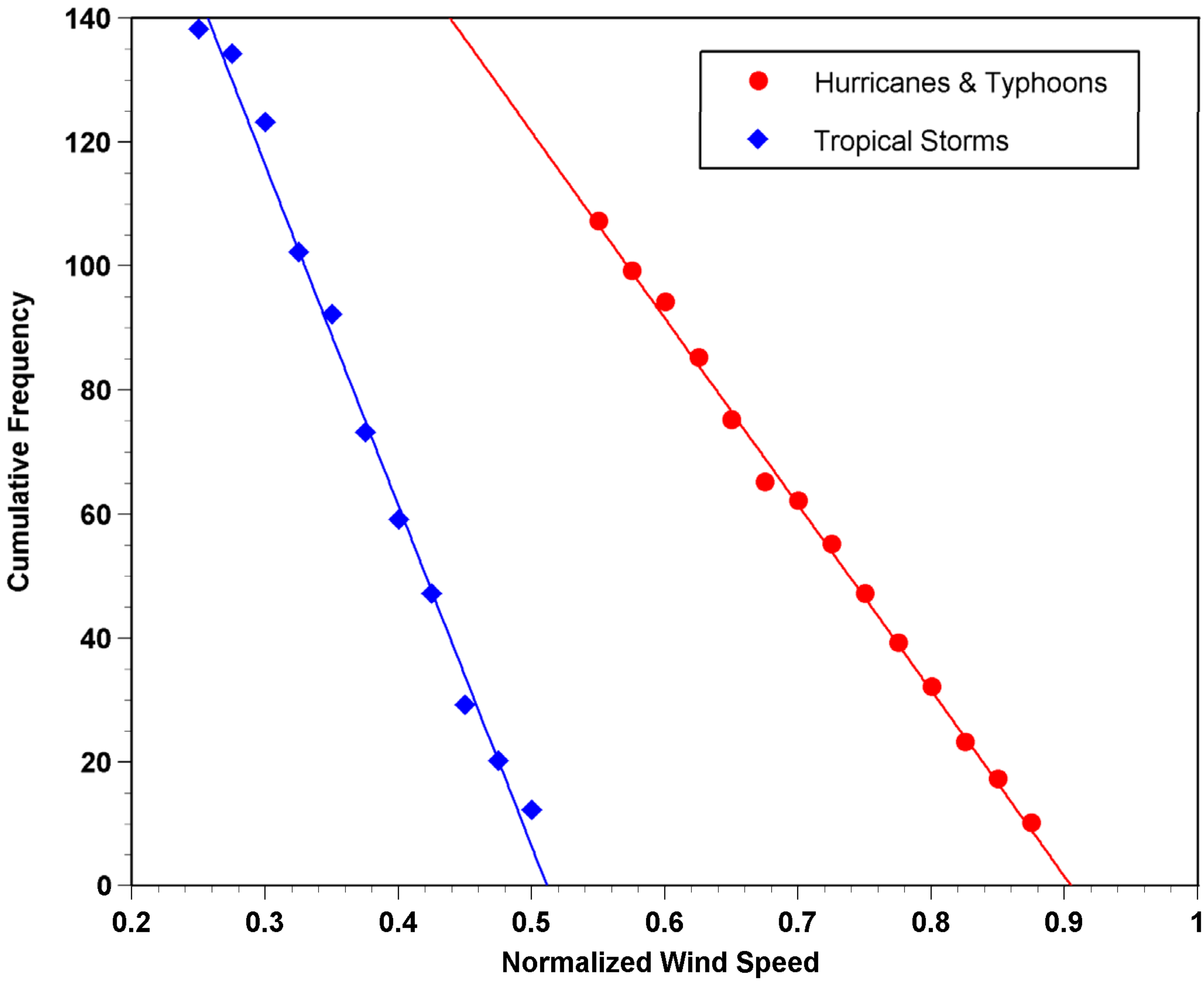



00 GMT 27 April 2007
40^{o}N
20^{o}N
0^{o}
20^{o}S
40^{o}S
0^{o}
60^{o}E
120^{o}E
180^{o}W
120^{o}W
60^{o}W
0^{o}
10
20
30
40
50
60
70
80
90

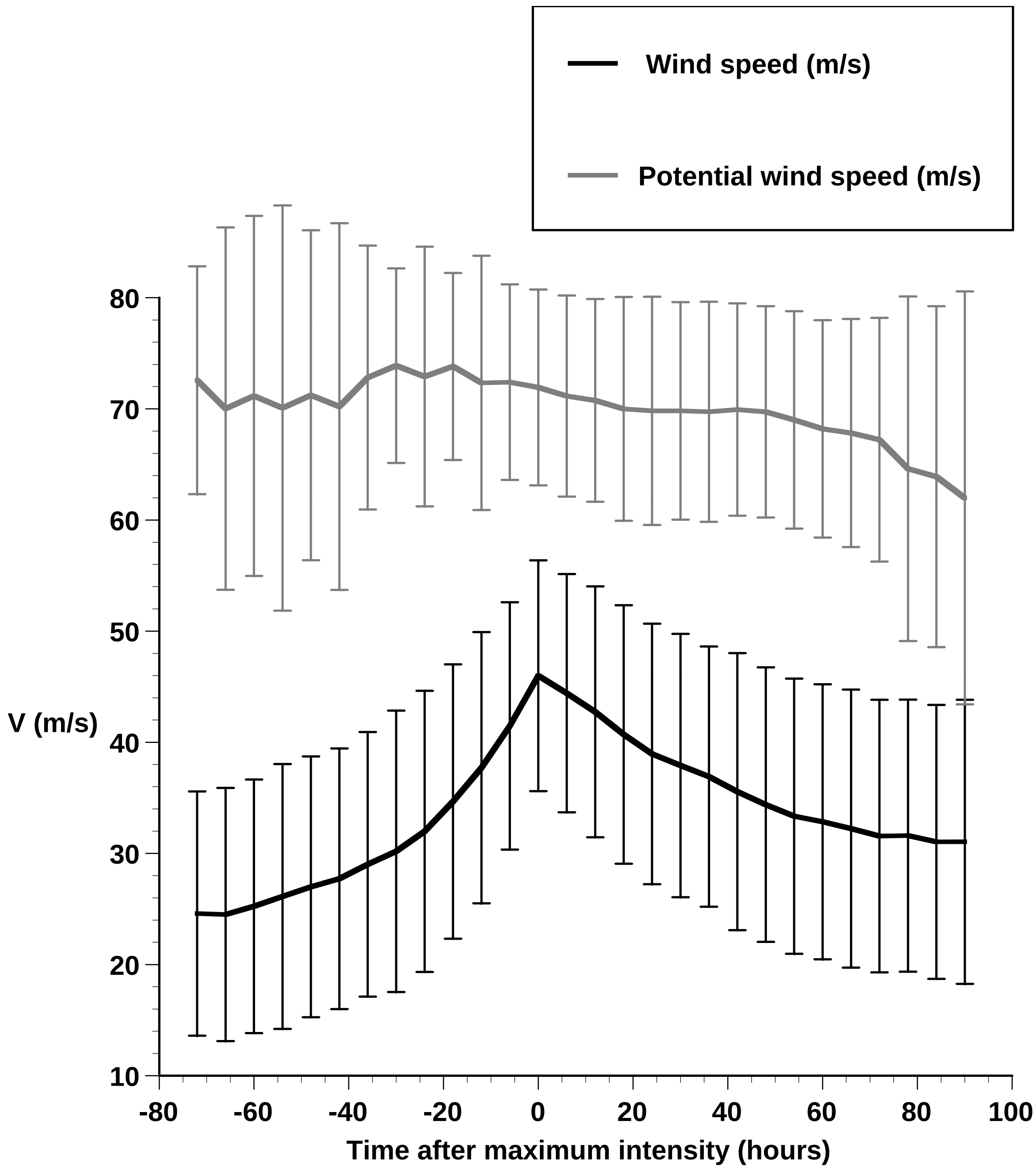
Relationship between potential intensity (PI) and intensity of real tropical cyclones



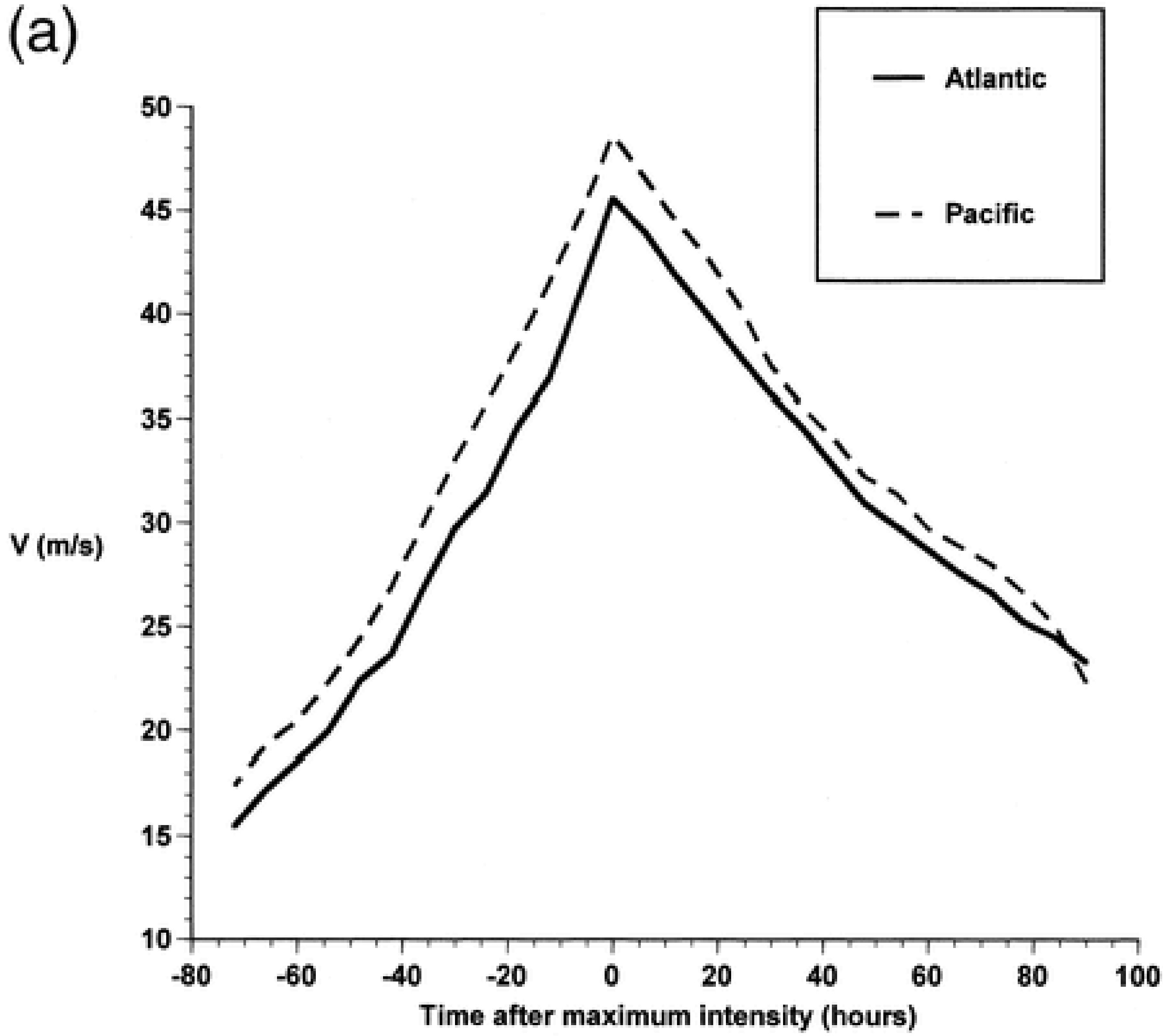
(a)
120
100
80
60
40
20
0
Cumulative Frequency
Number of events
120*(0.92-v)
237*(0.75-v)
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
Normalized Wind Speed



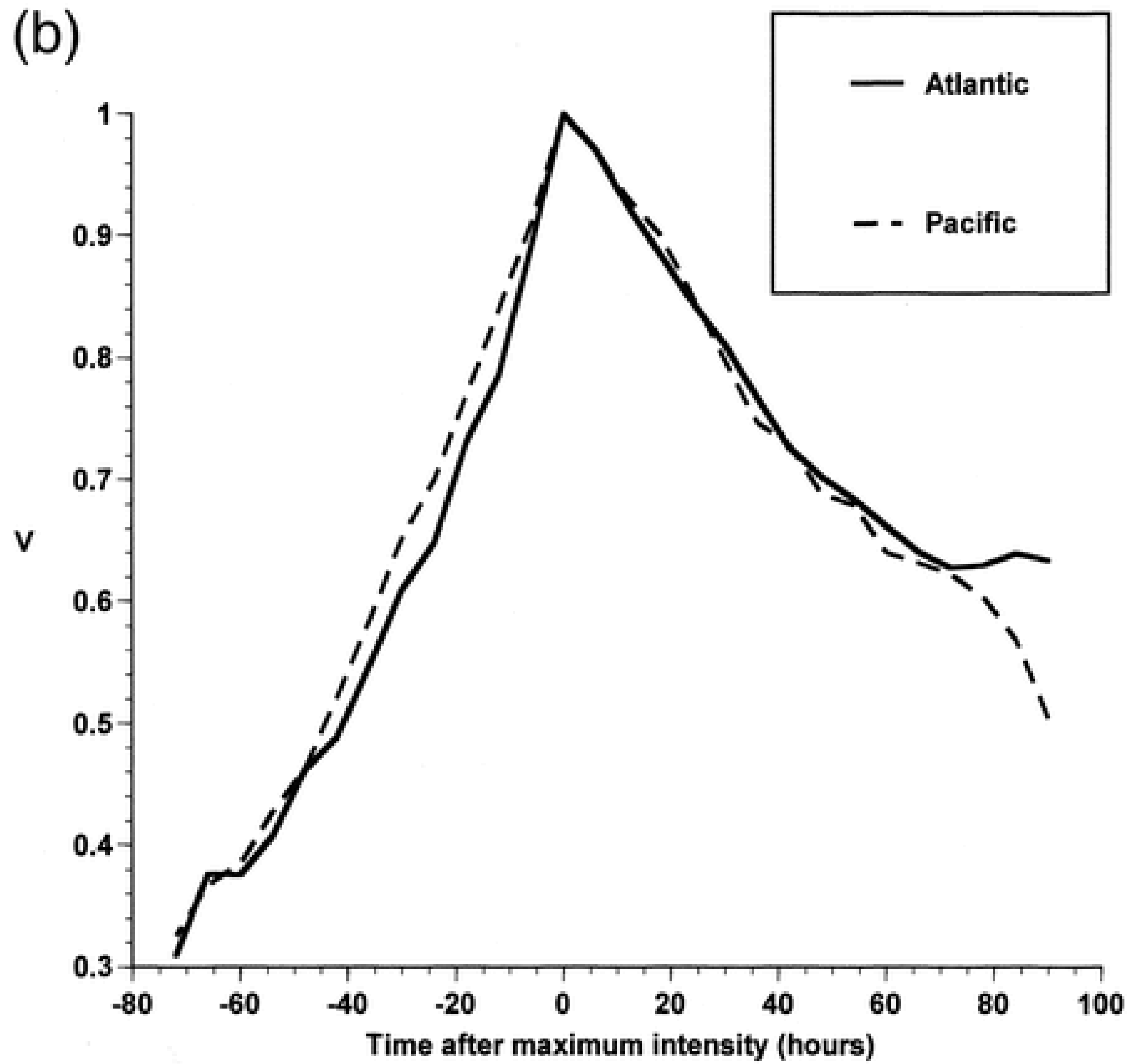
Hurricanes & Typhoons
Tropical Storms
Cumulative Frequency
Normalized Wind Speed
0
20
40
60
80
100
120
140
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1


Wind speed (m/s)
Potential wind speed (m/s)
V (m/s)
80
70
60
50
40
30
20
10
-80
-60
-40
-20
0
20
40
60
80
100
Time after maximum intensity (hours)

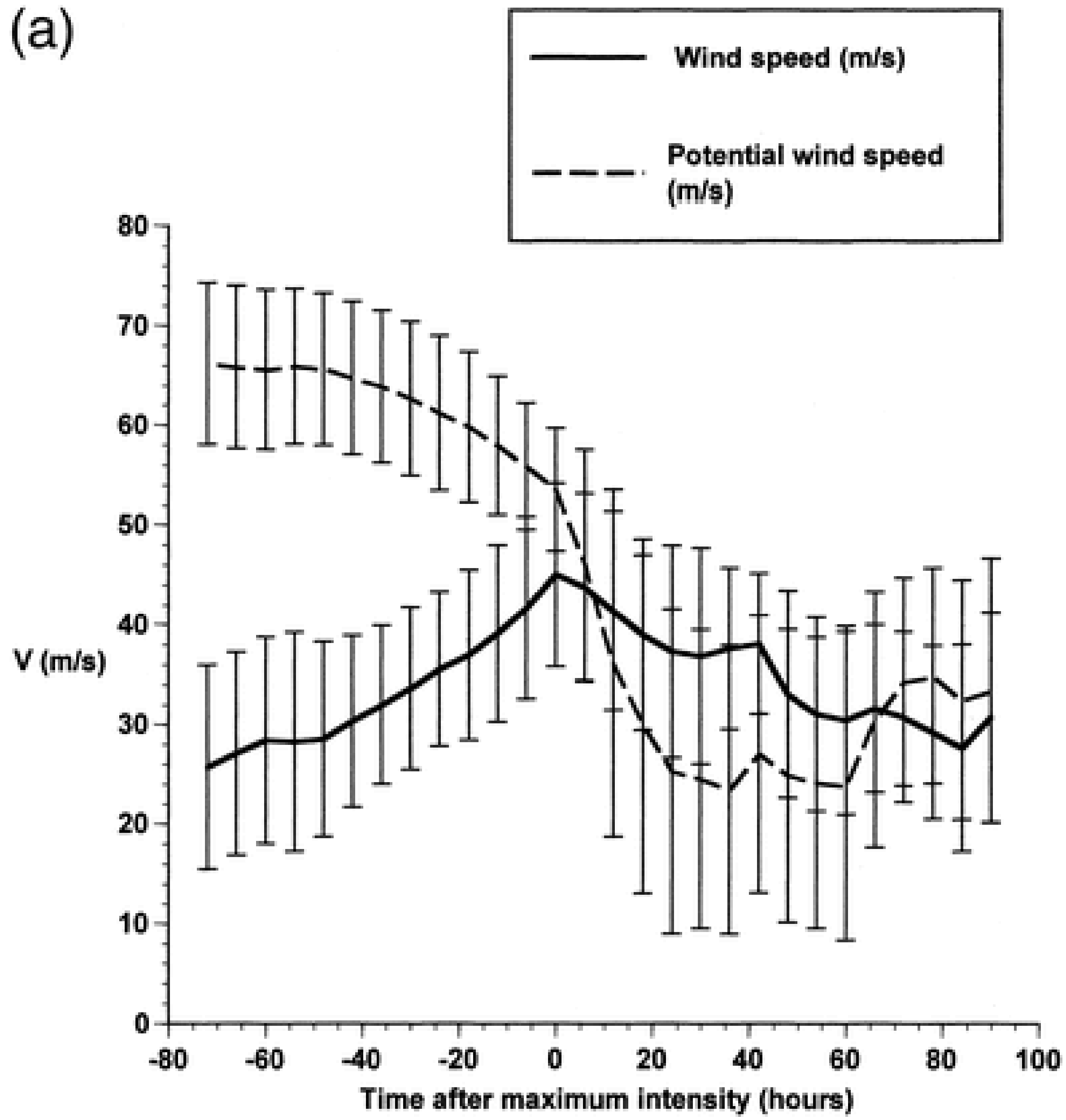
Evolution with respect to time of maximum intensity



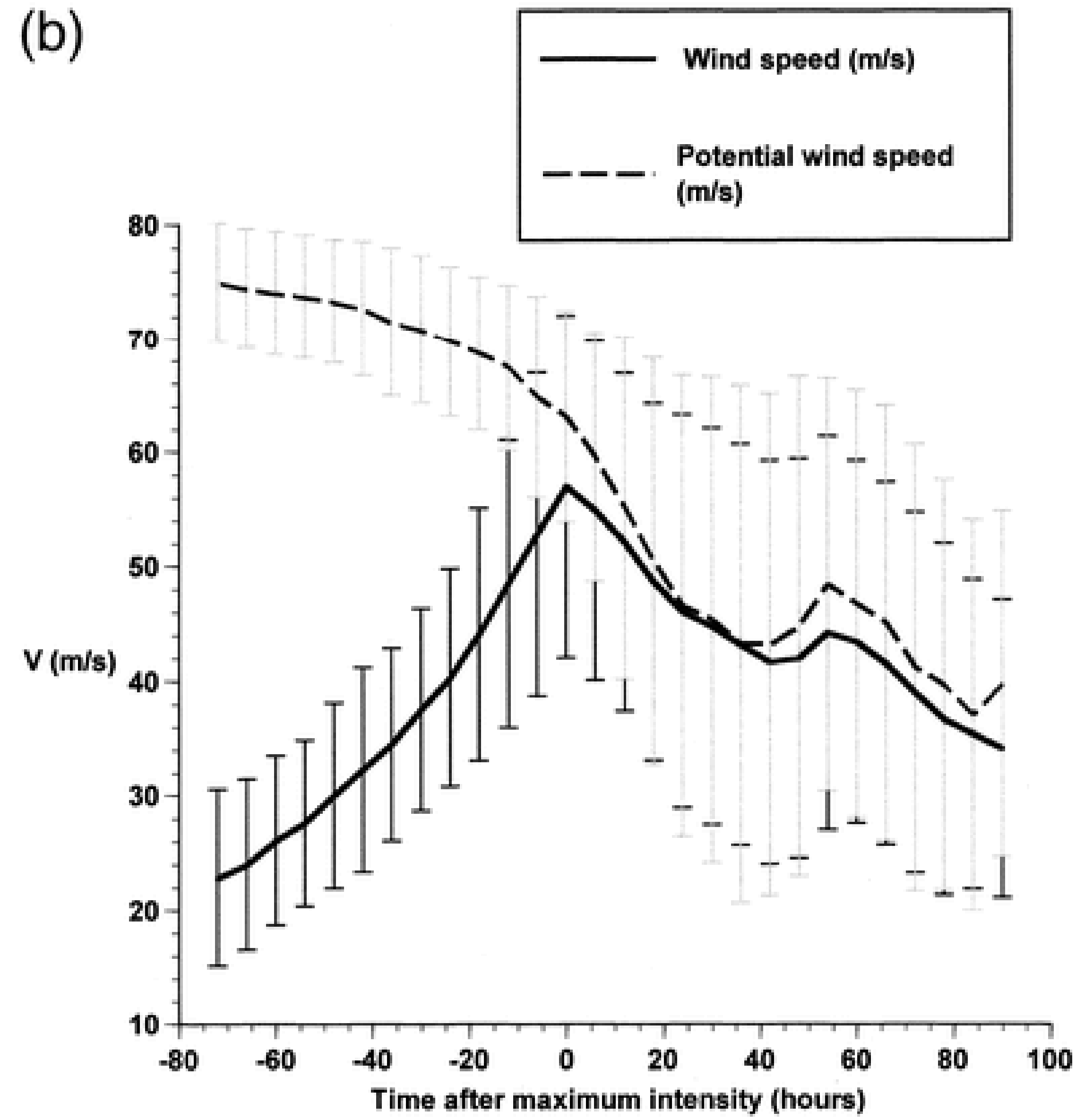
Evolution with respect to time of maximum intensity, normalized by peak wind



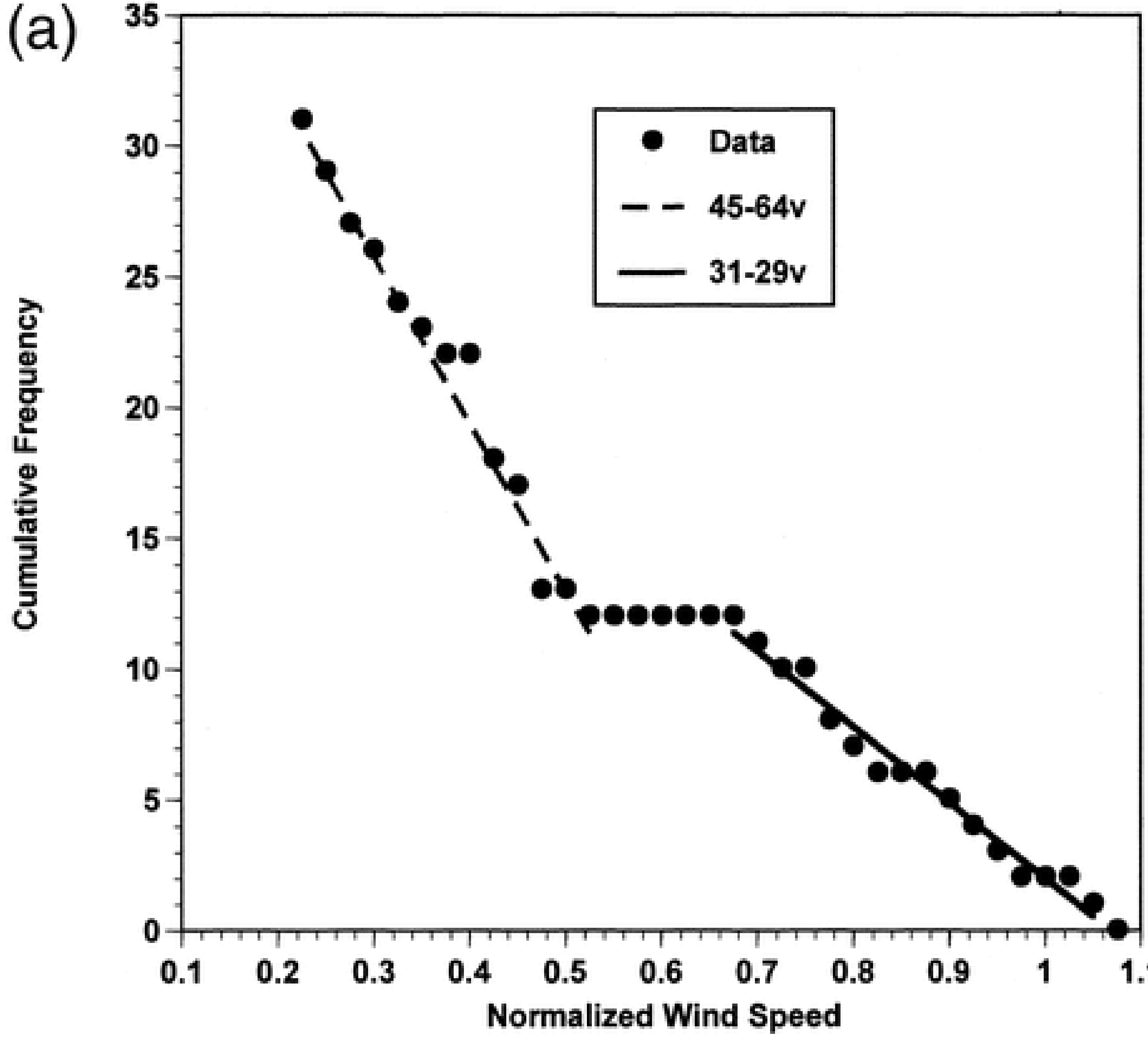
Evolution curve of Atlantic storms whose lifetime maximum intensity is limited by declining potential intensity, but not by landfall



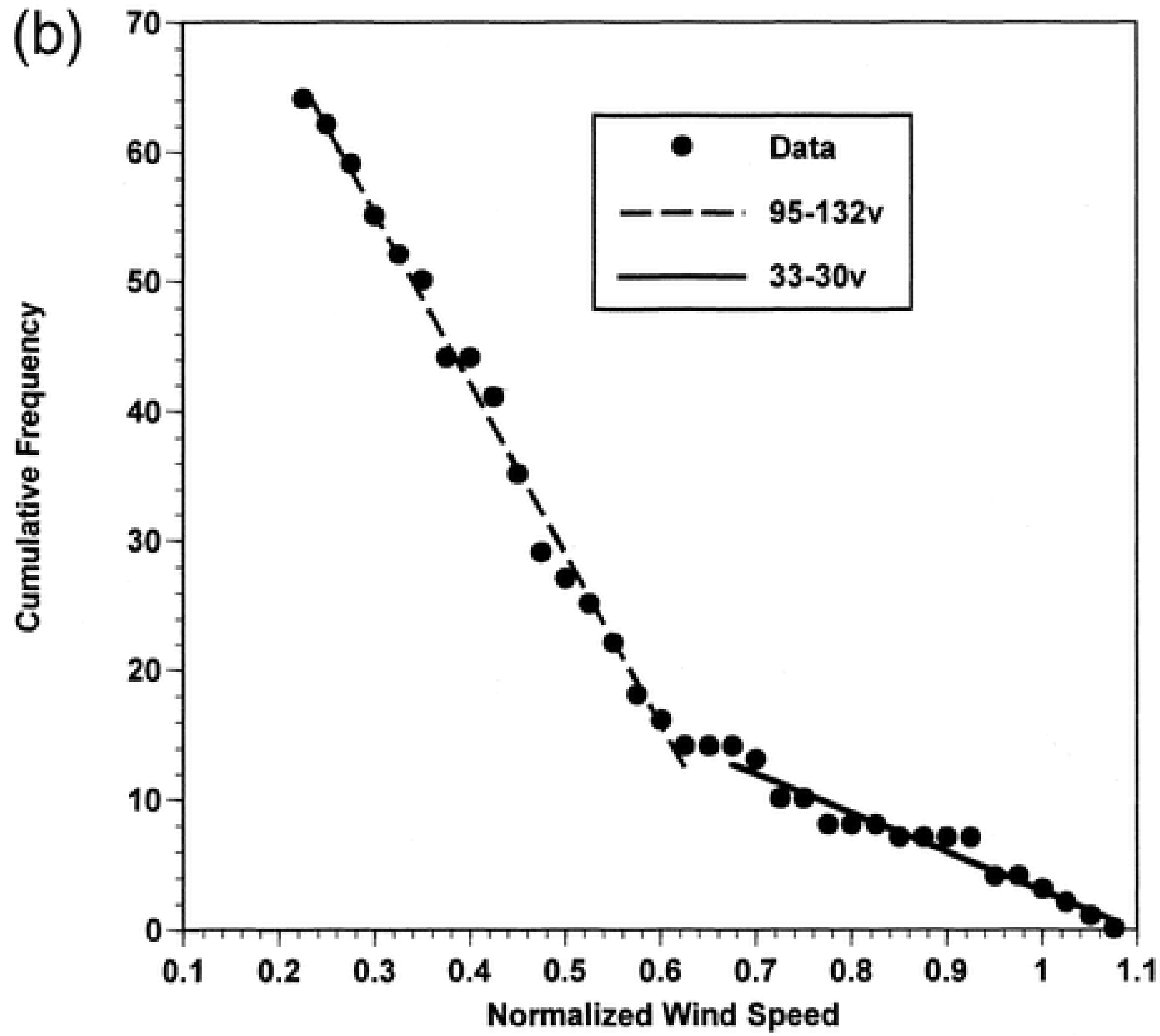
Evolution curve of WPAC storms whose lifetime maximum intensity is limited by declining potential intensity, but not by landfall



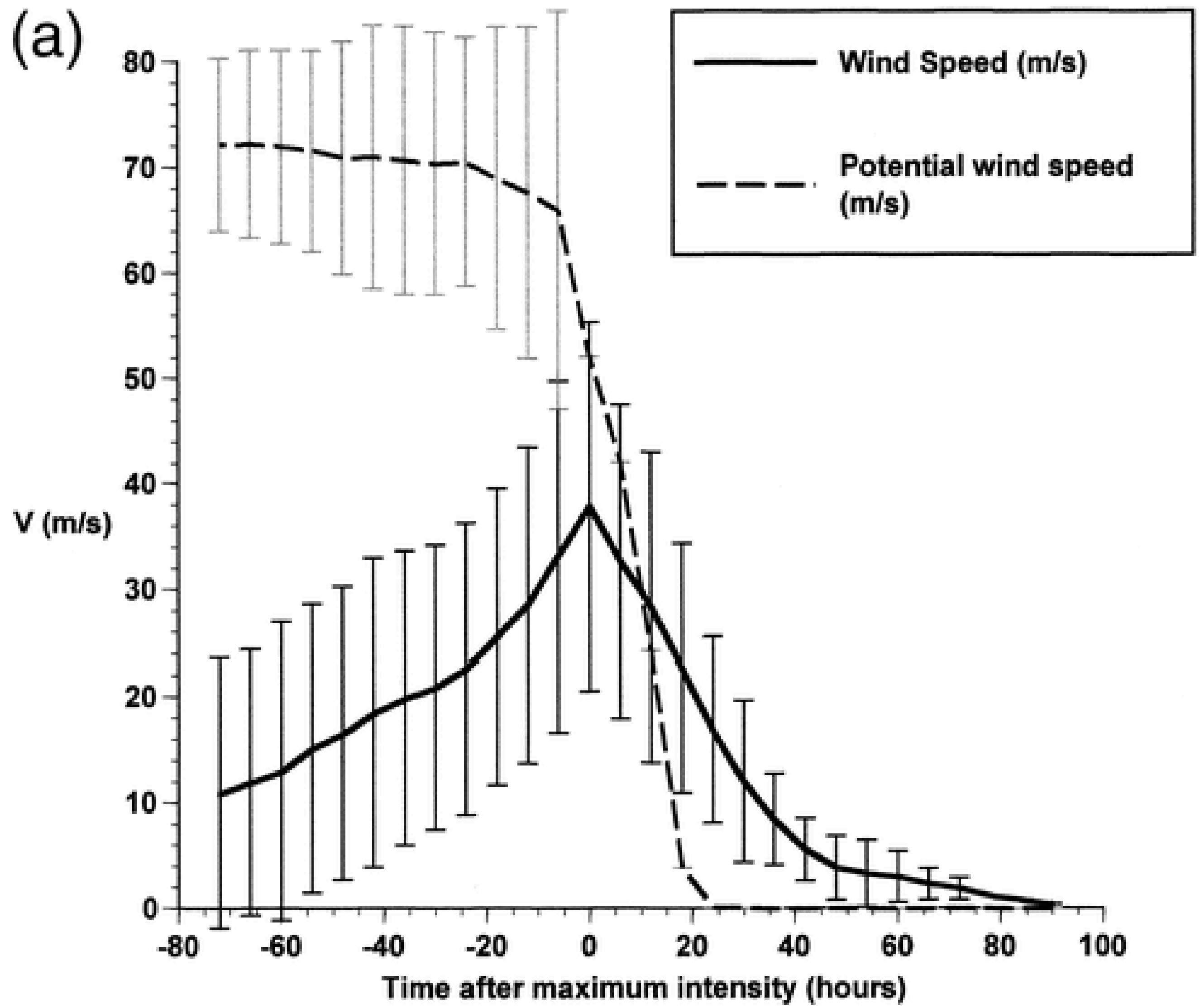
CDF of normalized lifetime maximum wind speeds of North Atlantic tropical cyclones of tropical storm strength (18 m s^{-1}) or greater, for those storms whose lifetime maximum intensity was limited by landfall. (a)



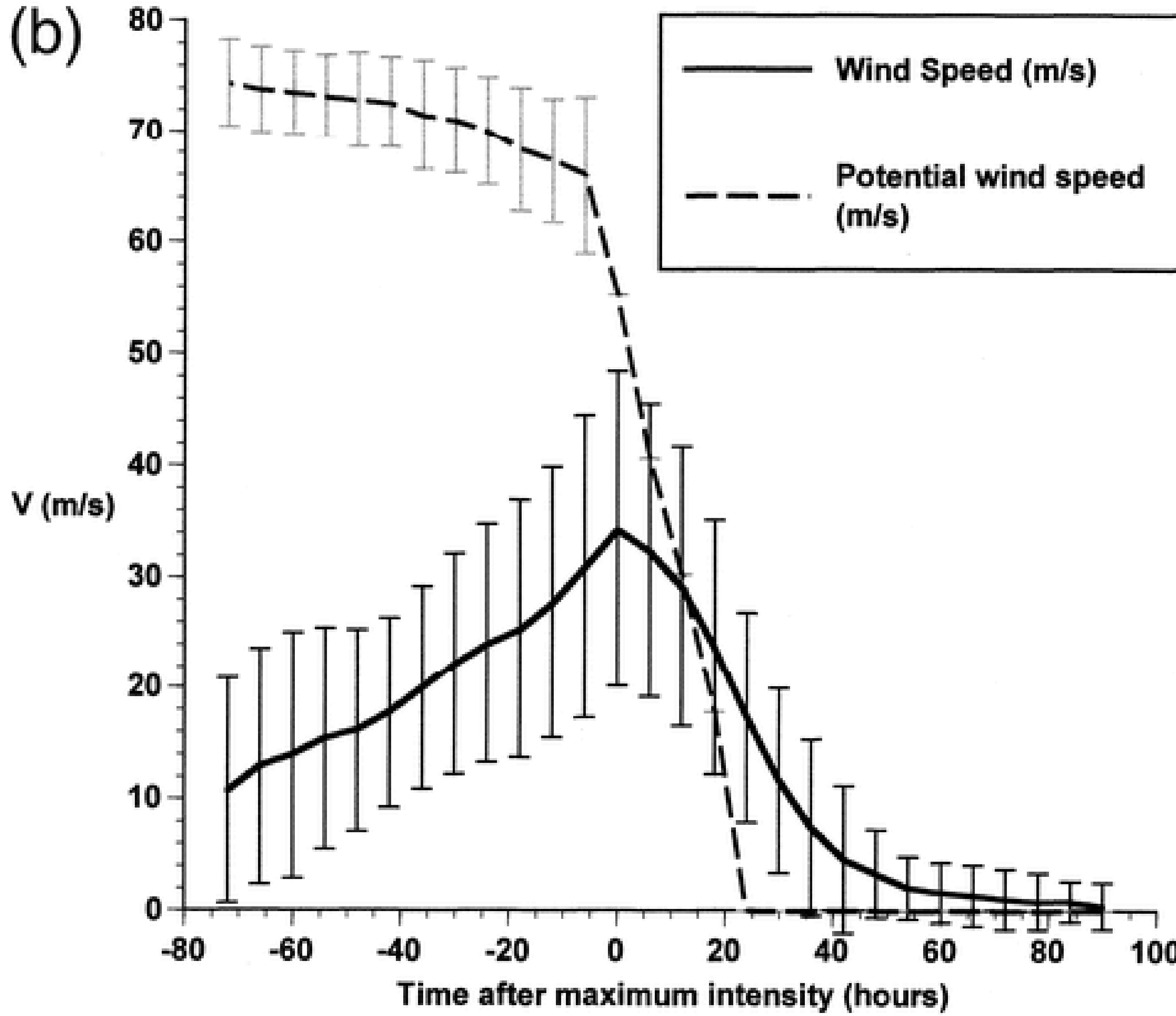
CDF of normalized lifetime maximum wind speeds of Northwest Pacific tropical cyclones of tropical storm strength (18 m s−1) or greater, for those storms whose lifetime maximum intensity was limited by landfall.



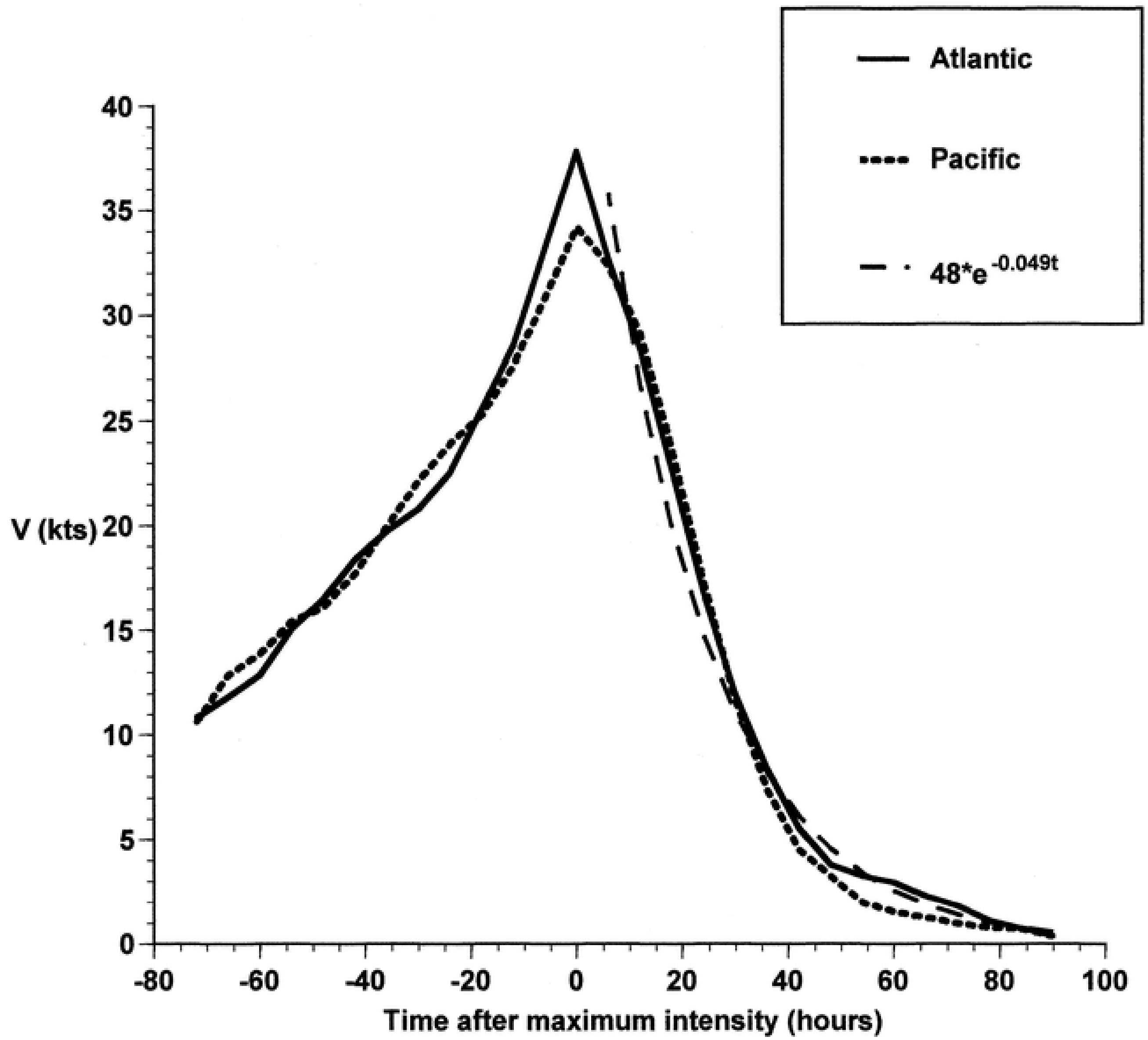
Evolution of Atlantic storms whose lifetime maximum intensity was limited by landfall



Evolution of Pacific storms whose lifetime maximum intensity was limited by landfall



Composite evolution of landfalling storms



12.811 Tropical Meteorology
Spring 2011