

14.01 Principles of Microeconomics Fall 2018
Final Exam Solutions

Long Question 1: Trade (40 minutes)

For this question, it is okay to have non-integer answers

Steel manufacturing in the US requires only one input: labor. Steel is produced by a representative firm under perfect competition according to the production function

$$S(L) = \frac{1}{2}L$$

Throughout this problem assume that the wage in the United States is 1.

- (1) (4 minutes) Suppose that United States can freely import and export steel. Foreign supply of steel is supplied with perfect elasticity at a price of 1. How much steel is produced in the US?

Solution: Zero. The US does not produce any steel. Note that under perfect competition, we must have $w = p \cdot MPL \implies 1 = \frac{p}{2} \implies p = 2$. However, the world price of steel is 1, so companies will choose to import steel rather than buying it from domestic producers.

Car manufacturers use steel and labor to make cars. Both steel and labor are variable inputs and cars are produced by a representative firm under perfect competition according to the production function

$$F(L, S) = (\sqrt{L} + \sqrt{S}).$$

- (2) (10 minutes) Now suppose the price of steel is p_s . Solve for the supply function of US car manufacturers (quantity supplied as a function of the price of cars, p , and the price of steel p_s).

Solution: Since both inputs are variable, the choice of labor and steel must satisfy

$$\frac{1}{p_s} = \frac{\frac{1}{2}L^{-\frac{1}{2}}}{\frac{1}{2}(S)^{-\frac{1}{2}}} = \left(\frac{S}{L}\right)^{\frac{1}{2}} \implies \sqrt{L} = p_s\sqrt{S} \implies L = p_s^2 S$$

The least-cost way of producing q cars is given by

$$q = \sqrt{L} + \sqrt{S} = \sqrt{S} + p_s\sqrt{S} = (1 + p_s)\sqrt{S}$$

so that

$$S(q) = \frac{q^2}{(1 + p_s)^2}, \quad L(q) = \frac{p_s^2 q^2}{(1 + p_s)^2}$$

Supply curve is given by $p = MC$, subject to the shutdown condition $p \geq AVC$. Note that the cost function is given by

$$C(q) = wL(q) + p_s S(q) = \frac{p_s^2 q^2}{(1 + p_s)^2} + \frac{p_s q^2}{(1 + p_s)^2} = \frac{p_s q^2}{(1 + p_s)^2} (1 + p_s) = \frac{p_s q^2}{(1 + p_s)}$$

so that

$$MC(q) = \frac{2qp_s}{1 + p_s}, \quad AVC(q) = \frac{qp_s}{(1 + p_s)}$$

Note that we always have $p = MC(q) > AVC(q)$ so that the supply curve is given by

$$p = \frac{2qp_s}{1 + p_s} \implies q = \frac{p(1 + p_s)}{2p_s}$$

- (3) (3 minutes) Assume the US freely trades steel, where the world price of steel is 1 as before. What is the supply curve for cars?

Solution: Using the equations from the previous exercise with $w = 1$ and $p_s = 1$, we get the inverse supply curve $p = q$

- (4) (8 minutes) Suppose that the supply of cars from foreign producers is perfectly elastic at $p = 100$ (the world price of cars is 100), and the US demand for cars is $Q = 1080 - 10P$.

- (a) (4 minutes) Supposing that there is free trade in the car market as well as the steel market, calculate the equilibrium price and quantity of cars consumed (purchased) in the United States. Does the US import or export cars and how many does the US import or export?

Solution: Since the US is open to trade, the equilibrium price of cars is 100 and the quantity of cars consumed domestically is 80. Now, at a price of 100, US producers make 100 cars, so the US exports 20 cars.

- (b) (4 minutes) Calculate consumer and producer surplus under trade.

Solution: Consumer surplus is given by $\frac{1}{2}(108 - 100)(80) = 320$, while producer surplus is given by $\frac{1}{2}(100)(100) = 5000$

- (5) (15 minutes) The US government is unhappy with steel imports and decides to impose a 200 percent tariff on imported steel so that the price of *imported* steel is now 3 when importing from abroad. (Continue to assume that the US domestic steel market operates in perfect competition with production function $S(L) = \frac{1}{2}L$)

- (a) (2 minutes) What is the price of domestic steel? Will car manufacturers choose to use domestic or foreign steel?

Solution: Since domestic steel is under perfect competition, the price of domestic steel will be $p_s = 2$. This is cheaper than imported steel, which has a price of 3 because of the tariff. Car manufacturers will choose to exclusively use domestic steel.

- (b) (5 minutes) Calculate the new equilibrium in the US market for cars, continuing to assume that cars are traded freely at a world price of 100. Does the US still export cars?

Solution: Since cars are still under free trade, the equilibrium price is still 100 and the equilibrium quantity is still 80. To figure out if the US exports or imports cars, we derive the new supply function of US manufacturers. Using the formula from part 2, we have that the new inverse supply curve is given by

$$p = \frac{4}{3}q$$

At a world price of 100, US car manufacturers produce 75 cars. The US is now importing 5 foreign cars.

- (c) (4 minutes) What happens to the consumer and producer surplus in the market for cars? (You can describe the changes qualitatively, you do not need to calculate the numbers)

Solution: Consumer surplus doesn't change because the price for consumers doesn't change. However, it is now more costly for producers to make cars, so producer surplus decreases.

- (d) (4 minutes) What does this exercise suggest about the costs of implementing tariffs on intermediate goods such as steel?

Solution: This exercise suggests inefficiencies can be greater when putting a tariff on an intermediate good rather than a final good because the cost of the tariff gets passed on through the rest of the supply chain. The exercise also suggests that in the case of intermediate goods, helping out intermediate good producers will come at the cost of hurting the competitiveness of other producers in the country.

Long Question 2: Redistribution and Efficiency (40 minutes)

Consider an economy with two types of individuals: skilled and unskilled workers. The only difference between the two is that the skilled have a higher hourly wage $w_s = 40$ than the unskilled do, $w_u = 10$. Suppose that there are 400 unskilled and 100 skilled workers in this economy.

Suppose that each individual has a utility function over consumption (c) and leisure (l) of the following form:

$$U(c, l) = \ln(c) + 2\ln(l)$$

where $l \in [0, 24]$.

- (a) (3 minutes) Write down the individual's budget constraint in terms of consumption and leisure. Draw the budget constraints for the skilled and unskilled workers in the same graph with leisure on the x -axis.

Solution: Individual j 's budget constraint is: $c + w_j l = 24w_j$ where $j = s, u$. (Need to add graphs)

- (b) (7 minutes) Solve for each individual's optimal leisure, labor, and consumption choice.

Solution: We can solve for the optimum by setting the marginal rate of substitution between c and l equal to their price ratio:

$$\frac{\partial U / \partial l}{\partial U / \partial c} = \frac{2c}{l} = w_j$$

Plugging back into the budget constraint and rearranging, we find $l = 16$ and $c = 8w_j$.

Unskilled individuals have wage $w_u = 10$, enjoy $l_u^ = 16$ hours of leisure, work $h_u^* = 24 - 16 = 8$ hours, and consume $c_u^* = 8 \times 10 = 80$. Skilled individuals have wage $w_s = 40$, enjoy $l_s^* = 16$ hours of leisure, work $h_s^* = 8$ hours, and consume $c_s^* = 320$.*

- (10 minutes) Now suppose that the government wants to redistribute from the skilled to the unskilled workers. It levies an income tax which collects 20% of each skilled worker's earnings and then uses the tax revenue to give an equal amount (lump-sum transfer) T to each unskilled worker. So only the skilled workers are taxed and only the unskilled workers receive the transfer.

- (a) (2 minutes) On the same set of axes, draw the new budget constraints faced by the two types of individuals.

Solution: For the skilled workers, the presence of the tax modifies the budget constraint to $c = \frac{4}{5}w(24 - l)$. This is the same budget constraint as an individual with wage $\frac{4}{5}w$ would face. In other words, imposing this tax is "like" reducing each individual's wage by 20%. Add graph.

For the unskilled workers, the lump-sum rebate is a pure income effect. We have an upward parallel shift of the budget constraint. However, it is impossible to consume more than 24 hours of leisure so the constraint becomes a flat vertical line at $l = 24$ for $c \leq T$

- (b) (6 minutes) Solve for each skilled individual's new optimal leisure, labor, and consumption. Does labor supply change? What is the intuition behind this result?

Solution: We can use the same solution from above. Instead of the wage $w_s = 40$ though, we'll have the (after tax) wages $w'_s = 32$. Thus skilled individuals enjoy $l_s^{**} = 16$ hours of leisure, work $h_s^{**} = 8$ hours again, but consume $c_s^{**} = 0.8 \times 320 = 256$.

Labor supply does not depend on the wage and so does not change. This happens because the income and substitution effects go in the opposite direction and are exactly equal in magnitude so they cancel each other out. If the wage decreases, because of the income effects individuals want to consume less leisure (it's a normal good), however the price of leisure which is the wage has decreased, so by the substitution effect leisure increases. In this case, these effects exactly cancel out.

- (c) (2 minutes) Compute the total tax revenue collected by the government from taxing the skilled individuals.

Solution: The total tax revenue collected by the government is given by 20% of income times the number of skilled individuals in the economy i.e.

$$R = 100 \times 0.2 \times 40 \times 8 = 6400$$

3. (a) (4 minutes) Suppose that for every tax dollar collected, 6.25 cents are lost due to administrative costs. Suppose that the government sets T so that it spends what it collects (the government balances its budget). How large is T ?

Solution: The government wants to balance the budget, so it sets:

$$400T = 0.9375R \Rightarrow T = 15$$

- (b) (10 minutes) Solve for each unskilled individual's new optimal leisure and consumption. Compare the results to the ones before the policy was enacted.

Solution: The constraint of the unskilled workers is now

$$c + wl = 24w + 15$$

The tangency condition yields:

$$\frac{\partial U / \partial l}{\partial U / \partial c} = \frac{2c}{l} = w_u = 10$$

the same as before. Substituting into the budget constraint we have:

$$l_u^{**} = 16 + \frac{30}{30} = 17$$

$$c_u^{**} = 5 \times 17 = 85$$

Since the household has a pure income effect, as mentioned earlier it enjoys more leisure and higher consumption.

- (c) (6 minutes) Now suppose that the government wants to maximize a utilitarian social welfare function. Would it prefer setting no taxes, as in part 1 of this section, or would it prefer to set an income tax + lump-sum transfer scheme as described in part 2 of this exercise?

Hint: You can have non integer answers.

Solution: A utilitarian social planner tries to maximize the sum of the utilities of unskilled and skilled individuals in society. When there are no taxes or transfers the utility of unskilled individuals is

$$U^nt_u = \ln 80 + 2\ln 16 \approx 9.927$$

and skilled individuals get a utility of

$$U^nt_s = \ln 320 + 2\ln 16 \approx 11.313,$$

which leads to a utilitarian welfare function with the following value:

$$W(\text{NoTaxes}) = 400 \times 9.927 + 100 \times 11.313 = 5102.14$$

Similarly, when there is an income tax of $\frac{1}{5}$ on skilled workers and the government makes an equal lump-sum transfer $T = 15$ to each unskilled individual, unskilled workers get a utility of

$$U^t_u = \ln 85 + 2\ln 17 \approx 10.109$$

and skilled individuals get a utility of

$$U^t_s = \ln 256 + 2\ln 16 \approx 11.09,$$

which leads to a utilitarian welfare function with the following value:

$$W(\text{Taxes}) = 400 \times 10.109 + 100 \times 11.09 = 5152.6$$

This shows that a utilitarian social planner prefers to have taxes.

True/False Question (20 minutes)

For each of the following statements, write whether it is True or False, and justify your answer. Points will be given based on your explanation.

1. (5 minutes) Consider a labor market in which there is a monopsony. Setting a binding minimum wage will always lead to an increase in employment.

Solution: False, a binding minimum wage could either increase or decrease labor. If the minimum wage is below the competitive wage (i.e. the competitive wage is the equilibrium wage when the monopsonist behaves as a price taker), then employment would increase. However, if the minimum wage is set above the competitive wage, then it could lead to a decrease in employment.

2. (5 minutes) BestMovies Inc. is the only movie theatre in town, so it acts as a monopolist. There are two groups of consumers: retirees and students. BestMovies can set a different price for each of these two groups. The inverse demand for movies of retirees is $p = P_r^D(q)$, and the inverse demand of students is $p = P_s^D(q)$. Suppose that the demand of retirees is higher than that of students: $P_r^D(q) > P_s^D(q) \forall q$. Therefore, BestMovies must optimally set a higher price for retirees than for students.

Solution: False, the monopolist will set the profit maximizing prices for the two groups of consumers based on their respective demand elasticities, not based on which demand is higher. It is possible that the demand curve of retirees is more elastic than that of students, so in this case the price for retirees will be lower even though their demand is higher.

3. (5 minutes) The consumption of fossil fuels produces externalities by polluting the environment. Therefore, an organization like OPEC that tries to promote collusion among oil producers could improve total social surplus.

Solution: True, the fact that fossil fuels produce externalities means that the surplus maximizing price should be higher than the competitive price. By promoting collusion between oil producers, a cartel like OPEC will tend to increase the price of oil. Therefore, in this case it is possible for collusion to increase total surplus, as the increase in prices will bring the price closer to the social optimum (i.e. the higher markup would be acting as a Pigouvian tax).

4. (5 minutes) Consider the market for roses on Valentine's day. Suppose that every couple in the US will buy exactly one rose regardless of the price, so the domestic demand is perfectly inelastic. The US is an importer of roses. If the government sets an import tariff, there will be no efficiency loss since the quantity consumed will not change.

Solution: False, although the quantity consumed will not change, domestic production of roses will increase. The cost of production of these roses (i.e. the roses that weren't produced at the international price, but are being produced with the tariff) is higher than the import price, so this creates a DWL. That is, it is inefficient for the US to produce roses if they could be imported at a cheaper price.

Long Question 3: Savings and Uncertainty (40 minutes)

Suppose that households live for two periods $t = 1, 2$ and die at the end of period 2. They have wealth in the first period $W > 0$ but no wealth in the second period. Their utility over consumption in the first and second period is given by

$$U(c_1, c_2) = \sqrt{c_1} + \beta\sqrt{c_2}$$

where c_1 is consumption in period 1, c_2 is consumption in period 2 and $\beta \in (0, 1]$ is a preference parameter. Buying 1 unit of consumption costs \$1 in both periods. The households can save by investing in a safe asset at the market interest rate r . Assume that the household gets no utility from leaving any money behind after death.

1. (4 minutes) Write down the household budget constraints for periods 1, 2. Then, write an expression for the household's intertemporal budget constraint in terms of today's dollars.

Solution: The budget constraint in period 1 is:

$$c_1 + s = W$$

The budget constraint in period 2 is

$$c_2 = (1 + r)s$$

Combining the two constraints by substituting s , we have

$$c_1 + \frac{c_2}{1 + r} = W$$

2. (8 minutes) How much of its income will the household consume in each of the two periods and how much will it save given the interest rate r ?

Solution: From the tangency condition, we have

$$\frac{2\sqrt{c_2}}{2\beta\sqrt{c_1}} = 1 + r \Rightarrow c_2 = \beta^2(1 + r)^2 c_1$$

Substituting for c_2 in the budget constraint yields:

$$c_1 + \frac{\beta^2(1 + r)^2 c_1}{1 + r} = W \Rightarrow c_1 = \frac{W}{1 + \beta^2(1 + r)}$$

and this implies that

$$c_2 = \frac{W(1 + r)^2 \beta^2}{1 + (1 + r)\beta^2}$$

Savings are equal to

$$s = W - c_1 = W - \frac{W}{1 + \beta^2(1 + r)} = W \frac{\beta^2(1 + r)}{1 + \beta^2(1 + r)}$$

3. (4 minutes) Does increasing β increase or decrease savings? What is the intuition behind this result?

Solution: Taking the partial derivative yields:

$$\frac{\partial s}{\partial \beta} = \frac{2\beta W(1+r)}{(1+\beta^2(1+r))^2} > 0$$

so a higher β implies higher savings. The intuition behind this result is that β is a preference parameter capturing the relative preference for second period consumption compared to first period. A higher β implies relatively higher utility from second period consumption and so the household wants to save more in order to consume more in the second period.

4. (4 minutes) Does a higher interest rate increase or decrease savings? Provide the intuition for this result.

Solution: Taking the partial derivative yields:

$$\frac{\partial s}{\partial r} = \frac{\beta^2 W}{(1+\beta^2(1+r))^2} > 0$$

Increasing the interest rate increases savings and decreases consumption in period 1. The intuition behind this result is that the substitution effect is larger than the income effect (in absolute value), so savings increase.

5. (20 minutes) Now suppose that $\beta = 1$ and that there are two types of households: rich and poor. They differ only in their first period wealth which is $W = k$ for the rich and $W = \frac{3k}{4}$ for the poor for some $k > 0$. Now, the households can only save by investing in a risky asset (the safe asset option of the previous questions is not available now). This asset requires an investment of exactly $\frac{k}{2}$ and will yield gross second period income equal to k with probability $1/3$ and gross second period income of 0 with probability $2/3$. Note that households can only buy exactly one unit of the risky asset. Households seek to maximize their expected utility of consumption.

- (a) (8 minutes) Will the poor households choose to invest in the risky asset in order to transfer resources to the next period? Why or why not?

Solution: If the poor households don't invest in the risky asset, they only consume their wealth in the first period and nothing in the second period. Their expected utility is

$$U_P^{NS} = \sqrt{3k/4} + \sqrt{0} = \sqrt{3k/4}$$

If they invest in the risky asset, then they consume

$$c_1 = \frac{3k}{4} - \frac{k}{2} = \frac{k}{4}$$

in the first period and their expected utility is:

$$EU_P^S = \sqrt{\frac{k}{4}} + \frac{1}{3}\sqrt{k} + \frac{2}{3}\sqrt{0} = \frac{\sqrt{k}}{2} + \frac{\sqrt{k}}{3}$$

Comparing the two utilities we have:

$$U_P^{NS} > EU_P^S \quad \frac{\sqrt{3k}}{2} > \frac{\sqrt{k}}{2} + \frac{\sqrt{k}}{3} \quad \frac{1}{3} < \frac{\sqrt{3}-1}{2} \Rightarrow 25 < 27$$

So the poor households will choose not to invest in the risky asset as their utility when they do so is lower than when they don't.

- (b) (12 minutes) Will the rich households choose to invest in the risky asset? Is your answer different from the previous subquestion? Provide an intuition for why or why not.

Solution: If the rich households don't invest in the risky asset, they only consume their wealth in the first period and nothing in the second period. Their expected utility is

$$U_R^{NS} = \sqrt{k} + \sqrt{0} = \sqrt{k}$$

If they invest in the risky asset, then they consume

$$c_1 = k - \frac{k}{2} = \frac{k}{2}$$

in the first period and their expected utility is:

$$EU_R^S = \sqrt{\frac{k}{2}} + \frac{1}{3}\sqrt{k} + \frac{2}{3}\sqrt{0} = \sqrt{\frac{k}{2}} + \frac{\sqrt{k}}{3}$$

Comparing the two utilities we have:

$$U_R^{NS} < EU_R^S \quad \sqrt{\frac{k}{2}} + \frac{\sqrt{k}}{3} > \sqrt{k} \quad \frac{1}{\sqrt{2}} > \frac{2}{3} \Rightarrow 18 > 16$$

So the rich households will choose to invest in the risky asset as their utility when they do so is higher than when they don't. Both household types are risk-averse but the risky asset requires an investment of a higher proportion of the poor households' income than the rich households' income which results in a lower period 1 consumption and thus makes the plan riskier and thus less appealing for them.

Long Question 4: Monopoly and Oligopoly (40 minutes)

For this question, it is okay to have non-integer answers.

Consider the perfectly competitive market for lobsters in the village of Rockport and suppose that there are many market stalls selling lobsters around the village. Assume that each of them faces a cost function of $C(q) = 4 + q^2$ if they decide to sell any lobsters, otherwise their cost is 0. Consider the market in the long run and the case where there is free entry. The inverse demand for lobsters is given by $p(Q) = 10 - Q$.

1. (2 minutes) What is the average total cost? And the marginal cost?

Answer. The average and marginal costs are both given by:

$$ATC(q) = \frac{4}{q} + q$$
$$MC(q) = 2q$$

2. Considering the market in the long run:

- a) (4 minutes) What is the supply function for a single market stall?

Answer. To obtain each firm's supply function, we use $p = MC(q)$, which implies $q = \frac{p}{2}$. The firm will be willing to supply a positive amount of lobsters if $p \geq \min_q ATC(q)$. The minimum of the ATC is achieved at $q = \sqrt{4} = 2$ and $\min_q ATC(q) = 2\sqrt{4} = 4$. Hence, each firm's supply curve is given by:

$$q^s(p) = \begin{cases} \frac{p}{2} & \text{if } p \geq 4 \\ 0 & \text{otherwise} \end{cases}$$

- b) (2 minutes) What is the market supply function?

Answer. From the results above, the market supply function is flat at $p = 4$ —since this is a perfectly competitive market with free entry.

- c) (6 minutes) What is the price in equilibrium? How many lobsters are sold? How many market stalls will there be?

Answer. Because of the flat supply curve, the equilibrium price is given by $p^e = 4$ and the equilibrium quantity is $Q^e = 10 - 2\sqrt{4} = 6$. Since every market stall produces $q^e = p^e/2 = 2$, there is a total of $N^e = 6/2 = 3$ market stalls.

Now assume that there is only one market stall, also known as the Lobster Roll, selling lobsters and that it has the same cost function above.

3. (4 minutes) What are the equilibrium price and quantity under monopoly?

Answer. The monopolist solves the problem: $\max_Q P(Q)Q - C(Q)$. The FOC yields $Q^m = \frac{10}{4} = 2.5$ and, therefore, $p^m = 10\left(1 - \frac{1}{4}\right) = 7.5$

4. (8 minutes) Suppose that another firm enters the lobster market (Lobster King), that has the same cost function above and engages in Cournot competition with Lobster Roll. Denote the quantity produced by Lobster King and Lobster Roll as q_K and q_R , respectively. What is the equilibrium price? What is the quantity produced by each firm?

Answer. The demand function can be rewritten as $P(q_K, q_R) = 10 - (q_K + q_R)$. Then, each firm is maximizing:

$$\max_{q_i} P(q_i, q_j)q_i - C(q_i)$$

The FOC gives the best response function for each firm i :

$$q_i^{BR}(q_j) = \frac{10 - q_j}{4}$$

By symmetry, each market stall is going to sell the same number of lobsters, so $q^c = q_K = q_R$. Rearranging, $q^c = \frac{10}{5} = 2$ and, hence, $p^c = 10 \left(1 - \frac{2}{5}\right) = 6$ and $Q^c = 4$.

5. (10 minutes) Suppose now that both market stalls form a cartel. That is, they agree on the quantity that they will produce and the price they will charge to maximize joint profits. What is the new equilibrium price and quantity? How many lobster does each market stall produce?

Answer. Notice that, even though firms have the same cost function, this is not linear in the quantity produced. Hence, this equilibrium will not be equal to the monopoly equilibrium. When both firms form a cartel, the maximization problem becomes:

$$\max_{q_R, q_K} (10 - q_R - q_K)(q_R + q_K) - (4 + q_R^2) - (4 + q_K^2)$$

Taking the FOC with respect to each quantity, we obtain that $q_R = (5 - q_K)/2$ and $q_K = (5 - q_R)/2$. By symmetry $q = q_R = q_K$, so $q^a = 5/3$, $Q^a = 10/3$ and $p^a = 20/3$.

6. (4 minutes) Rank each of the models (perfect competition, monopoly, Cournot and cartelization) according to consumer surplus and explain the intuition behind it. Explain the intuition. You do not need to make any calculations for this part.

Answer. We expect the ranking to be $PC > Cournot > Cartel > M$. The more competition, the higher the quantity and the lower the price; so it is better for consumers. The cartel is better for consumers than monopoly because the cost function is strictly convex and they can split the quantity between both firms so the cost of producing the same quantity is lower under the cartel.

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14.01 Principles of Microeconomics
Fall 2018

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