14.01 Fall 2010
Problem Set 4

1. (27 points) For each of the following production functions, sketch a representative isoquant (2 points). Calculate the marginal product for each input, and indicate whether each marginal product is diminishing, constant, or increasing (3 points). Calculate the marginal rate of technical substitution for each function (2 points). Also indicate whether the function exhibits constant, increasing, or diminishing returns to scale (2 points).

   (a) $F(L, K) = LK^3$

   (b) $F(L, K) = L + 3K$

   (c) $F(L, K) = (\min\{L, K\})^{\frac{1}{3}}$

2. (21 points) Consider the production function $f(L, K) = 2L^{\frac{1}{4}}K^{\frac{1}{4}}$.

   (a) (15 points) Find the associated (long run) total, average, and marginal cost curves.

   (b) (6 points) Sketch the total, average, and marginal cost curves.

3. Problem removed due to copyright restrictions. This content is presented in audio form in the Solution Video for Problem Set 4, Problem 3.

4. (28 points) You run a cost-minimizing firm with production function $f(L, K) = [\min\{L, K\}]^{\frac{1}{3}}$, where L is labor and K is capital. Assume that you are a price-taker in the input markets: you pay w for each unit of labor you hire and r for each unit of capital (where w and r are set exogenously), and face no costs other than those from labor and capital.

   (a) (15 points) Assuming that you can freely choose both labor and capital (i.e., the "long run problem"), derive expressions for your cost-minimizing conditional input demands, $L^*(r, w, Q)$ and $K^*(r, w, Q)$. Confirm that the conditional input demand functions are "homogeneous of degree zero" in w and r; that is,

$$\begin{cases} L^*(tr, tw, Q) = L^*(r, w, Q) \\ K^*(tr, tw, Q) = K^*(r, w, Q) \end{cases} \quad \text{for all } t > 0$$

   (b) (8 points) What will happen to your conditional demand for labor if there is an increase in the wage rate, assuming that r and Q remain the same? Explain in one sentence why your answer makes intuitive sense.

   (c) (5 points) Use your answers from (a) to write down an expression for your total cost function $TC(r, w, Q)$. Is this function "homogeneous of degree one" in w and r; that is, does $TC(tr, tw, Q) = t \cdot TC(r, w, Q)$?