MIT 14.11: Math Module 0
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Math Preliminaries

In this session we will cover the basic mathematical notation and theory required to understand the game theory we will teach in this course. If you have had a few college level math or economics courses, most of the material presented will be repetitive. Most of this material can also be found in the Appendix of Osborne's text.

1 Numbers

The **natural numbers**($\mathbb{N}$) are the positive whole numbers. $\mathbb{N} = \{1, 2, 3, ...\}$

The **integers** ($\mathbb{Z}$) are the natural numbers, the negatives of the natural numbers, and zero.
$\mathbb{Z} = \{..., -3, -2, -1, 0, 1, 2, 3, ...\}$

The **rational** numbers ($\mathbb{Q}$) are the numbers that can be expressed as a quotient of two integers.
E.g.: $\frac{1}{3} \in \mathbb{Q}$, $\frac{22}{7} \in \mathbb{Q}$, etc.

The **real** numbers ($\mathbb{R}$) are all the rational and irrational numbers, or, all numbers that represent a quantity along a continuous line.
E.g.: $\frac{1}{3} \in \mathbb{R}, \pi \in \mathbb{R}, e \in \mathbb{R}$

2 Sets

A **set** is a collection of distinct entities or objects regarded as a unit.

A set is **finite** if all of its members can be listed. Usually we list members in curly braces: $\{A = a_1, a_2, a_3\}$.

If an object belongs to a particular set, we write $a_1 \in A$.

If every member of some set B is also a member of A, B is a **subset** of A. Note that every set is a subset of itself, and that for two sets to be equal, both must be a subset of the other.
E.g.: $B = \{a_1, a_2\} \subset A$.

The set of all members of two sets A and B is the **union** of the two sets, denoted $A \cup B$.
E.g.: $A \cup B = \{a_1, a_2, a_3\}$.

The set of all members both in A and in B is the **intersection** of the two sets, denoted $A \cap B$.
E.g.: $A \cap B = \{a_1, a_2\}$.

A **partition** of a set A is the collection of subsets of A such that every member of A appears in exactly one of the sets.
E.g.: A has five possible partitions:

- $\{\{a_1\}, \{a_2\}, \{a_3\}\}$
- $\{\{a_1, a_2\}, \{a_3\}\}$

- $\{\{a_1, a_3\}, \{a_2\}\}$
- $\{\{a_1\}, \{a_2, a_3\}\}$
- $\{\{a_1, a_2, a_3\}\}$

A set is **infinite** if we cannot list all of its members. To describe an infinite set, we describe a property that characterizes its members. Suppose we wish to describe a set C made up of the positive integers. We can write:

$$C = \{c \in C : c \in \mathbb{Z} \text{ and } c > 1\}$$

Which is read "the set of all c in C such that c is an integer and c is greater than 1."

3 Functions

A **function** is a relationship or expression relating one or more variables.

The **domain** of a function is the set of possible values of the input of a function.

The **range** of a function is the set of possible values of the output of a function.

E.g.: The function $f : A \to D$ is defined $f(a_1) = 1, f(a_2) = 2, f(a_3) = 3$. We say the domain of f is the set A and the range of f is the set $D = \{1, 2, 3\}$.

4 Probability

The **probability** of an event taking place is a measure of how likely it is for that event to take place.

A **probability distribution** is an assignment of probabilities to events. In any probability distribution, the sum of the probabilities of all possible events must be 1.

If the probability that two events, E and F both occur is $\Pr(E \cap F) = \Pr(E)\Pr(F)$, then events E and F are **independent**.

If the probability that two events, E and F both occur is $\Pr(E \cap F) = \Pr(E) + \Pr(F)$, then events E and F are **mutually exclusive**.

A **lottery** is a probability distribution over outcomes. The elements of a lottery correspond to probability that a certain outcome arises out of a given state.

If the events of a probability distribution are numerical (E.g. payoffs to players in a strategic game), the **expected value** of the lottery is the average value of the lottery. So if a player gets payoff x_1 with probability p_1, payoff x_2 with probability p_2 and so on, up to payoff x_n with probability p_n the expected value of this lottery is:

$$E = \sum_{i=1}^{n} p_n x_n.$$

5 Conditional Probability

Conditional probability of an event E is the probability of E occurring given that another event, F is known to have occurred.

The **prior belief** is the initial probability that an event E will occur.

The **posterior belief** is the modified probability that E will occur, given another event F is known to have occurred.

We calculate the probability $\Pr(E|F)$ of E **conditional on** F by **Bayes Rule** :

$$Pr(E|F) = \frac{Pr(E \cap F)}{Pr(F)}$$

Given n mutually exclusive events $F_1, ..., F_n$, the **law of total probability** gives the total probability that an event E will occur:

$$Pr(E) = Pr(E|F_1)Pr(F_1) + ... + Pr(E|F_n)Pr(F_n)$$

Often we will not be given all the information needed to calculate $\Pr(E|F)$ in the formulation above. The law of total probability gives us conditional probability in terms of $\Pr(E)$ and $\Pr(F|E)$:

$$Pr(E|F) = \frac{Pr(E)Pr(F|E)}{Pr(E)Pr(F|E) + Pr(\text{ not } E)Pr(F|\text{ not } E)}$$

Some of these topics will be integral to our use of game theory, while others will be used less frequently. If you have *any* questions about these preliminary math topics, please do not hesitate to ask Zoë. Understanding the math behind the theory is the only way to understand how our insights and explanations of social behavior are *ultimate*. No question is too basic to ask.

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