14.12 Game Theory Midterm 2
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Instructions. This is an open book exam; you can use any written material. You may use a calculator. You may not use a computer or any electronic device with wireless communication capacity. You have one hour and 20 minutes. There are three questions. The breakdown of points within each question is specified below. Please answer each question in a separate blue book. Be sure to put your name on each book. Good luck!

1. This problem deals with the game depicted below, where θ is some parameter.

	A	B	C
X	6, 2θ+2	0,0	0,-θ
Y	0,0	2,θ	9,-θ

(a) (10 points) If $\theta = -1$, find all of the SPE of the 8-times repeated version of this game. Explain how you know you found them all.

(b) (15 points) Now suppose instead that $\theta = 2$ and consider a T-times repeated version of this game. For each of the following, state whether the proposed strategies could be played in the **first round** of a subgame perfect equilibrium or not. If they can, describe a full set of SPE strategies that implement it.

1. T=2, and (Y,C)
2. T=2 and (X,B)
3. T=3 and (X,C)

(c) (10 points) Now suppose that θ is known by player 2 but not by player 1, who believes that there is a 1/2 chance that $\theta = -1$ and a 1/2 chance that $\theta = 2$. Find and describe all of the Bayesian Nash Equilibria of the game.

2. Alice and Bob jointly own a dollar which they can have only if they agree on a division. In every round, one of the two will be the "offerer" and the other will be the "receiver." At the beginning of round 1, they flip a coin to determine who gets to be offerer. At the beginning of every round after round 1, there is a computer program that generates outcomes $\{S, R, E\}$ with probabilities p_S, p_R and p_E, respectively, where $p_S + p_R + p_E = 1$ and $p_E > 0$. If the computer generates S, the offerer stays the SAME as the in the last round. If the computer generates R, the player who was the RECEIVER in the previous round gets to be the offerer. In any given round:

   - If it is the second round or later and the outcome is E, then the game ends with payoff vector 0.
   - If Alice is the offerer, then
     - Alice offers a division $(1 - x, x)$, where x is the share of Bob,
     - Bob decides whether to accept the offer;
     - if offer is accepted the game ends with payoff vector $(1 - x, x)$; otherwise the game proceeds to the next round.
   - If Bob is the offerer
     - Bob offers a division $(x, 1 - x)$, where x is the share of Alice,
     - Alice decides whether to accept the offer;
     - if offer is accepted the game ends with payoff vector $(x, 1 - x)$; otherwise we proceed to the next round.

   Consider the strategy profile

   s^* : Alice always offers $(1 - x_B, x_B)$ and accepts an offer $(x, 1 - x)$ if and only if $x \geq x_A$.
   Bob always offers $(x_A, 1 - x_A)$ and accepts an offer $(1 - x, x)$ if and only if $x \geq x_B$.

   (a) (10 points) Explain precisely why $x_B = 1$, $x_A = 0$ is not a subgame perfect equilbirium.

   (b) (20 points) Find a subgame-perfect equilibrium of the game, and use the single deviation principle to prove that it is indeed a subgame perfect equilibrium.

3. Consider the following infinitely repeated game.

- There is an Incumbent and a (potential) Entrant.
- Each date t has two stages:
  - At the first stage, the Entrant decides whether to Enter or Stay Out.
  - If the Entrant decides to Enter, it bears an entry cost of $k = 0.1$. The two players then play a standard "Cournot" game:
    * Each of the two firms simultaneously decide on a non-negative output q_E and q_I.
    * The demand curve is given by $p = 1 - Q$, where $Q = q_E + q_I$, so the price of output is given by $p = 1 - Q$. (Notice that we allow negative prices here; but you should be able to ignore that for the sake of this problem.)
    * The marginal cost of production is zero, so the profits are given by:

$$\begin{aligned} \pi_I &= q_I p \\ \pi_E &= q_E p - k. \end{aligned}$$

  - If the Entrant Stays Out, it gets a payoff of 0, and the Incumbent decides on a production level q_I to produce, facing the same demand curve.
- The game is infinitely repeated, with a discount factor $\delta \in (0, 1)$.
- Notice that the entrant bears the cost k in each round in which she enters.

(a) (10 points) Find the subgame perfect equilibrium of the non-repeated version of this game (the game that takes place at any t).

(b) (10 points) Prove that the following strategy profile is a subgame perfect equilibrium for sufficiently high δ, and find the minimal $\widehat{\delta}$ for which it is:

- The Entrant Enters every round.
- Every producer produces $q = \frac{1}{4}$, so long as no producer has ever produced a quantity other than $\frac{1}{4}$.
- If any producer has ever produced a quantity other than $\frac{1}{4}$, "trigger" to playing the subgame perfect equilibrium from part (a) in every period.

(c) (10 points) For $\delta = .9$, is there a subgame perfect equilibrium in which entry is always deterred (i.e., in which the Entrant never Enters)? If so, find one. If not, explain why not.

(d) (5 points) Describe strategies that can implement the same *outcome* as part (b) for some $\delta < \widehat{\delta}$.

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