**14.123 Microeconomics III—Problem Set 3**

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**Instructions.** You are encouraged to work in groups, but everybody must write their own solutions. Each question is 33 points. Good Luck!

1. Problem 3 in Problem Set 2.

2. Bob has just retired and has $w_0$ dollars. His utility from a consumption stream $(c_0, c_1, \ldots)$ is

$$\sum_{t=0}^{n} \delta^t u\left(c_t\right),$$

where $u : R \to R$ is a von Neumann-Morgenstern utility function with constant relative risk aversion $\rho > 1$. For each $t$, he dies in between periods $t$ and $t+1$ with probability $p$, in which case he gets 0 utility.

   (a) Take $n = 1$, and find the optimal consumption stream $c^*$ with $c_0^* + c_1^* \leq w_0$.

   (b) Take $n = \infty$, and find the optimal consumption stream $c^*$ with $c_0^* + c_1^* + \cdots \leq w_0$.

   (c) What would be your answer to part (b) if $\rho = 1$?

   (d) Solve part (c), assuming instead that Bob can get $r_t$ from each dollars saved at $t$, i.e., $w$ dollars saved at $t$ becomes $wr_t$ dollars at $t+1$, where $(r_t)$ is i.i.d. with $r_t > 0$ and $\delta E\left[\log r_t\right] \in (0, 1)$.

3. For any real-valued random variables $X$ and $Y$ and any increasing function $g : R \to R$, prove or disprove the following statements.

   (a) If $X$ first-order stochastically dominates $Y$, then $g\left(X\right)$ first-order stochastically dominates $g\left(Y\right)$.

   (b) If $X$ second-order stochastically dominates $Y$, then $g\left(X\right)$ second-order stochastically dominates $g\left(Y\right)$.

   (c) If $X$ first-order stochastically dominates $Y$, then $X$ first-order stochastically dominates $\alpha X + (1-\alpha) Y$ for every $\alpha \in [0, 1]$.

4. Ann has constant absolute risk aversion $\alpha > 0$ and initial wealth $w$. She can buy shares from two divisible assets that are sold at unit price. One of assets pays a dividend $X \sim N\left(2\mu, \sigma^2\right)$ and the other pays a dividend $Y \sim N\left(\mu, \sigma^2\right)$ where $X$ and $Y$ are independently distributed and $\mu > 1$. She can buy any amount of shares from each asset, and she can keep some of her initial wealth in cash. Find the optimal portfolio for Ann.

# 14.123 Microeconomic Theory III

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