14.124
Holmstrom
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HOMEWORK 3

QUESTION 1

A principal hires an agent at date 0. At date 1, the agent will face one of two tasks, task A or task B. Neither the principal nor the agent knows at date 0 which task the agent will face at date 1. It will be task A with probability p and task B with probability 1−p. The agent will learn the task at date l; the principal will never learn which task the agent faced.

The agent's effort into either task is private information. The agent's cost of effort when performing task A is $c_A(e) = e^2$ and her cost of effort when performing task B is $c_B(e) = ae + be^2$, where a is a parameter that can be positive or negative and b is positive. The principal's benefit from task A is B(e) = e. The principal does not benefit from task B. The principal and the agent are both risk neutral.

The principal and the agent observe a signal of effort $x = e + \varepsilon$, where ε is a noise term with zero mean (given risk neutrality, the specific distribution is unimportant.) The signal x is observed regardless of which task the agent is performing. The principal pays the agent with a linear contract $s(x) = \alpha x + \beta$. The agent's reservation utility is normalized to zero. The fixed payment β can be negative.

a. Set up the program that selects the Pareto optimal parameters α and β given the agent's incentive compatibility and participation constraints. (Note that effort levels are different depending on which task the agent ends up performing).

b. What is the optimal value of α?

c. Suppose the principal can rule out task B. If task B is ruled out, x is identically zero when task B would have come up (task A still comes up with probability p). Will it ever be optimal not to rule out task B?

*QUESTION 2 (from final 2014)

Consider a monopsonistic firm facing a continuum of workers. These workers can be of two types θ_L and θ_H with $0 < \theta_L < \theta_H < 1$. The fraction of workers of each type is p_L and p_H. Workers of type θ_i that are paid wage w and asked to work h hours receive utility

$$U = u(w - \theta_i h),$$

where u is a strictly concave increasing utility function with u(0) = 0. The hours of work h must fall in the (normalized) interval [0, 1]. Both type of workers have u(0) as their opportunity cost of working. The value to the firm from hiring a worker of type θ_i at wage w and hours h is

$$\pi = h/\theta_i - w.$$

The workers know their cost parameter θ_i, the firm does not.

a. What is the profit maximizing first-best contract in this situation (ie. when the firm can identify the two types and offer separate contracts to each). Can this contract be implemented?

b. Set up the program that identifies the profit maximizing second-best solution.

c. Use a diagram to identify binding constraints. Characterize the solution to the second-best program as precisely as you can (utilizing the diagram if you wish). One can give an exact answer.

d. When will the low type not be hired at all?

QUESTION 3.

An agent produces output for a principal according to the production function $y = e + \sigma$, where $e \geq 0$ is the agent's choice of input and σ is a stochastic productivity parameter that takes on the value σ_H with probability p ($0 < p < 1$) and the value $\sigma_L < \sigma_H$ with probability (1-p); $\sigma_i > 0$ for i = L,H.

The principal can only observe the output y, not the input e nor the productivity parameter σ. The agent can observe σ before choosing his input x.

The agent's utility function is $u(m,x) = m - c(x)$, where m is money and c is a strictly convex and increasing cost function with $c(0) = 0$. The principal is risk neutral and values profit (that is the difference between output y and the payment to the agent w). The principal offers the agent a contract w(y), which the agent can reject or accept *after observing the value of* σ. The agent's reservation utility is the same in either state σ and is normalized to 0.

a. Set up the program that maximizes the principals expected profit subject to the agent's incentive compatibility and individual rationality constraints (participation constraints).

b. Show that only one of the individual rationality constraints and one of the incentive compatibility constraints will bind. (You can provide an algebraic or a geometrically based argument.)

c. Assume now that $\sigma \in [0,1]$ is a continuous parameter. Write down a formula for $w(\sigma)$ that implements the first best choice function $e^*(\sigma)$ for the agent.

Hint: For each type σ the agent can be viewed as choosing y rather than e. It is easier to consider y the agent's choice variable.

*QUESTION 4

Consider the following regulation problem. A firm produces a public good with the cost function

$$c(x,\theta) = \theta x^2/2$$

where x is the output and θ is a cost parameter that only the firm knows. The social benefit is $b(x) = x$. The government has to decide on an optimal incentive scheme for the firm. If $p(x)$ is the payment for x, the firm's profit is $p(x) - c(x,\theta)$. The firm always has the option not to produce, which yields profit 0.

a. Suppose the government wants to maximize the sum of social benefits and the firm's profits. Show that in this case there is a simple subsidy scheme that maximizes the government's objective and thus achieves the first-best outcome.

b. Suppose instead that the government is only interested in maximizing the social benefit $b(x)$ net of the payment $p(x)$ to the firm. Assume the cost parameter θ can take two values, $\theta = 1$ and $\theta = 2$ with Prob $(\theta = 1) = p$ and Prob $(\theta = 2) = 1 - p$. Set up a program that solves the government's second-best problem. Draw a diagram that shows the nature of the second-best solution, including the constraints that are binding, the level of firm profit and the second-best distortions in the choice of x.

c. Assume now that θ is continuously distributed on the interval [1,2]. Suppose the government wants to implement the solution $x(\theta) = 2 - \theta$. What payment scheme should it use given the objective in part b?