Global Games

14.126 Game Theory

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Motivation

- Multiple equilibria exist in settings with strategic complementarities
  - Investment/Development
  - Search
  - Bank runs
  - Currency attacks
- Global Games: introducing a certain type of incomplete information leads to a unique equilibrium prediction.

A partnership game

	Invest	Not-Invest
Invest	θ,θ	$\theta - 1, 0$
Not-Invest	$0,\theta - 1$	$0,0$

θ is common knowledge

$\theta < 0$

	Invest	Not-Invest
Invest	θ,θ	θ –1, 0
Not-Invest	0,θ–1	0,0

θ is common knowledge

$\theta > 1$

	Invest	NotInvest
Invest	θ,θ	$\theta - 1, 0$
NotInvest	$0,\theta - 1$	$0,0$

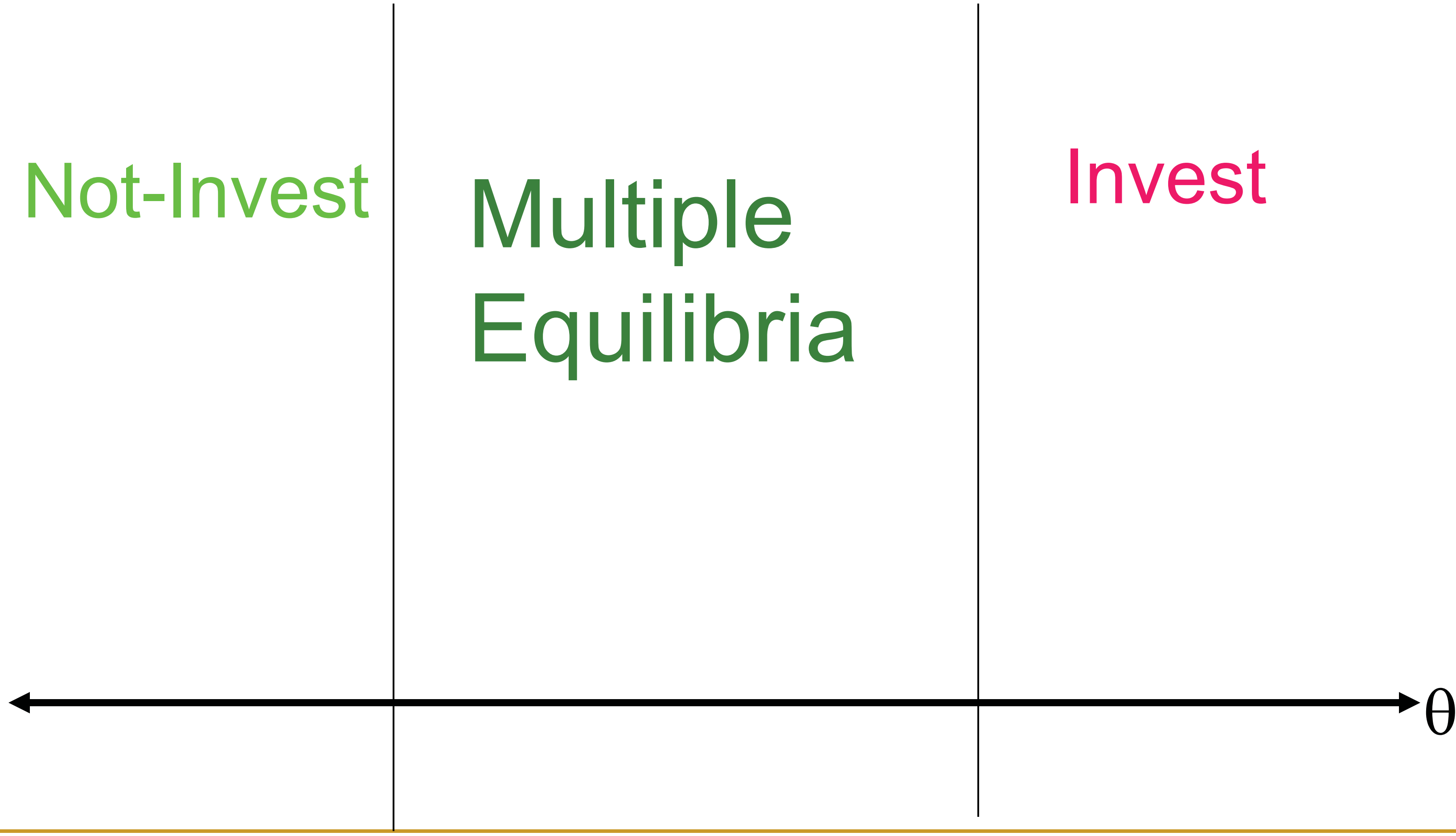
θ is common knowledge

$0 < \theta < 1$

Multiple Equilibria

	Invest	Not-Invest
Invest	θ,θ	$\theta - 1, 0$
Not-Invest	$0,\theta - 1$	$0,0$

θ is common knowledge



θ is not common knowledge

- θ is uniformly distributed over a large interval
- Each player *i* gets a signal

$$x_i = \theta + \varepsilon \eta_i$$

  - (η_1, η_2) is bounded
  - Independent of θ
  - iid with continuous F (common knowledge)
  - $E[\eta_i] = 0$

Recall: Monotone supermodular games

- $G = (N,T,A,u,p)$
- $T = T_0 \times T_1 \times \ldots \times T_n$ ($\subseteq$ R^M)
- A_i compact sublattice of R^K
- $u_i : A \times T \to$ R
  - $u_i(a,.)$: $T \to$ R is measurable
  - $u_i(.\ ,t)$: $A \to$ R is continuous, "bounded", supermodular in a_i, has increasing differences in a and in (a_i,t)
- $p(.|t_i)$ is increasing function of t_i—in the sense of 1st-order stochastic dominance (e.g. p is affiliated).
- Theorem: There exist BNE s^* and s^{**} such that
  - For each BNE s, $s^* \geq s \geq s^{**}$.
  - Both s^* and s^{**} are isotone.

Conditional Beliefs given x_i

$$\theta =_d x_i - \varepsilon\eta_i$$

- i.e. $\Pr(\theta \le \theta' | x_i) = 1\text{-}F((x_i\text{-}\ \theta')/\varepsilon) := G(\theta' | x_i)$

$$x_j =_d x_i + \varepsilon(\eta_j\text{-}\eta_i)$$

- $\Pr(x_j \le x_j' | x_i) = \Pr(\varepsilon(\eta_j - \eta_i) \le x_j' - x_i)$
- $\Pr(\theta \le \theta', x_j \le x_j' | x_i) =$ $\int 1_{\{\theta \le \theta'\}} F((x_j' - \theta)/\varepsilon) dG(\theta | x_i)$ decreasing in x_i because integrand decreasing in θ and $G(\cdot | x_i)$ FOSD $G(\cdot | x_i')$ whenever $x_i \ge x_i'$
- $E[\theta | x_i] = x_i$

Payoffs

	Invest	Not-Inv
Invest	θ	θ -1
Not-Inv	0	0

- Invest > Not-Invest
- $U_i(a_i,a_j,\theta,x)$ is supermodular.
- Monotone supermodular
- There exist greatest and smallest rationalizable strategies, which are
  - Bayesian Nash Equilibria
  - Monotone (isotone)

Monotone BNE

- Best response

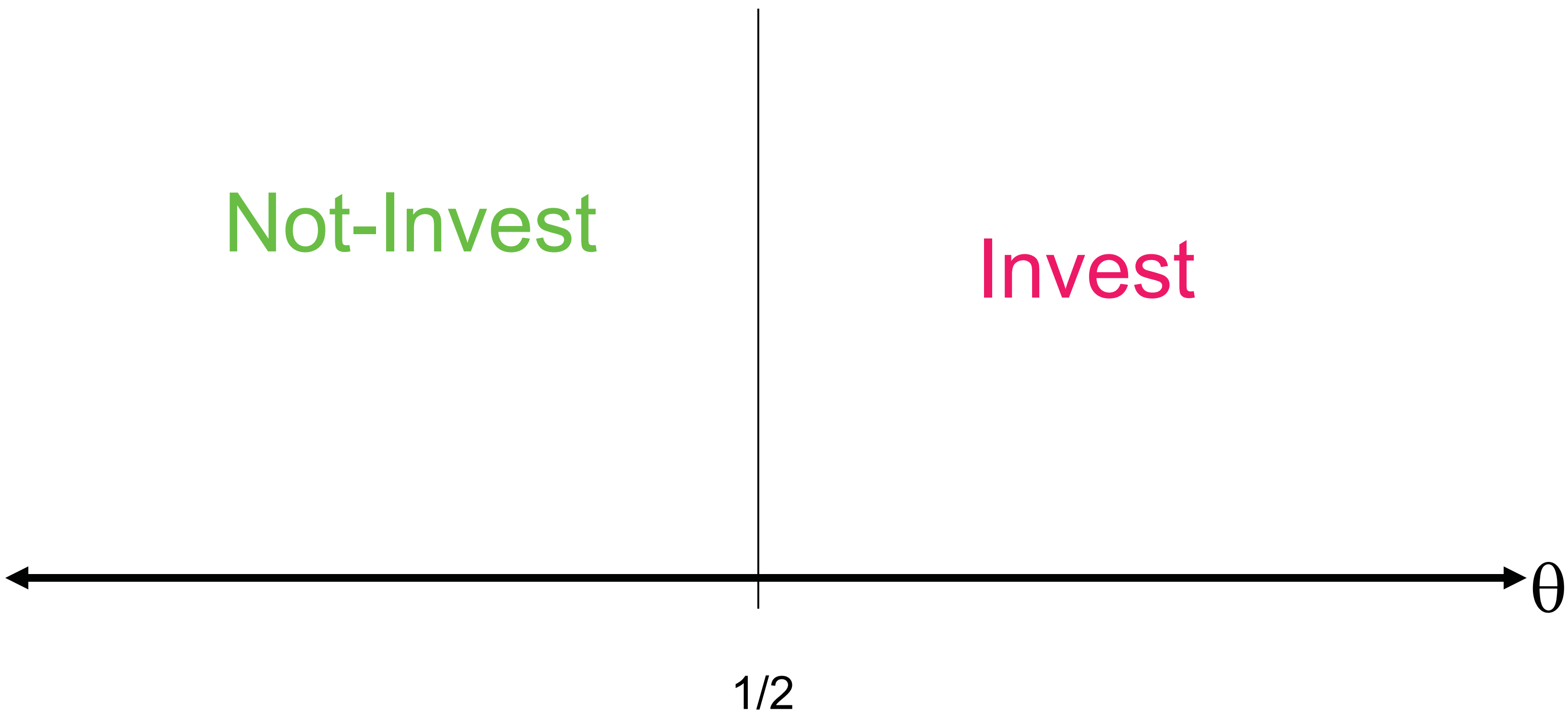
  Invest iff $x_i \geq \Pr(s_j = \text{Not-Invest}|x_i)$
- Assume supp(θ) = [a,b] where $a < 0 < 1 < b$.
- $x_i < 0 \Rightarrow s_i(x_i)$ = Not Invest
- $x_i > 1 \Rightarrow s_i(x_i)$ = Invest
- A cutoff x_i^* s.t.
  - $x_i < x_i^* \Rightarrow s_i(x_i)$ = Not Invest; $x_i > x_i^* \Rightarrow s_i(x_i)$ = Invest
- Symmetry: $x_1^* = x_2^* = x^*$
- $x^* = \Pr(s_j = \text{Not-Invest}|x^*) = \Pr(x_j < x^*|x_i = x^*) = 1/2$
- "Unique" BNE

Questions

- What is the smallest BNE?
- What is the largest BNE?
- Which strategies are rationalizable?
- Compute directly.

θ is not common knowledge but the noise is very small

It is very likely that



Risk-dominance

- In a 2 x 2 symmetric game, a strategy is said to be "risk dominant" iff it is a best reply when the other player plays each strategy with equal probabilities.

	Invest	Not-Invest
Invest	θ,θ	$\theta-1, 0$
Not-Invest	$0,\theta-1$	$0,0$

Invest is RD iff

$$0.5\theta + 0.5(\theta-1) > 0$$

$$\Leftrightarrow \theta > 1/2$$

Players play according to risk dominance

Carlsson & van Damme

Risk Dominance

	A	B
A	u_{11},v_{11}	u_{12},v_{12}
B	u_{21},v_{21}	u_{22},v_{22}

	A	B
A	g_1^a, g_2^a	0,0
B	0,0	g_1^b, g_2^b

- Suppose that (A,A) and (B,B) are NE.
- (A,A) is risk dominant if

  $$(u_{11}-u_{21})(v_{11}-v_{12})$$

  - $> (u_{22}-u_{12})\ (v_{22}-v_{21})$

- Affine transformation: g_1^a...
- (A,A) risk dominant if

  $$g_1^a\, g_2^a > g_1^b\, g_2^b$$

- i is indifferent against $\underline{s}_j$; (A,A) risk dominant if

  $$\underline{s}_1 + \underline{s}_2 < 1$$

Dominance, risk-dominance regions

- Dominance region

$$D_i^a = \{(u,v) |\ g_i^a > 0,\ g_i^b < 0\}$$

- Risk-dominance region

$$R^a = \{(u,v) |\ g_1^a > 0, g_2^a > 0;\ g_1^b,\ g_2^b > 0 \Rightarrow \underline{s}_1 + \underline{s}_2 < 1\}$$

Model

- $\Theta \subseteq \Re^m$ is open; (u,v) are continuously differentiable functions of θ w/ bounded derivatives;
- prior on θ has a density h which is strictly positive, continuously differentiable, bounded.
- Each player i observes a signal

$$x_i = \theta + \varepsilon\eta_i$$

  - (η_1, η_2) is bounded,
  - Independent of θ,
  - Admits a continuous density

Theorem

- Suppose that
  - x is on a continuous curve C $\subseteq \Theta$
  - $(u(c),v(c)) \in R^a$ for each c$\in$C
  - $(u(c),v(c)) \in D^a$ for some $c \in C$.
- Then A is the only rationalizable action at x when ε is small.

"Public" Information

- $\theta \sim N(y,\tau^2)$ and $\varepsilon\eta_i \sim N(0,\sigma^2)$
- Given x_i,

$$\theta \sim N(rx_i+(1-r)y,\ \sigma^2 r)$$
$$x_j \sim N(rx_i+(1-r)y,\ \sigma^2(r+1))$$
$$r = \tau^2/(\sigma^2+\tau^2)$$

- (Monotone supermodularity) monotone symmetric NE w/cutoff x^c:

$$rx^c + (1-r)y = \Pr(x_j \le x^c \mid x_i = x^c) = \Phi\left(\frac{(1-r)(x^c - y)}{\sigma\sqrt{r+1}}\right)$$

- Unique monotone NE (and rationalizable strategy) if

$$rx^c + (1-r)y - \Pr(x_j \le x^c \mid x_i = x^c)$$

is increasing in x^c whenever zero, i.e.,

$$\sigma^2 < 2\pi\tau^4(r+1)$$

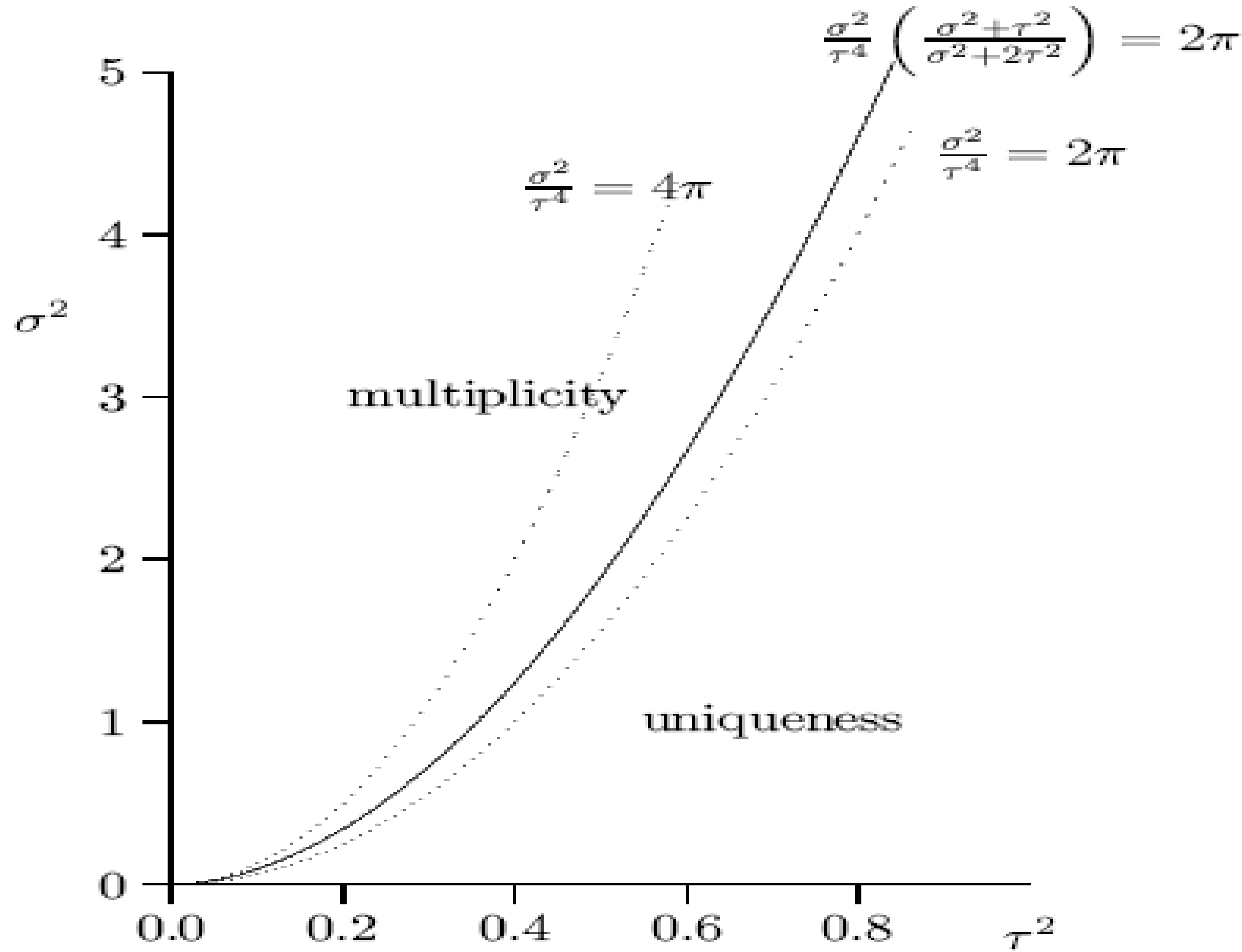


Figure 3.1: Parameter Range for Unique Equilibrium

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