14.127 Lecture 4

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1 Bounded Rationality

Three reasons to study:

- Hope that it will generate a unified framework for behavioral economics

- Some phenomena should be captured: difficult-easy difference. It would be good to have a metric for that

- Artificial intelligence

Warning – a lot of effort spend on bounded rationality since Simon and few results.

Three directions:

- Analytical models

  – Don't get all the fine nuances of the psychology, but those models are tractable.

- Process models, e.g. artificial intelligence

  – Rubinstein direction. Suppose we play Nash, given your reaction function, my strategy optimizes on both outcome and computing cost. Rubinstein proves some existence theorems. But it is very difficult to apply his approach.

- Psychological models

  - Those models are descriptively rich, but they are unsystematic, and often hard to use.

Human - computer comparison

- Human mind 10^{15} operations per second

- Computer 10^{12} operations per second

- Moore's law: every 1.5 years computer power doubles

- Thus, every 15 years computer power goes up 10^3

- If we believe this, then in 45 years computers can be 10^6 more powerful than humans

- Of course, we'll need to understand how human think

1.1 Analytical models

- Bounded Rationality as noise. Consumer sees a noisy signal $\tilde{q} = q + \sigma\varepsilon$ of quantity/quality q.

- Bounded Rationality as imperfect monitoring of the state of the world. People don't think about the variables all the time. They look up variable k at times $t_1, ..., t_n$

- Bounded Rationality as adjustment cost. Call by θ the parameters of the world.

  - Now I am doing a_0 and $\kappa = $ cost of decision/change

– I change my decision from a_0 to $a^* = \arg\max u\left(a, \theta_t\right)$ iff

$$u\left(a^*, \theta_t\right) - u\left(a_0, \theta_t\right) > \kappa$$

1.1.1   Model of Bounded Rationality as noise

- Random utility model – Luce (psychologist) and McFadden (econometrician who provided econometric tools for the models)

  - n goods, $i = 1, ..., n$.

  - Imagine the consumer chooses

  $$\max_i q_i + \sigma_i \varepsilon_i$$

  - What's the demand function?

- Definition. The Gumbel distribution G is

$$F(x) = P(\varepsilon < x) = e^{-e^{-x}}$$

and have density

$$f(x) = F'(x) = e^{-e^{-x} - x}.$$

- If ε has the Gumbel distribution then $E\varepsilon = \gamma > 0$, where $\gamma \simeq 0.59$ is the Euler constant.

- Proposition 1. Suppose ε_i are iid Gumbel. Then

$$P\left(\max_{i=1,..,n} \varepsilon_i + q_i \leq \ln n + q^* + x\right) = e^{-e^{-x}}$$

with q^* defined as $e^{q^*} = \frac{1}{n}\sum e^{q_i}$. This means that

$$M_n = \max_{i=1,..,n} \varepsilon_i + q_i =^d \ln n + q^* + \eta$$

and η is a Gumbel.

Proof of Proposition 1.

- Call $I = P\left(\max_{i=1,..,n} \varepsilon_i + q_i \leq y\right).$

- Then

$$I = P\left((\forall i)\, \varepsilon_i + q_i \leq y\right) = \Pi_{i=1}^{n} P\left(\varepsilon_i + q_i \leq y\right)$$

- Thus,

$$\ln I = \sum P\left(\varepsilon_i + q_i \leq y\right)$$

and

$$\ln P\left(\varepsilon_i + q_i \leq y\right) = \ln P\left(\varepsilon_i \leq y - q_i\right) = -e^{-(y-q_i)}.$$

- Thus

$$\ln I = \sum -e^{-(y-q_i)} = -e^{-y} \sum e^{q_i}$$

- Using

$$e^{q^*} = \frac{1}{n} \sum e^{q_i}$$

we have

$$\ln I = -e^{-y} n e^{q^*} = -e^{-[y-\ln n - q^*]}$$

which proves that I is a Gumbel. QED

Demand with noise

- Demand for good $n+1$ equals

$$D_{n+1}\left(q_1, ..., q_{n+1}\right) = P\left(\varepsilon_{n+1} + q_{n+1} > \max_{i=1,..,n} \varepsilon_i + q_i\right)$$

where q_i is total quality, including the disutility of price.

- Proposition 2.

$$D_{n+1}\left(q_1, ..., q_{n+1}\right) = \frac{e^{q_{n+1}}}{\sum_{i=1}^{n+1} e^{q_i}}.$$

In general,

$$D_j = P\left(\varepsilon_j + q_j > \max_{i \neq j} \varepsilon_i + q_i\right) = \frac{e^{q_j}}{\sum_{i=1}^{n+1} e^{q_i}}$$

Proof of Proposition 2.

- Observe that $\sum_{j=1}^{n+1} D_j = 1$.

- Note

$$D_{n+1}\left(q_1, ..., q_{n+1}\right) = P\left(\varepsilon_{n+1} > \max_{i=1,..,n} \varepsilon_i + q_i'\right)$$

where $q_i' = q_i - q_{n+1}$.

- Thus,

$$D_{n+1}\left(q_1, ..., q_{n+1}\right) = Ee^{-e^{-\left(\varepsilon_{n+1} - \ln n - q^*\right)}}$$

- Call $a = -\ln n - q^*$. Then

$$D_{n+1}\left(q_1, ..., q_{n+1}\right) = E e^{-e^{-(\varepsilon_{n+1}+a)}}$$
$$= \int e^{-e^{-(x+a)}} f(x)\, dx = \int e^{-e^{-(x+a)}} e^{-e^{-x}-x}\, dx$$
$$= \int e^{-e^{-(x+a)}-e^{-x}-x}\, dx = \int e^{-e^{-x}\left(e^{-a}+1\right)-x}\, dx$$

- Call $H = 1 + e^{-a}$ and re-write the above equation as

$$D_{n+1}\left(q_1, ..., q_{n+1}\right)$$
$$= \int e^{-e^{-x-\ln H}-x}\, dx$$
$$= \int e^{-e^{-x-\ln H}-(x-\ln H)} e^{-\ln H}\, dx$$

- Note that

$$\int_a^b e^{-e^{-y}-y}dy = \left[e^{-e^{-y}}\right]_a^b$$

- Thus

$$D_{n+1}(q_1, ..., q_{n+1}) = e^{-\ln H}\left[e^{-e^{-x-\ln H}}\right]_{-\infty}^{+\infty} dx$$

$$= \frac{1}{H} = \frac{1}{1+e^{-a}} = \frac{1}{1+e^{\ln n+q^*}} = \frac{1}{1+ne^{q^*}}$$

$$= \frac{1}{1+\sum_{i=1}^n e^{q_i'}} = \frac{e^{q_{n+1}}}{e^{q_{n+1}}+e^{q_{n+1}}\sum_{i=1}^n e^{q_i'}} = \frac{e^{q_{n+1}}}{\sum_{i=1}^{n+1} e^{q_i}}$$

QED

Demand with noise cont.

- This is called "discrete choice theory".

  - It is exact for Gumbel.

  - It is asymptotically true for almost all unbounded distributions you can think off like Gaussian, lognormal, etc.

- Dividing total quality into quality and price components

$$D_1 = P\left(q_1 - p_1 + \sigma\varepsilon_1 > \max_{i=2,\ldots,n} q_i - p_i + \sigma\varepsilon_i\right)$$

where ε_i are iid Gumbel, $\sigma > 0$.

- Then

$$D_1 = P\left(\frac{q_1 - p_1}{\sigma} + \varepsilon_1 > \max_{i=2,\ldots,n} \frac{q_i - p_i}{\sigma} + \varepsilon_i\right) = \frac{e^{\frac{q_1 - p_1}{\sigma}}}{\sum_{i=1}^{n} e^{\frac{q_i - p_i}{\sigma}}}$$

- This is very often used in IO.

Optimal pricing. An application – example

- Suppose we have n firms, $n \gg 1$.

- Firm i has cost c_i and does

$$\max_i \left(p_i - c_i\right) D_i \left(p_1, ..., p_n\right) = \pi_i$$

- Denote the profit by π_i and note that

$$\ln \pi_i = \ln (p_i - c_i) + \frac{q_i - p_i}{\sigma} + \ln \left(\sum e^{\frac{q_j - p_j}{\sigma}} \right)$$

and

$$\frac{\partial}{\partial p_i} \ln \pi_i = \frac{1}{p_i - c_i} - \frac{1}{\sigma} + \frac{-e^{-\left(\frac{q_i - p_i}{\sigma}\right)}}{\sum e^{\frac{q_j - p_j}{\sigma}}}$$

$$= \frac{1}{p_i - c_i} - \frac{1}{\sigma} + O\left(\frac{1}{n}\right)$$

- So

$$\frac{1}{p_i - c_i} - \frac{1}{\sigma} \simeq 0$$

and unit profits

$$p_i - c_i = \sigma$$

- Thus decision noise is good for firms' profits. See Gabaix-Laibson "Competition and Consumer Confusion"

- Evidence: car dealers sell cars for higher prices to women and minorities than to white men. Reason: difference in expertise. There is lots of other evidence of how firms take advantage of consumers. See paper by Susan Woodward on mortgage refinancing markets: unsophisticated people are charged much more than sophisticated people.

What about non-Gumbel noise?

- Definition. A distribution is in the domain of attraction of the Gumbel if and only if there exists constants A_n, B_n such that for any x

$$\lim_{n \to \infty} P\left(\max_{i=1,\dots,n} \varepsilon_i \leq A_n + B_n x \right) = e^{-e^{-x}}.$$

when ε_i are iid draws from the given distribution.

- Fact 1. The following distributions are in the domain of attraction of a Gamble: Gaussian, exponential, Gumbel, lognormal, Weibull.

- Fact 2. Bounded distributions are not in this domain.

- Fact 3 Power law distributions ($P\left(\varepsilon > x\right) \sim x^{-\zeta}$ for some $\zeta > 0$) are not in this domain.

- Lemma 1. For distributions in the domain of attraction of the Gumbel $F(x) = P(\varepsilon < x)$ take $\bar{F} = 1 - F(x) = P(\varepsilon \geq x)$, and $f = F'$. Then A_n, B_n are given by

$$\bar{F}(A_n) = \frac{1}{n}$$

$$B_n = \frac{1}{nf(A_n)}$$

- Lemma 2

$$\lim_{n \to \infty} P\left(\max_{i=1,\dots n} \varepsilon_i + q_i \leq A_n + B_n y + q_n^*\right) = e^{-e^{-y}}$$

with

$$e^{q_n^*/B_n} = \frac{1}{n}\sum e^{q_i/B_n}$$

- Proposition.

$$D_1 = P\left(q_1 - p_1 + \sigma\varepsilon_1 > \max_{i=2,\ldots,n} q_i - p_i + \sigma\varepsilon_i\right)$$

For $n \to \infty$, $\lim D_1/\bar{D}_1 = 1$ where

$$\bar{D}_1 = \frac{e^{\frac{q_1-p_1}{\sigma}}/B_n\sigma}{\sum_{i=1}^n e^{\frac{q_i-p_i}{\sigma}}/B_n\sigma} \simeq D_1.$$

- Example. Exponential distribution $f(x) = e^{-(x+1)}$ for $x > -1$ and equals 0 for $x \leq -1$. then, for $x > -1$

$$\bar{F}(x) = P(\varepsilon > x) = \int_x^\infty e^{-(x+1)} dy$$
$$= \left[-e^{-(x+1)}\right]_x^\infty = e^{-(x+1)} = f(x).$$

Thus

$$\bar{F}(A_n) = \frac{1}{n},$$

and

$$A_n = -1 + \ln n$$

and

$$B_n = \frac{1}{nf(A_n)} = 1$$

- Example 2. Gaussian. $f(x) =$, $\bar{F}(x) = \int_x^\infty \frac{1}{\sqrt{2\pi}} e^{-\frac{s^2}{2}} ds$. For large x,

  the cumulative $\bar{F}(x) \sim \frac{e^{-\frac{x^2}{2}}}{\sqrt{2\pi}x}$. Result

$$A_n \sim \sqrt{q \ln n}$$

$$B_n \sim \frac{1}{\sqrt{2 \ln n}}$$

Optimal prices satisfy

$$\max_i \frac{\left(p_i - c_i\right) e^{\frac{q_i - p_i}{B_n \sigma}}}{\sum e^{\frac{q_j - p_j}{B_n \sigma}}} = \max \left(p_i - c_i\right) \bar{D}_1 = \pi_i$$

- Same as for Gumbel with $\sigma' = B_n \sigma$.

- Thus

$$p_i - c_i = B_n \sigma$$

- Examples

  - Gumbel

$$p_i - c_i = \sigma$$

  - Exponential noise

$$p_i - c_i = \sigma$$

  - Gaussian

$$p_i - c_i = \frac{1}{\sqrt{2 \ln n}} \sigma$$

  and competition almost does not decrease markup (beyond markup when there are already some 20 firms).

- Example. Mutual funds market.

  - Around 10,000 funds. Fidelity alone has 600 funds.

  - Lots of fairly high fees. Entry fee 1-2%, every year management fee of 1-2% and if you quit exit fee of 1-2%. On the top of that the manager pays various fees to various brokers, that is passed on to consumers.

  - The puzzle – how all those markups are possible with so many funds?

  - Part of the reason for that many funds is that Fidelity and others have incubator funds. With large probability some of them will beat the market ten years in a row, and then they can propose them to unsophisticated consumers.

- Is it true that if competition increases then price goes always down?

  - Not always. For lognormal noise $B_n \sim e^{\sqrt{\ln n}}$ and so

  $$p_i - c_i = \sigma e^{\sqrt{\ln n}}.$$

1.1.2 Implications for welfare measurement (sketch)

- Assume no noise and rational consumers.

- Introduce a new good which gets an amount of sales

$$Q = pD$$

  where D is demand and p is price..

- The welfare increase is

$$\psi(\eta) Q$$

  where η is the elasticity of demand, the utility of consuming D is $D^{1-\frac{1}{\eta}}$, and $\psi(\eta)$ denotes $\frac{1}{\eta-1}$.

- If there is confusion, the measured elasticity $\hat{\eta}$ is less than the "true" elasticity as

$$-\frac{\partial \ln D_i}{\partial p_i} = \frac{1}{\sigma B_n} \sim \hat{\eta}$$

- Thus, the imputed welfare gain $\psi(\hat{\eta}) Q$ will be bigger than the true welfare gain $\psi(\eta) Q$.