14.127 Lecture 6

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0.0.1 Shrouded attributes. A continuation

- Rational guys

$$U_i = q - p + \max\left(V - \widehat{p}, V - e\right) + \sigma \varepsilon_i$$
$$= q - p + V - \min\left(\widehat{p}, e\right) + \sigma \varepsilon_i = \bar{U}_i + \sigma \varepsilon_i$$

- Rational demand for good 1

$$D_1 = P\left(U_1 > \max_{i=2,\ldots,n} U_i\right)$$
$$= P\left(\bar{U}_1 + \sigma \varepsilon_1 > \max_{i=2,\ldots,n} U_i\right)$$

- We look for symmetric equilibrium with $(p_i, \widehat{p}_i) = (p^*, \widehat{p}^*)$ for $i = 2, \ldots, n$.

- Profits

$$\pi_1 = D_1 \left(p + \widehat{p} 1_{\widehat{p} \leq e} \right)$$

- Assume $c = 0$ and $\widehat{c} = 0$

- Call α – fraction of rational guys.

- Then

$$\pi = \alpha \left(p + \widehat{p} 1_{\widehat{p} \leq e} \right) D(A) + (1 - \alpha) \left(p + \widehat{p} 1_{\widehat{p} \leq V} \right) D(-p + p^*)$$

where $A = -p - \min(\widehat{p}, e) + p^* + \min(\widehat{p}^*, e)$

- The slope of profit

$$\frac{\partial \pi}{\partial p} = \alpha D\left(A\right) - \alpha \left(p + \widehat{p} 1_{\widehat{p} \leq e}\right) D'\left(A\right) + \left(1 - \alpha\right) D\left(A\right) - \left(1 - \alpha\right) \left(p + \widehat{p} 1_{\widehat{p} \leq}\right.$$

$$= \alpha D\left(0\right) - \alpha \left(p + \widehat{p} 1_{\widehat{p} \leq e}\right)' D\left(0\right) + \left(1 - \alpha\right) D\left(0\right) - \left(1 - \alpha\right) \left(p + \widehat{p} 1_{\widehat{p} \leq V}\right.$$

$$= D\left(0\right) - \left(p + \widehat{p} \widehat{Q}\right) D'\left(0\right)$$

where $\widehat{Q}$ denotes the fraction of consumers that buy the add-on.

$$\widehat{Q} = \alpha 1_{\widehat{p} \leq e} + \left(1 - \alpha\right) 1_{\widehat{p} \leq V}$$

- Hence

$$p + \widehat{p}\widehat{Q} = \frac{D\left(0\right)}{D'\left(0\right)} = \mu$$

thus the total industry profits do not depend by the add-on structure (α, V, e).

- **Proposition.** An optimum $\widehat{p} \in \{e, V\}$.

- Proof.

  - If $e < \widehat{p} < V$, then it is better to set $\widehat{p} = V$.

  - If $0 \leq \widehat{p} < e$, then it is better to increase $\widehat{p}$ to e and lower p by $e - \widehat{p}$. Then rational demand stay the same, and irrational demand increases. QED

- So, $\widehat{p} \geq e$ and $\min(e, \widehat{p}) = e$, and

$$
\begin{aligned}
\pi_1 &= D\left(-p + p^*\right) \left[\alpha\left(p + \widehat{p}1_{\widehat{p} \leq e}\right) + (1 - \alpha)(p + \widehat{p})\right] \\
&= D\left(-p + p^*\right) \left[p + \widehat{p}\left(\alpha 1_{\widehat{p} \leq e} + 1 - \alpha\right)\right] \\
&= D\left(-p + p^*\right) [p + F]
\end{aligned}
$$

- If $\widehat{p} = V$ then $F = V(1 - \alpha)$. if $\widehat{p} = e$ then $F = e$. Consequently:

  - $\widehat{p} = V$ iff $V(1 - \alpha) > e$, or in other words, when

  $$
  \alpha < \alpha^* = 1 - \frac{e}{V}
  $$

  - $\widehat{p} = e$ iff

  $$
  \alpha > \alpha^* = 1 - \frac{e}{V}
  $$

- Take $\alpha < \alpha^*$. Then

$$p = \mu - \widehat{p}\widehat{Q} + c = \mu - V\alpha\left(1 - \alpha\right) + c$$

So if $\mu = 0$ then

$$p < c$$

and the base good is a loss leader.

1 Advertising. Does it solve the problem?

Furthermore, manufacturers in a competitive equipment market have incentives to avoid even this inefficiency by providing information to consumers. A manufacturer could capture profits by raising its [base-good] prices above market levels (i.e., closer to cost), lowering its aftermarket prices below market levels (i.e., closer to cost), and informing buyers that its overall systems price is at or below market. In this fashion, the manufacturer could eliminate some or all of the deadweight loss, attract consumers by offering a lower total cost of ownership, and still capture as profits some of the eliminated deadweight loss. In other words, and unlike traditional monopoly power, the manufacturers have a direct incentive to eliminate even the small inefficiency caused by poor consumer information (Shapiro 1995, p. 495).

1.1 Curse of education (& Advertising)

- Advertising makes more consumers anticipate the aftermarket.

- "When you choose a printer, remember to take into account the aftermarket for printer cartridges. Our cartridge prices are low relative to our competitors."

- If no firm advertises, then α is low.

- If at least one firm advertises, α rises to $\alpha' \in (\alpha, 1]$.

- We take the prices from the previous proposition and call them the *Shrouded Market Prices*.

- **Proposition**. If $\alpha < \alpha^\dagger = 1 - \frac{e}{V}$, the Shrouded Market Prices support an equilibrium in which firms choose not to advertise.

- Proof. Consider firm 1. It can advertise and force α to become $\alpha' > \alpha$ and charge $p, \widehat{p}$. It optimizes $\widehat{p} \in \{e, V\}$.

$$\pi_1 = D\left(-p + p^*\right)\left(p + \max\left(e, \left(1 - \alpha'\right)V\right)\right) = D\left(-p + p^*\right)\left(p + F\right)$$

Since $\alpha < \alpha^\dagger$ and $\alpha < \alpha'$ so $F\left(\alpha\right) > F\left(\alpha'\right)$. Thus

$$\max_p \pi_1 < \max_p D\left(-p + p^*\right)\left(p + F\left(\alpha\right)\right) = \text{pre-deviation profit}$$

Why doesn't Shapiro's advertising argument apply?

- Sophisticated consumers would rather pool with the naive consumers at firms with high add-on prices than defect to firms with marginal cost pricing of both the base good and the add-on.

- Consider the case in which the firm has no market power, so $\mu = 0$.

- If sophisticate gives her business to a firm with *Shrouded Market Prices*,

  sophisticate surplus $= -p + V - e = (1 - \alpha) V + V - e > (1 - \alpha) V + V - (1 - \alpha)$

- By contrast, if sophisticate gives her business to a firm with zero mark-ups on both the base good and the add-on, the sophisticate surplus will only be V.

- This preference for pooling reflects the sophisticate cross-subsidization by naive consumers.

- Sophisticates benefit from "free gifts" ($25 start-up deposit, DVD player) and avoid high fees.

1.2   Robustness:

- Our no-advertising results don't change if $\alpha > \alpha^\dagger$.

- We can change the timing of the advertising decision without affecting the "no-advertising" results.

- In the paper, we also generalize to the case of continuous choices and imperfect add-on substitutes.

1.3   Inattention

- Ricardo Reis "Inattentive Consumers" – job market paper. See also Mankew-Reis "Sticky Information"

- Instead of paying attention to stock market every day pay attention to them every D periods

- Revise your forecasting with Poisson probability λ

- Optimal price if last thinking was at $t - j$.
$$x_t^j = E_{t-j} p_t^*$$

- Sticky information but not sticky prices: prices change every period.

1.3.1 Other aspect of bounded rationality:

- Euler equation for $u = \dfrac{c^{1-\frac{1}{\psi}}}{1-\frac{1}{\psi}}$

$$E\left(\frac{\Delta c_t}{c_t}\right) = \psi r_t + A$$

- Empirically, $\hat{\psi} \simeq 0$.

- Rational model people say that is consistent, because $\gamma = \frac{1}{\psi}$ is very high.

- Other explanation. Suppose people don't pay much attention to r_t.

  - Then estimated $\hat{\psi} \simeq 0$ when we regress changes in consumption on the interest rate.

  - But deep parameter ψ doesn't need to be close to 0

1.3.2 Stock Market "6D bias"

- **Proposition.** If people look at the stock market every D periods and the econometrician estimates the Euler equation

$$E\left(\beta\left(\frac{c_{t+1}}{c_t}\right)^{-\gamma}\tilde{R}_{t+1}\right) = 1.$$

  then the estimate is $\hat{\gamma} = 6D\gamma$ for $D \geq 1$.

- D is the proportion of time between readjustment to one of econometrician's observations. E.g. $D =$ year/quarter. Then $\hat{\gamma} = 24\gamma$.

- Maybe that's why we have high estimates of $\hat{\gamma}$.

- Note that people may re-optimize as often as econometrician takes snapshot and $\hat{\gamma}$ is still time too large due to continuity of the underlying price process.

- The formulas get complicated when people re-optimize more often then econometrician's data.

1.3.3 A model of optimal time between changes T_D

- Re-optimizing every $T_D = D$ periods leads to a loss in utility

$$U = E \int_0^\infty e^{-\rho t} u\left(c_t\right) dt.$$

- How big is the loss?

$$V\left(w, D\right) = U_{\text{imperfect policy}} < U^*|_{D=0, w_0} = V\left(w_0, 0\right)$$

- We will measure the loss by Λ_c such that

$$V\left(w_0, D\right) = V\left(\left(1 - \Lambda_c\right) w_0, 0\right).$$

- **Proposition.** $\Lambda_c = \frac{\gamma}{2} \left\langle \left(\frac{\Delta c}{c} \right)^2 \right\rangle = \frac{\gamma}{4} \theta^2 \sigma^2 D$

  - where $\Delta c_t = c^{\text{policy}} - c^{\text{optimal}}$,

  - σ − volatility of stock returns,

  - θ − share of stock in your portfolio

- Say every time you look at your stock market account, you pay psychic cost q.

- Total cost

$$\Lambda_q = q \sum_{n \geq 0} \beta^{nD} = q \sum_{n \geq 0} e^{-\delta nD} = \frac{q}{1 - e^{-\delta D}}$$

- When δ is small then

$$\Lambda_q = \frac{q}{\delta D}$$

- Optimal D is one that minimizes

$$\min_{D} \Lambda_c + \Lambda_q = \frac{\gamma}{4}\theta^2\sigma^2 D + \frac{q}{\delta D} = AD + \frac{B}{D}$$

- Hence

$$D = \sqrt{\frac{B}{A}} = \sqrt{\frac{q/\delta}{\frac{\gamma}{4}\theta^2\sigma^2}}$$

- Mini theory of mental accounts

$$D_a = 2\sqrt{\frac{q}{\gamma\delta}\frac{1}{\theta_a\sigma_a}}$$

on mental account a.

- You pay attention to mental accounts that are important (θ) and volatile (σ).

- An extension of Ricardo Reis paper would for example endogenize all those things.

2 Behavioral game theory

- Colin Camerer "Behavioral Game Theory" (book that summarizes the field)

- When you play the game – you rarely start with Nash, but in games of low complexity you often converge to Nash

- The question: what happens before you reach Nash?

- "p-beauty" contest. Player i picks a number in $[0, 100] \ni x_i$.

- The winners is the one who comes closest to $p\bar{x} = p\frac{1}{N}\sum x_i$

- There is a unique Nash equilibrium with $\bar{x} = 0$.

- In practice, you should not play $x_i = 0$. Over time it converges to 0, and then it becomes boring and people start playing some random thing if stakes are low.

2.0.4   Quantal response equilibrium QRE

- Notation

  - $i -$ player

  - $j -$ strategy

  - s_i^j

- Payoff

$$\pi_1\left(s_1^{j_1}, ..., s_n^{j_n}\right) = \pi_1\left(s_1^{j_1}, s_{-1}^{j'}\right)$$

- **Definition.** QRE is a profile of weights $\left(p_i^j\right)$ such that

$$p_i^j = \frac{e^{\lambda \pi_i^j}}{\sum e^{\lambda \pi_i^{j'}}}$$

where $\pi_i^j =$ "expected value of strategy j" equals

$$\sum_{j'} \pi_i\left(s_i^j, s_{-i}^{j'}\right) p^{j'} = \sum_{j'} \pi_i\left(s_i^j, s_{-i}^{j'}\right) \prod_{i' \neq i} p_{i'}^{j'}$$

- This theory is nice empirically. For doing theory is too difficult to use – you need to solve for big fixed points. No close forms.