Networks

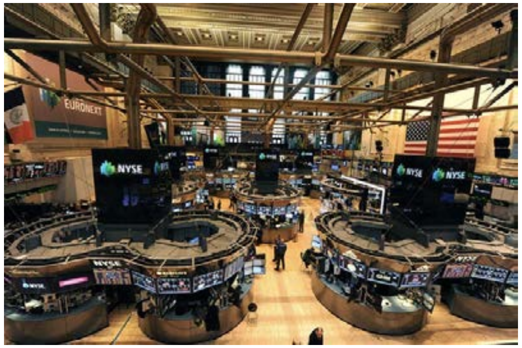
6.207J/14.15J

Sources

- Lecture slides
- Networks: An Introduction by Mark Newman (required)
- Networks, Crowds and Markets by Easley and Kleinberg
- Matthew Jackson, Social and Economic Networks, Princeton University Press

Networks

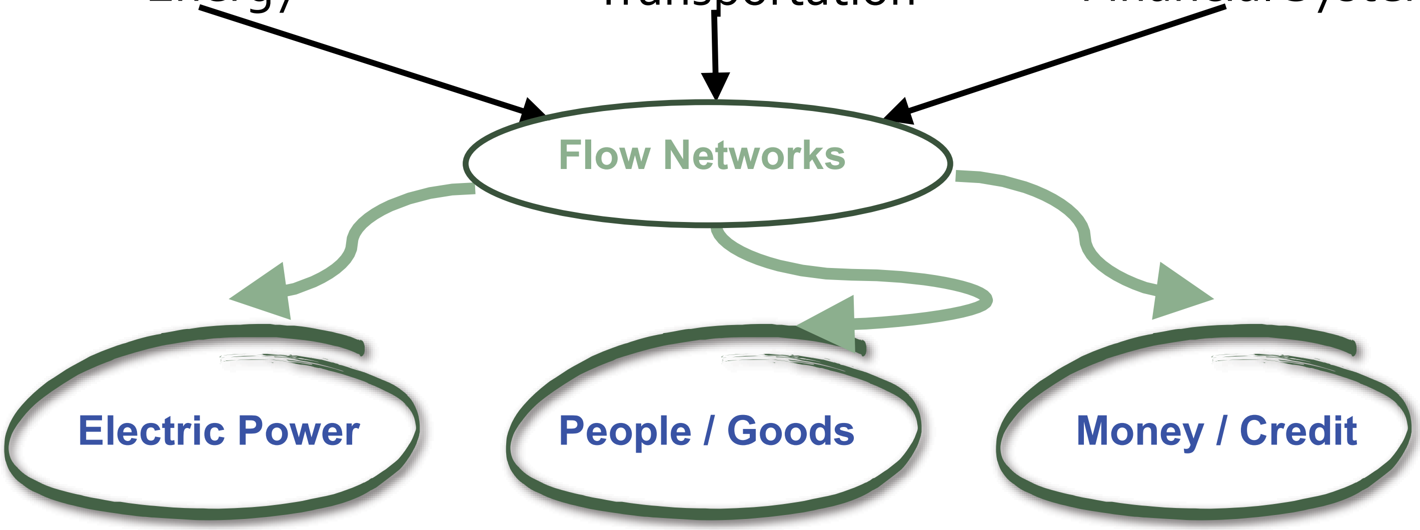
- What are networks? Why study networks
  - Facilitators of flow (of something) among distributed entities



Energy

Transportation

Financial Systems



Social, Economic, and Information Networks (This course)

- Social and economic networks, these units (nodes) are individuals or firms, links are information/interaction between nodes
- At some broad level, the study of networks can encompass the study of all kinds of interactions.
- Information transmission. Web links. Information exchange. Trade.
- Credit and financial flows. Friendship. Trust. Spread of epidemics. Diffusion of ideas and innovation.

Networks

- What are networks? Why study networks
  - Facilitators of flow (of something) among distributed entities

- Networks are a representation of interaction structure among units.
  - Physical (Communication, Power Grid, Transportation, Internet)
  - Virtual (information) (social interaction, business interactions, www)

- Commonalities
  - Nodes (cell phones, people, firms ..)
  - Links (EM waves, transmission lines, Cables, social interactions)

- Tools for studying networks
  - Graphs
  - Dynamics
  - Games

Examples: Telegraph Network

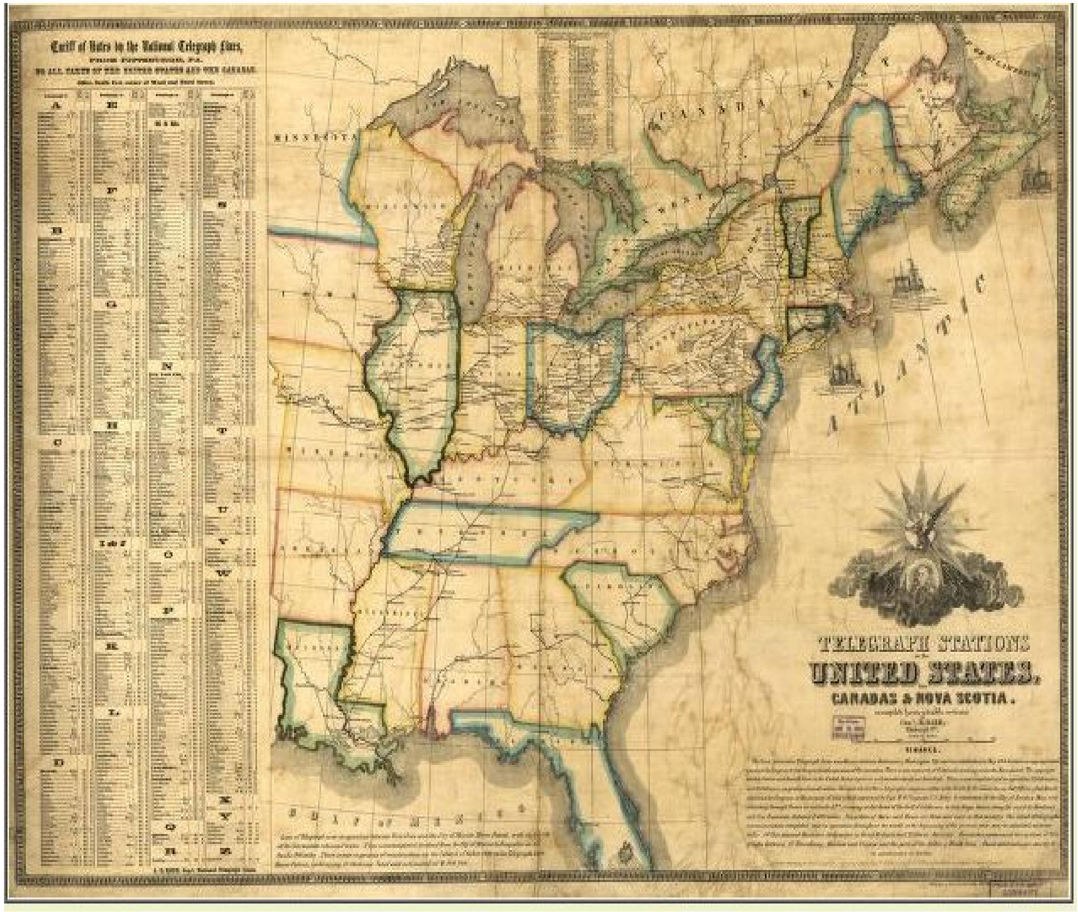


Figure: Telegraph network in US during Civil War. Arguably, shaped entire political career of Lincoln as well as the outcome of Civil War.

Examples: Telephone

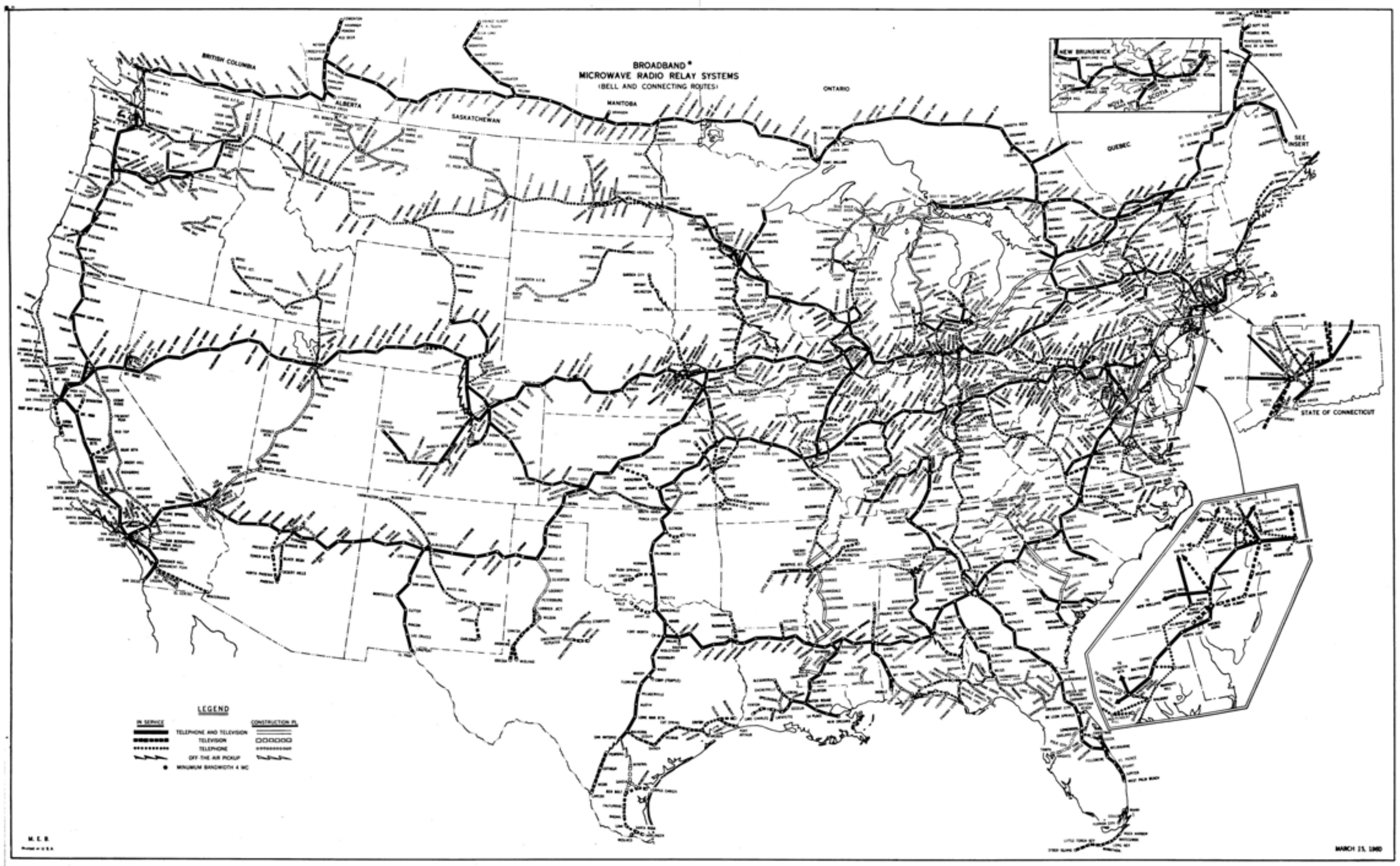


Figure: Telephone network in 1960, designed to withstand nuclear war.

- The human voice carries too far as it is.. and now you fellows come along and seek to complicate matters...
  - Mark Twain on the invention of the telephone.

Examples: Power Grid

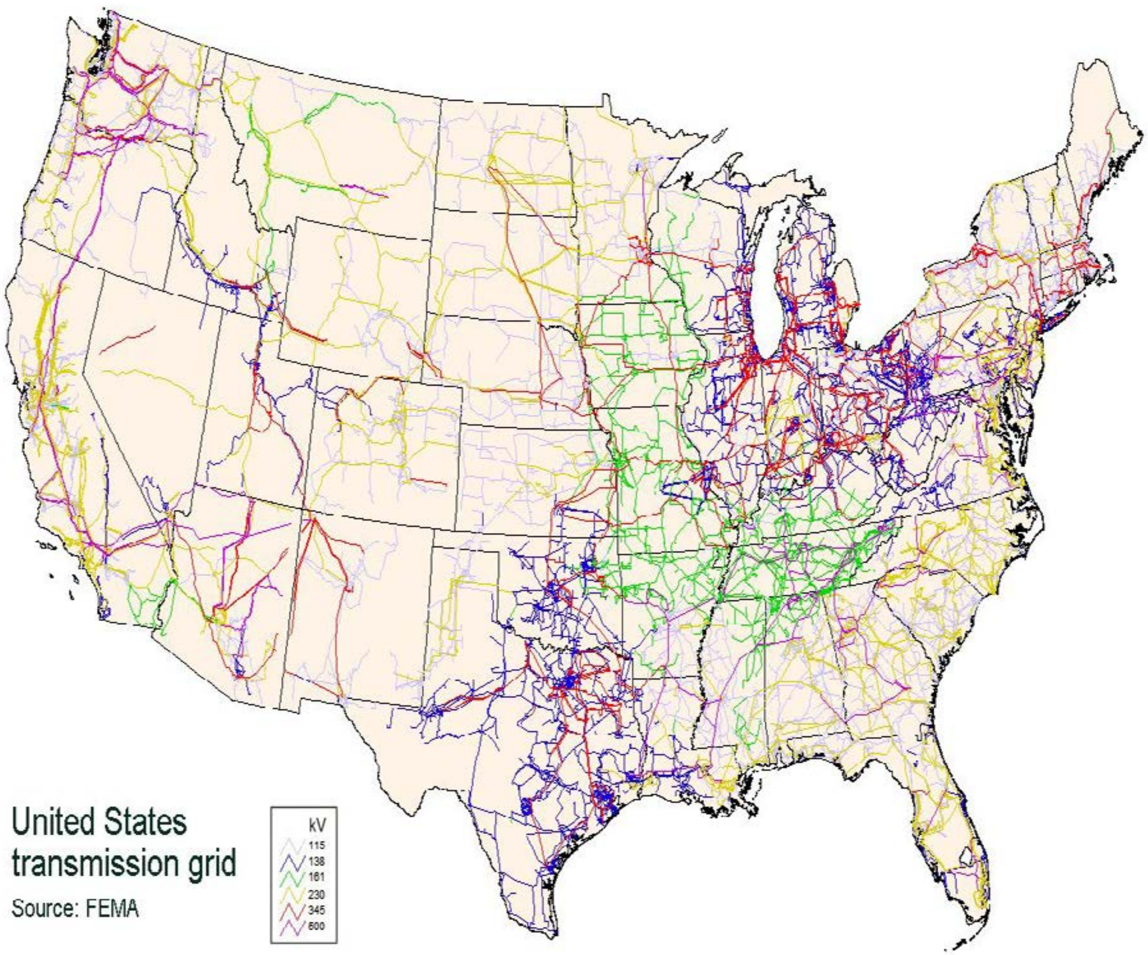


Figure: Map of US Power Grid.

Example: World Wide Web

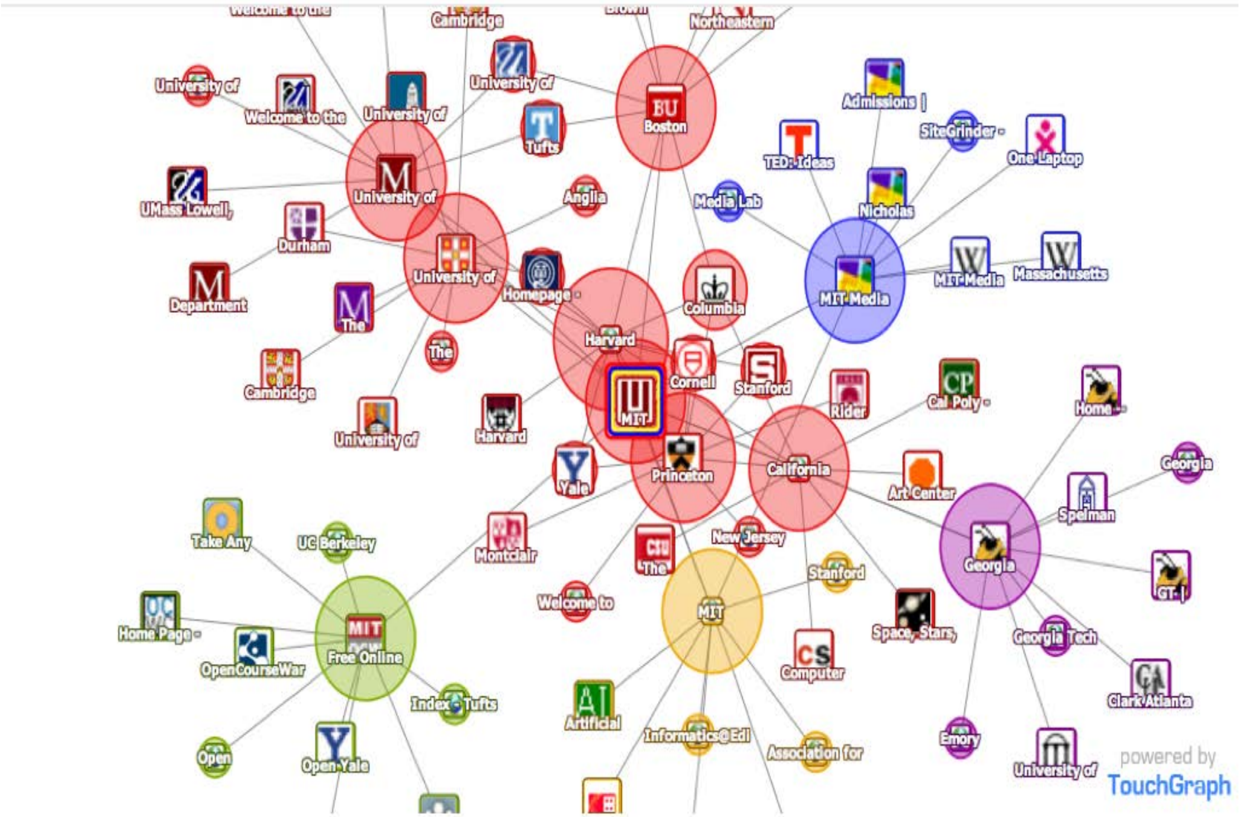
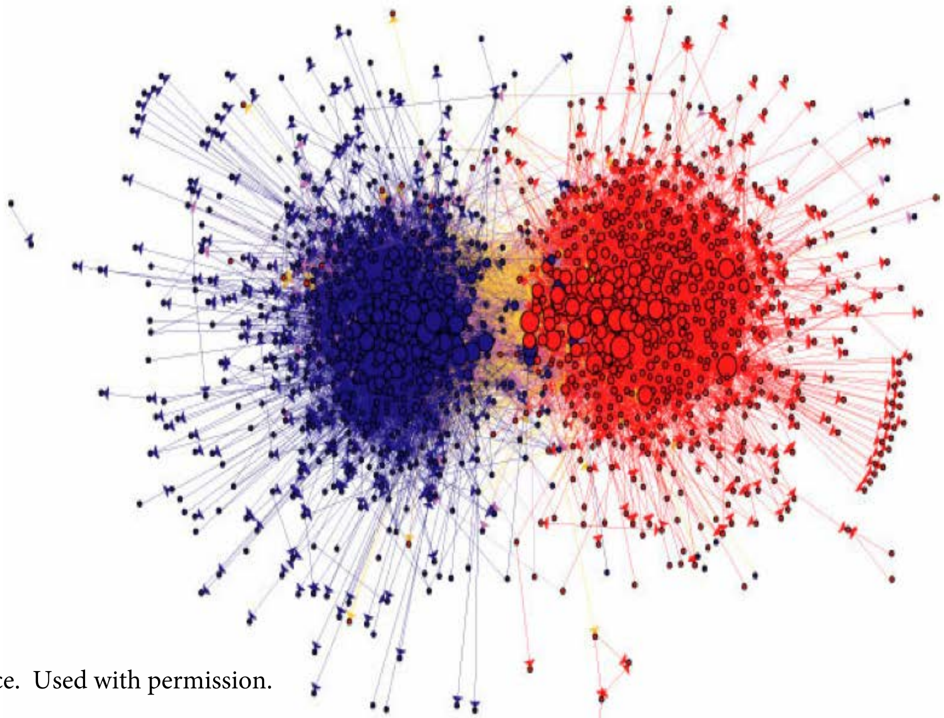


Figure: The web link structure centered at http://web.mit.edu (touchgraph)

Example: Content Networks



Courtesy of Lada Adamic and Natalie Glance. Used with permission.

Figure 1 in Adamic, Lada, and Natalie Glance. "The Political Blogosphere and the 2004 U.S. Election: Divided They Blog." In International Conference on Knowledge Discovery and Data Mining, Proceedings of the 3rd International Workshop on Link Discovery, Chicago, Illinois, 2005. New York, NY: Association for Computing Machinery (ACM), 2005, pp. 36-43. ISBN-13: 9781595932150. ISBN-10: 1595932151.

Figure: The network structure of political blogs prior to the 2004 U.S. Presidential election reveals two natural and well-separated clusters (Adamic and Glance, 2005)

Example: Social Networks

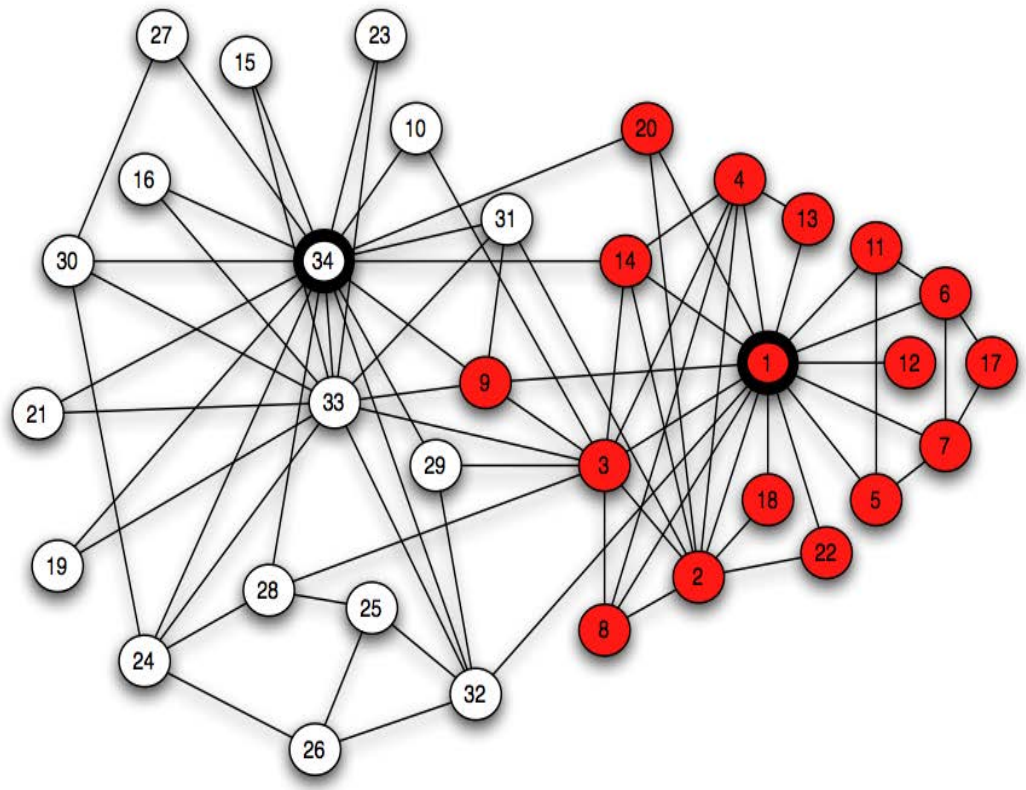


Figure: The social network of friendships within a 34-person karate club provides clues to the fault lines that eventually split the club apart (Zachary, 1977)

Adapted from Figure 1 (p. 456) in Zachary, Wayne W. "An Information Flow Model for Conflict and Fission in Small Groups." Journal of Anthropological Research 33, no. 4 (1977): 452-473.

Example: Financial Network

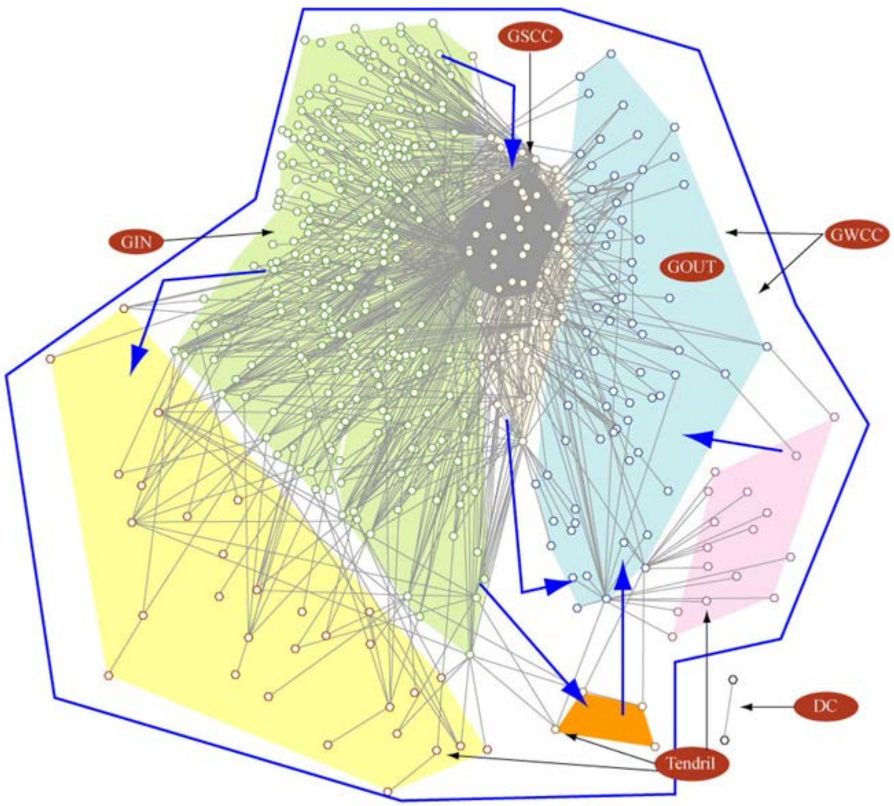


Figure: The network of loans among financial institutions can be used to analyze the roles that different participants play in the financial system, and how the interactions among these roles affect the health of individual participants and the system as a whole. (Bech and Atalay 2008)

Example: Infection Network

Courtesy of Valdis E Krebs. Used with permission.
For further information, see:
Andre, McKenzie, Kashef Ijaz, Jon D. Tillinghast, Valdis E. Krebs, Lois A. Diem, Beverly Metchock, Theresa Crisp, and Peter D. McElroy. "Transmission Network Analysis to Complement Routine Tuberculosis Contact Investigations." American Journal of Public Health 97, no. 3 (March 2007): 470–477.

Figure: The spread of an epidemic disease (such as the tuberculosis outbreak shown here) is another form of cascading behavior in a network. The similarities and contrasts between biological and social contagion lead to interesting research questions. (Andre et al. 2007)

Example: Influence Network

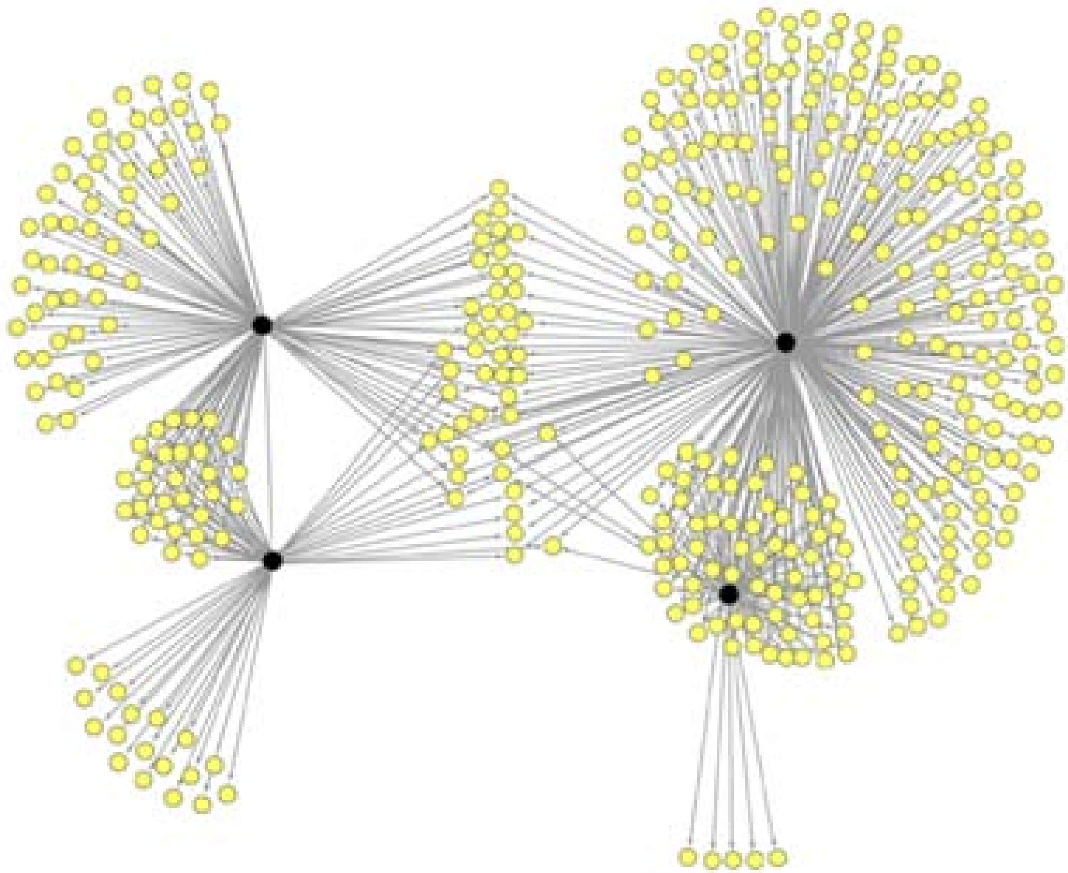


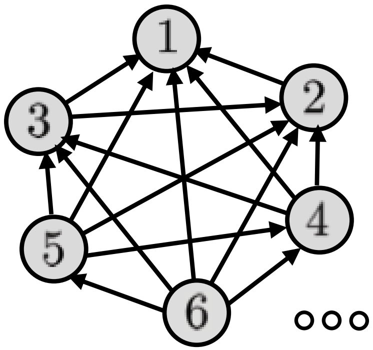
Figure: When people are influenced by the behaviors of their neighbors in the network, the adoption of a new product or innovation can cascade through the network structure. Here, e-mail recommendations for a Japanese graphic novel spread in a kind of informational or social contagion. (Leskovec et al. 2007)

So, what are we interested in?

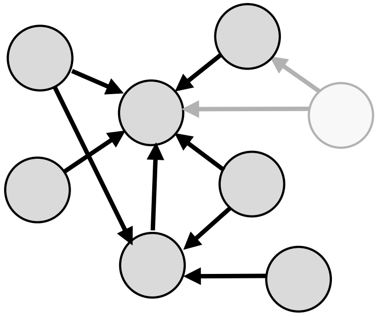
- Relevant measure for networks
  - Important to understand when these measures apply
  - Examples: degree distribution, centrality, diameter/conductance, modularity
- How can we generate interesting stochastic networks
  - Several stochastic models
  - Each has its own properties: connectivity, power laws, etc.
- Role of Structure and Dynamics on Behavior of Networks
  - Distributed Learning
  - Distributed computation
  - Diffusion (Information Spread, Epidemics)
- Decisions on Networks
  - Strategic Decisions: Game theory, network games
  - Social and economic networks

Random Networks

- Erdos Renyi



- Preferential attachment



Epidemics

- Simple Infection Model
  - SI Model on a random network

- Erdos-Renyi Random Graph

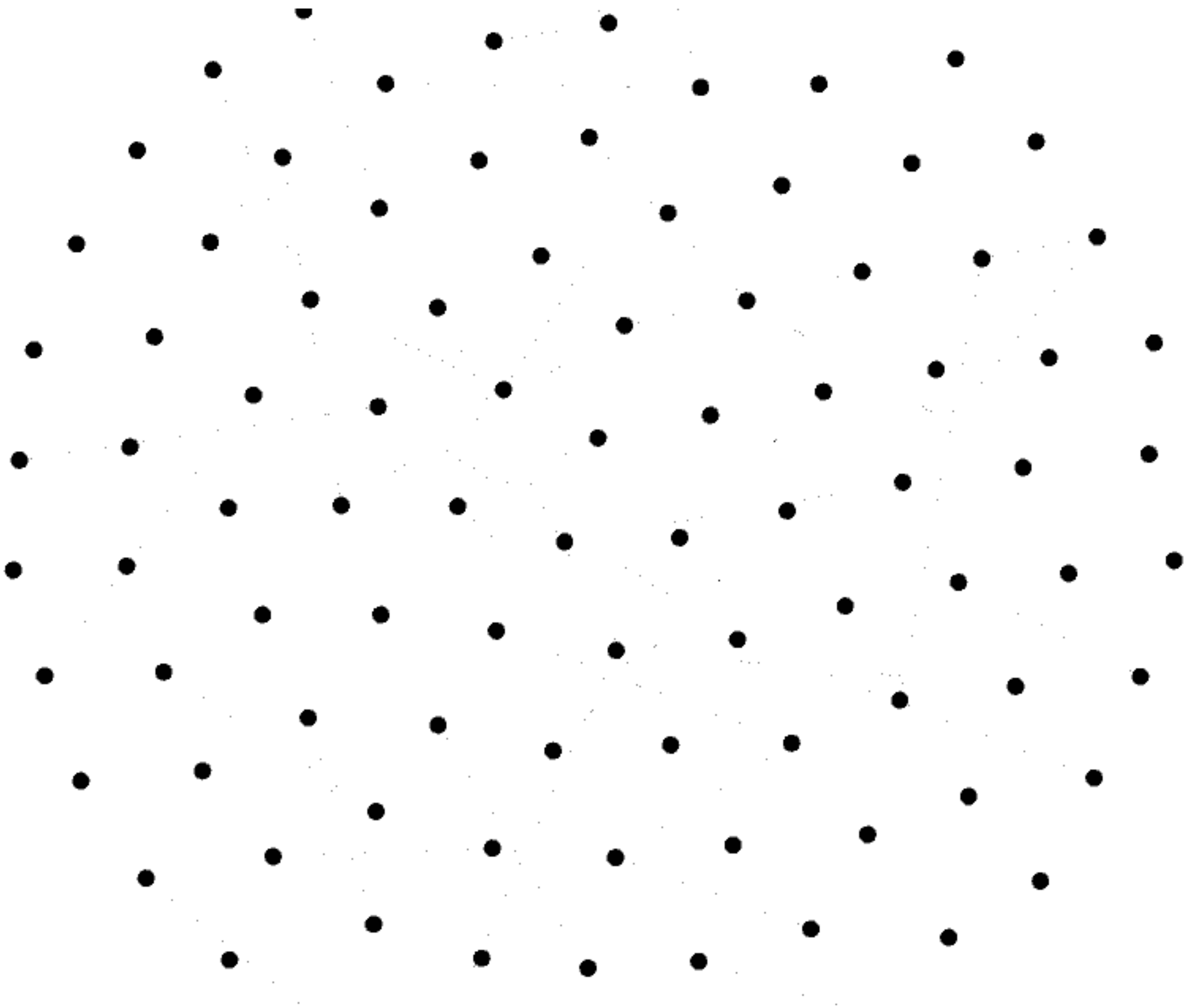
- What do we cover:
  - How do these networks depend on the underlying probability of connection
  - Sparse, large connected components, fully connected

- Who is the most infectious agent?
- Who does the disease Spread?

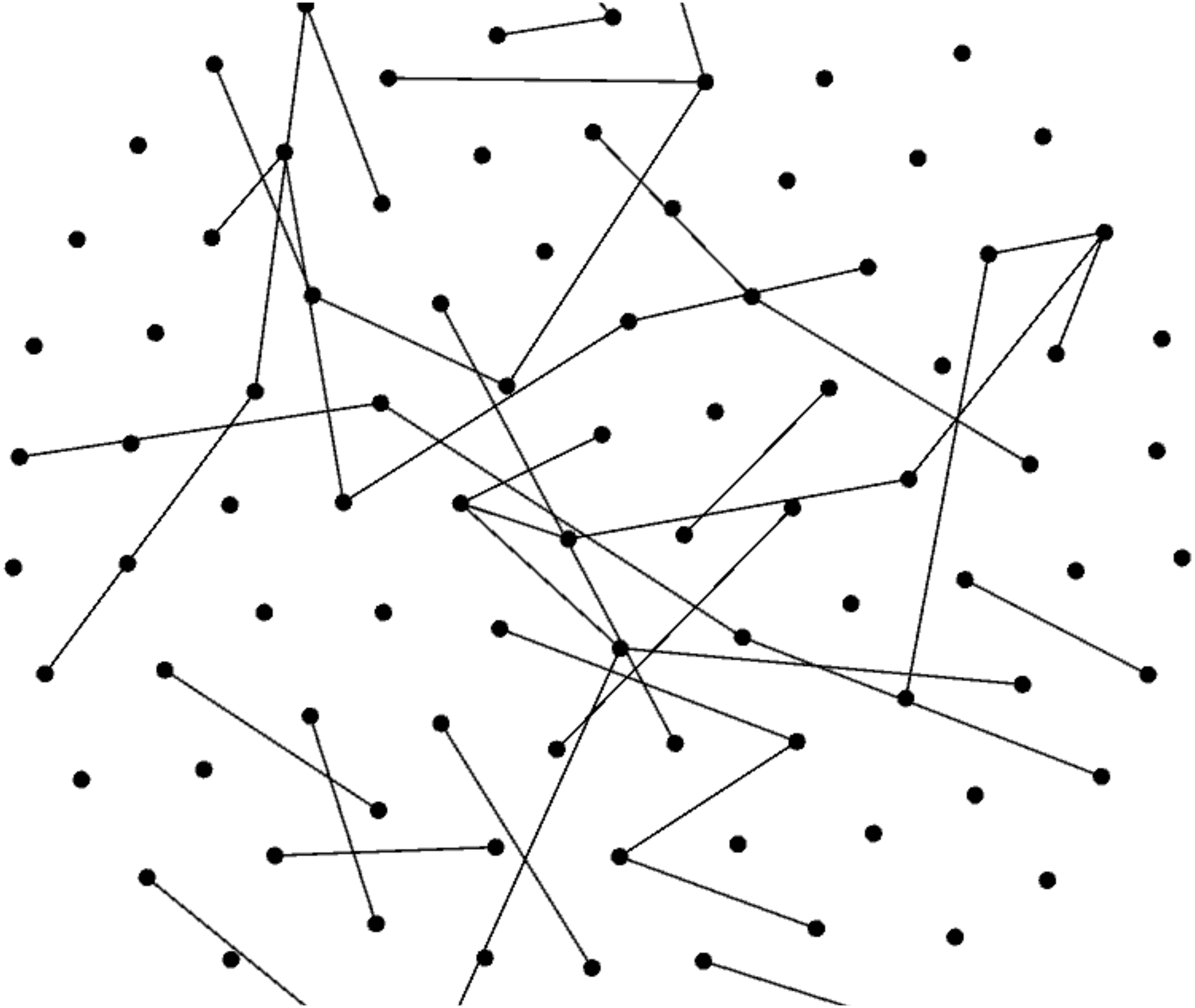
The Tipping Point: M. Gladwell

The Tipping Point is that magic moment when an idea, trend, or social behavior crosses a threshold, tips, and spreads like wildfire. Just as a single sick person can start an epidemic of the flu, so too can a small but precisely targeted push cause fashion trend, the popularity of a new product, or a drop of crime rate.

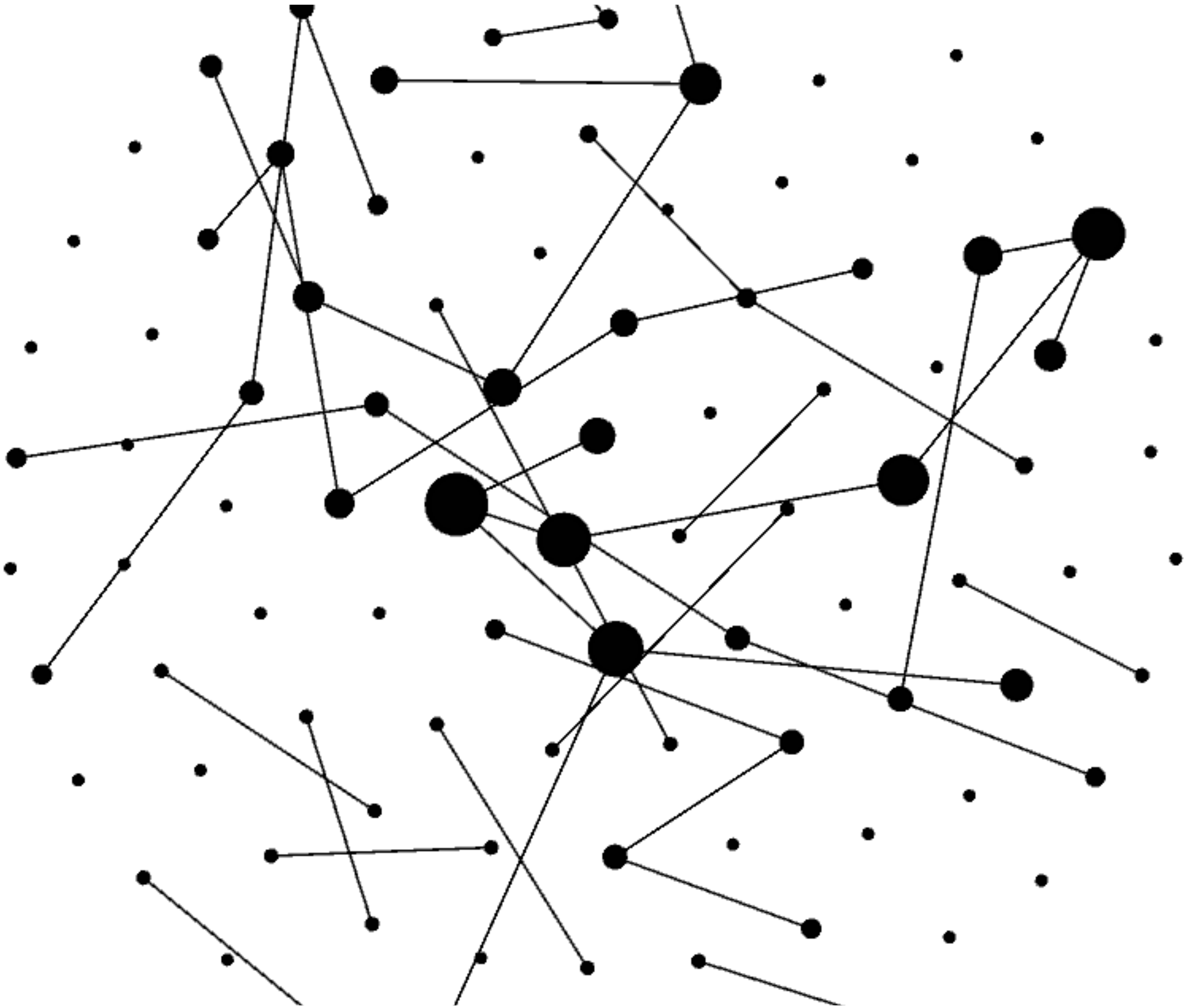
Agents



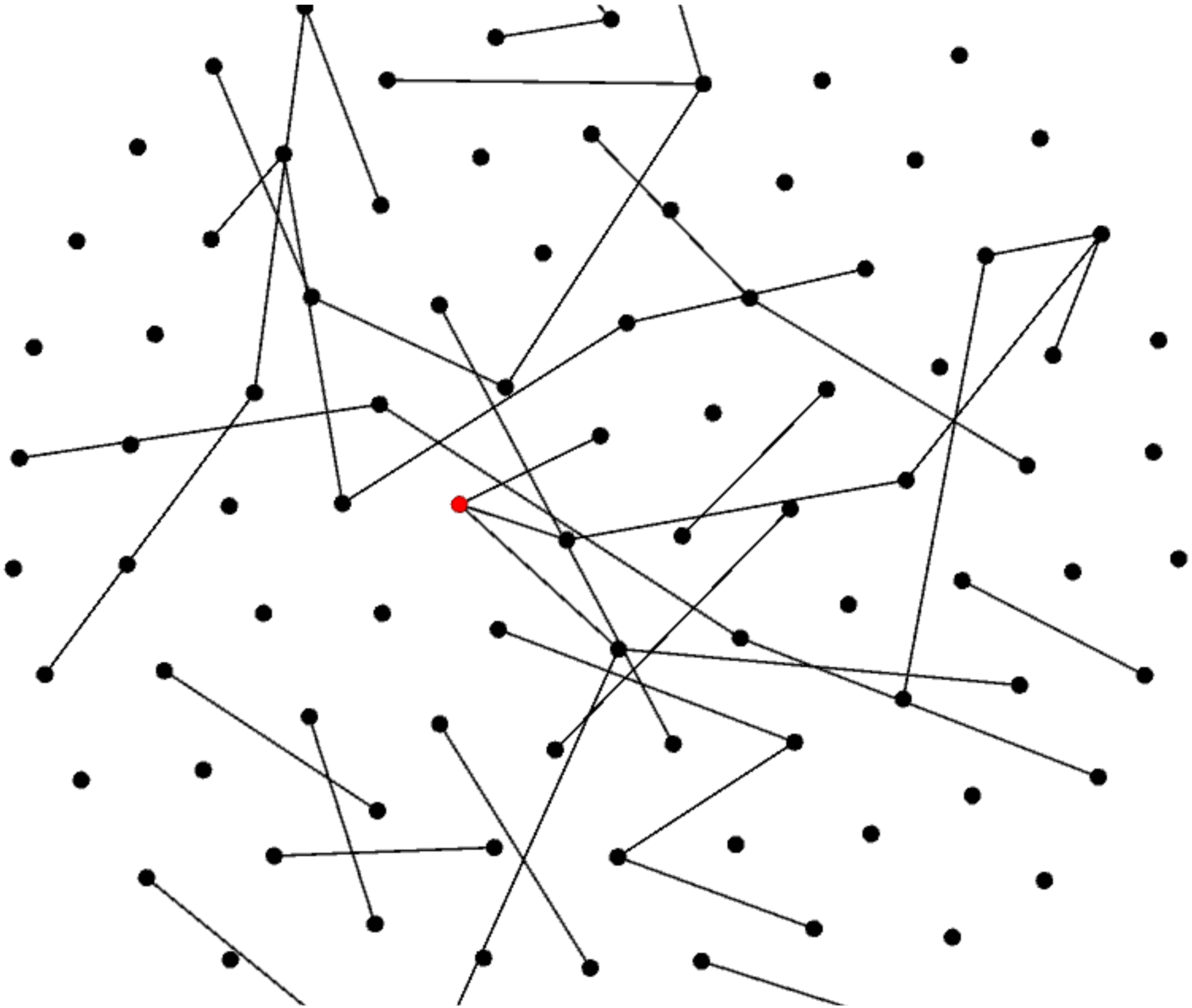
Erdos Reni, p = 0.009



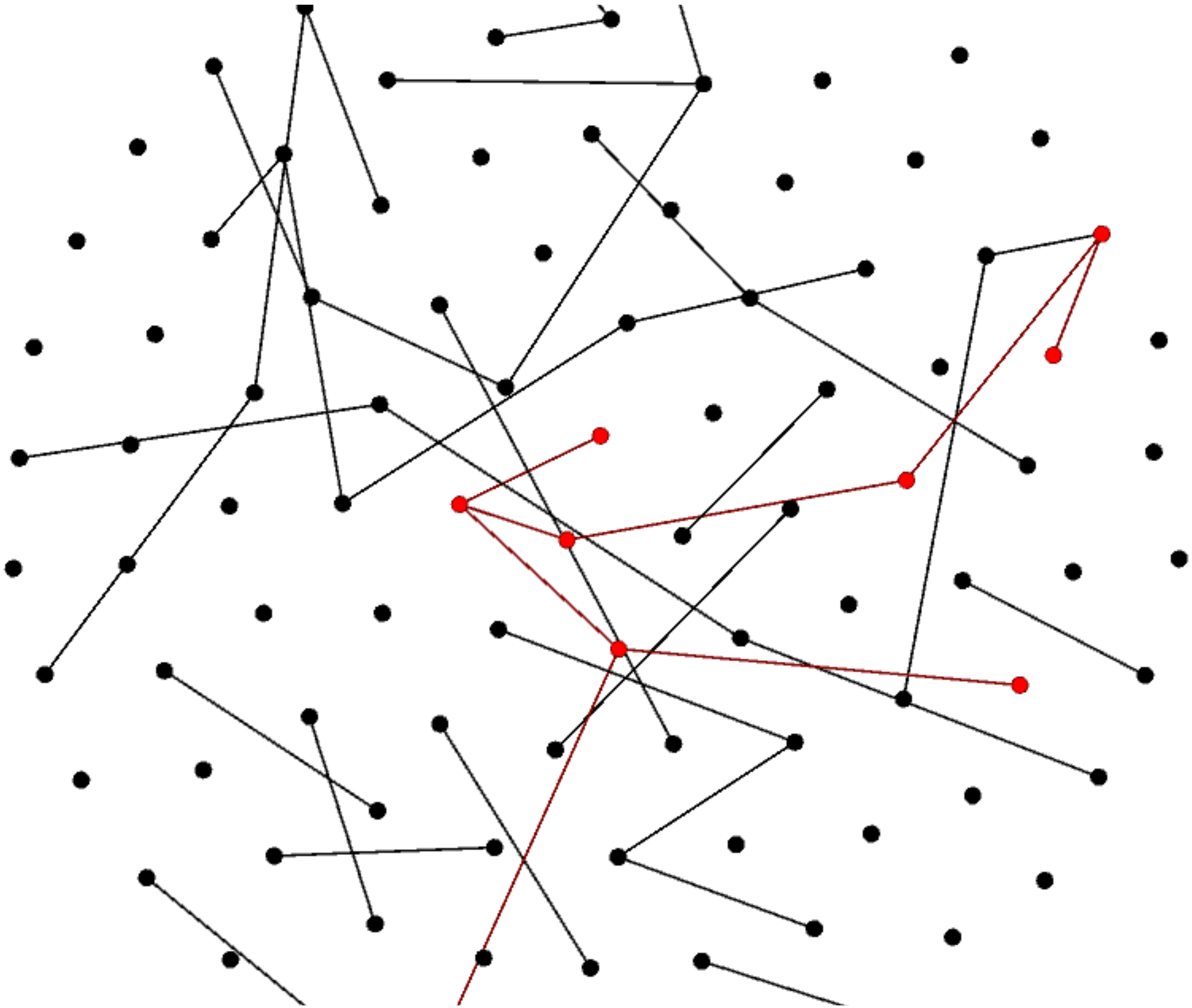
Eigenvector centrality



Seeding the Disease



T=10



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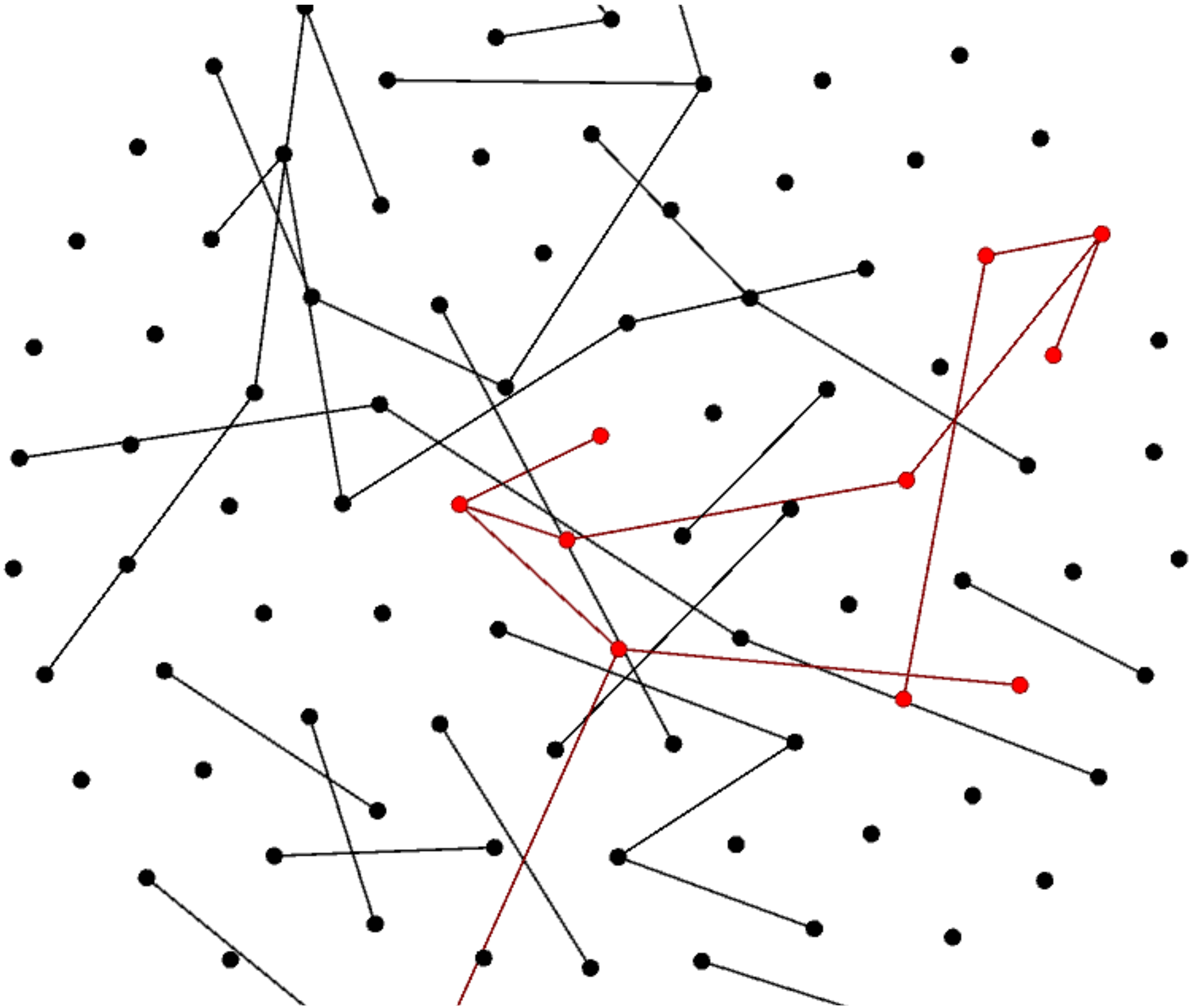
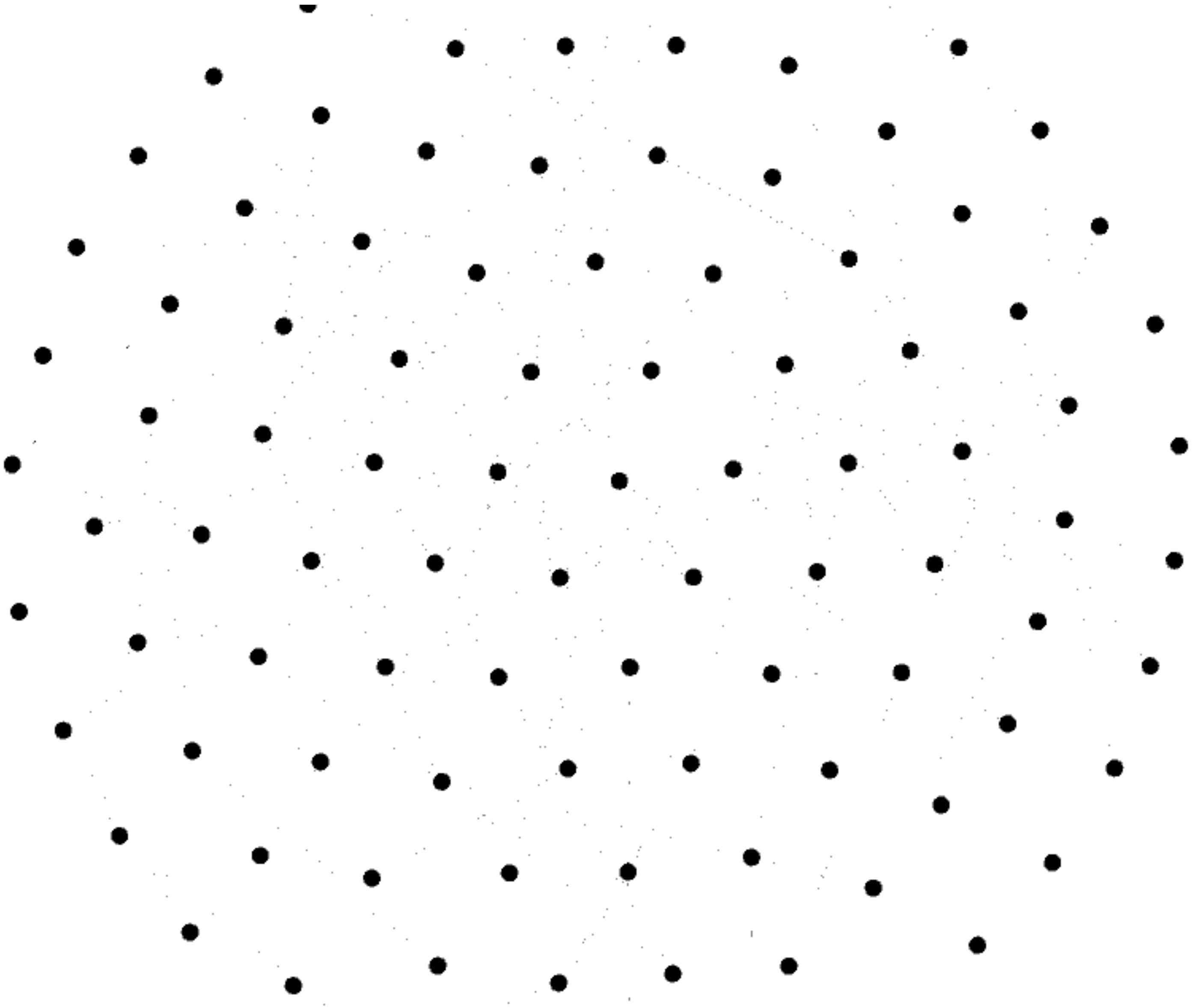
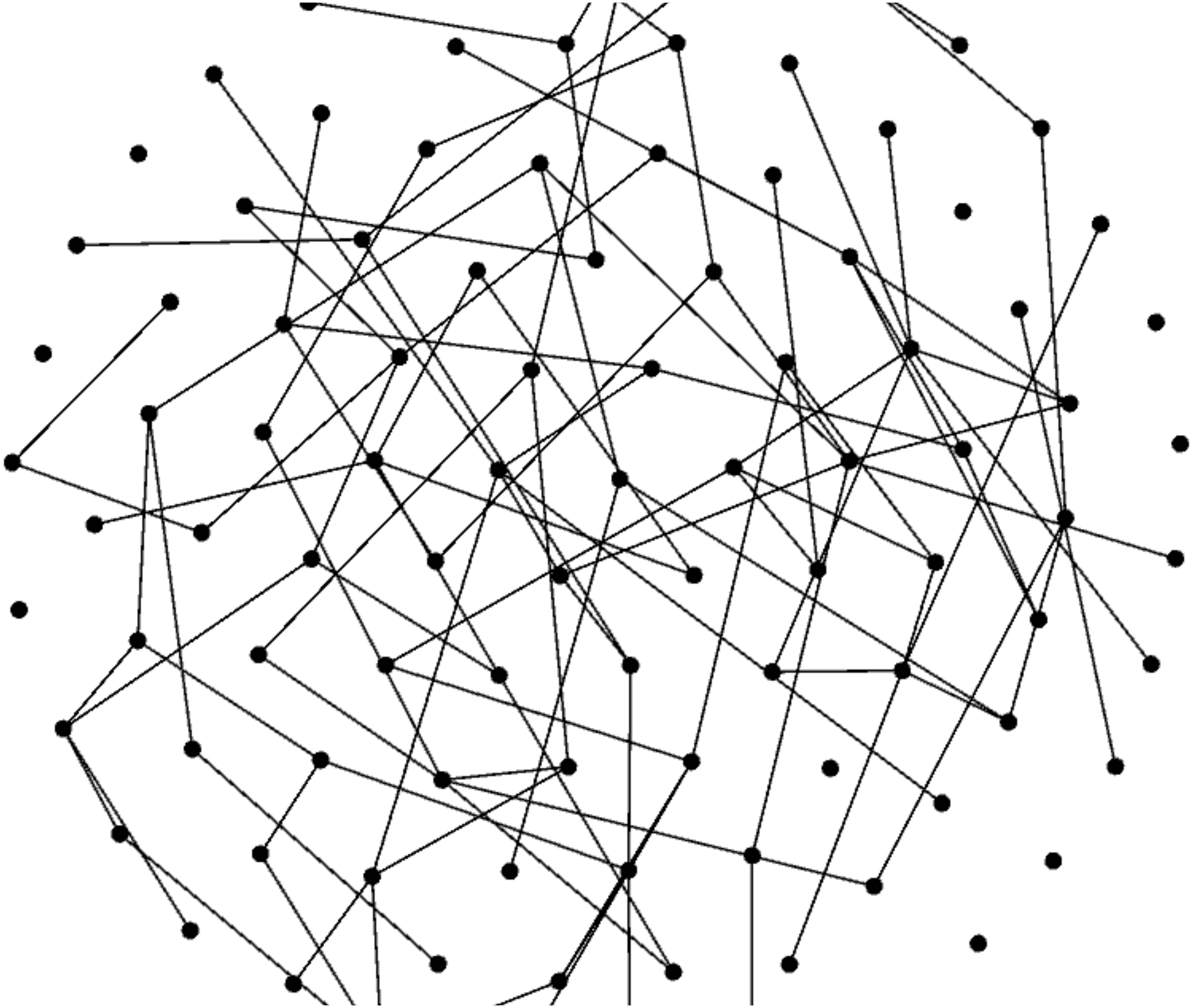


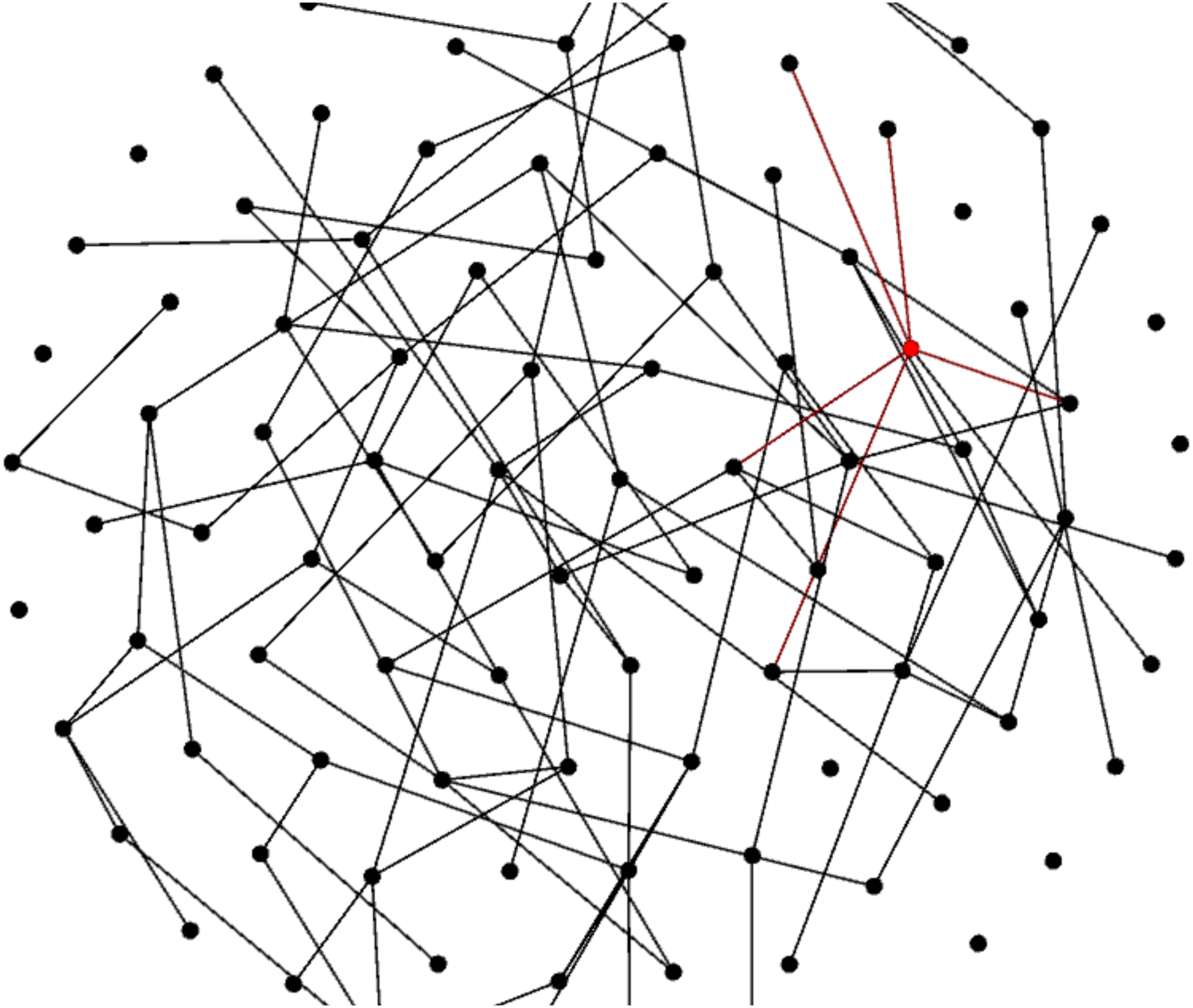
Same Agents



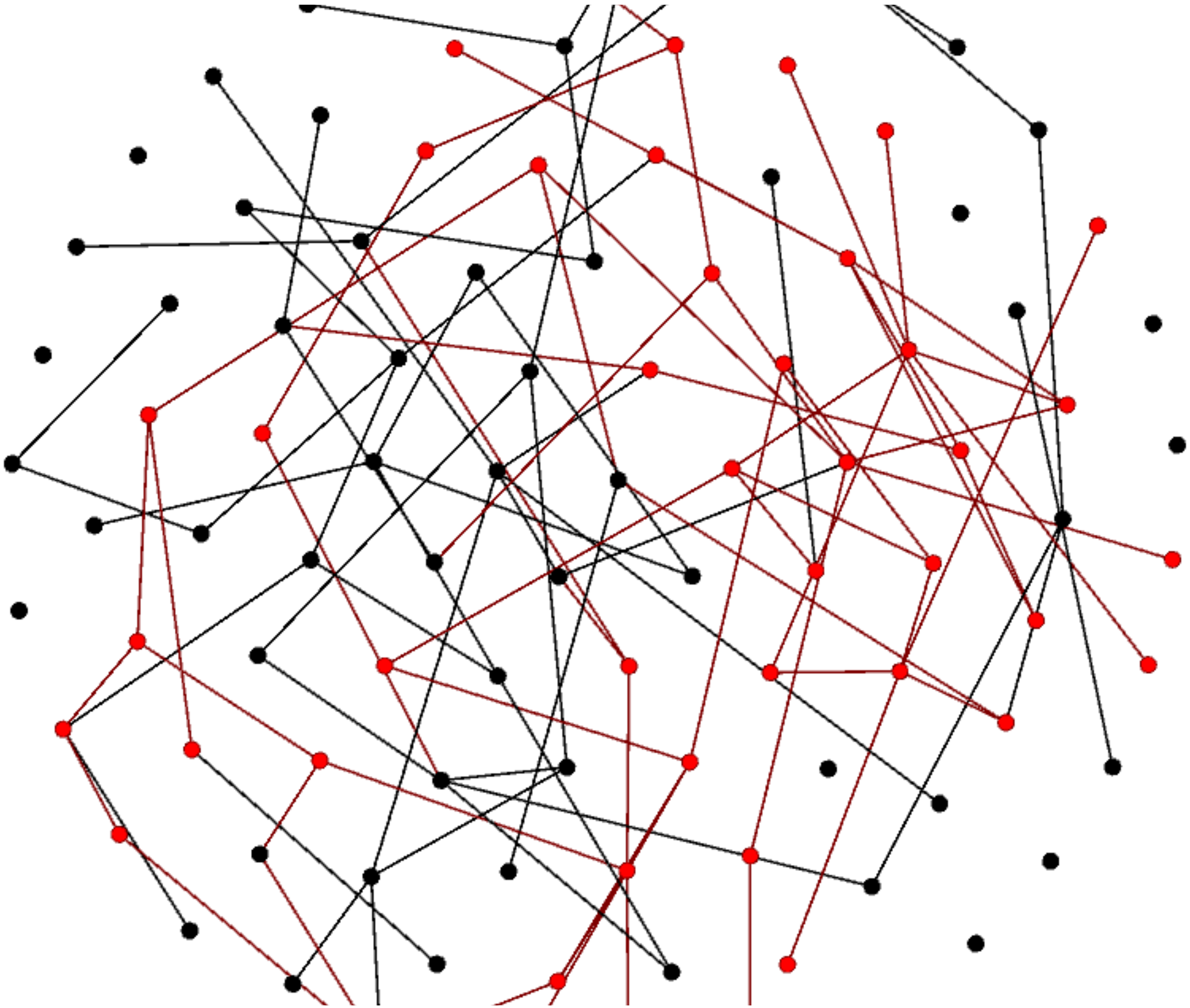
P=.02



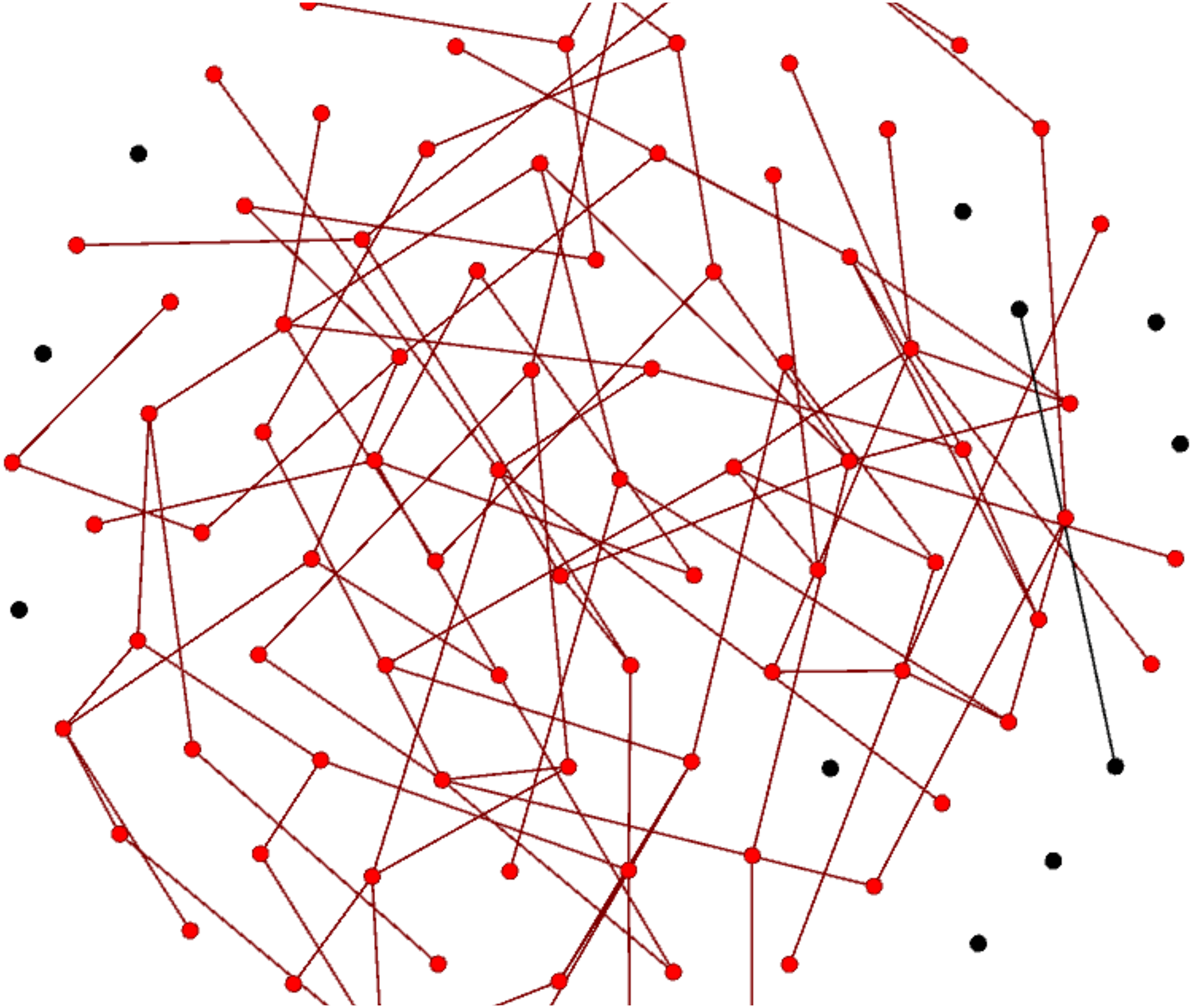
seeding



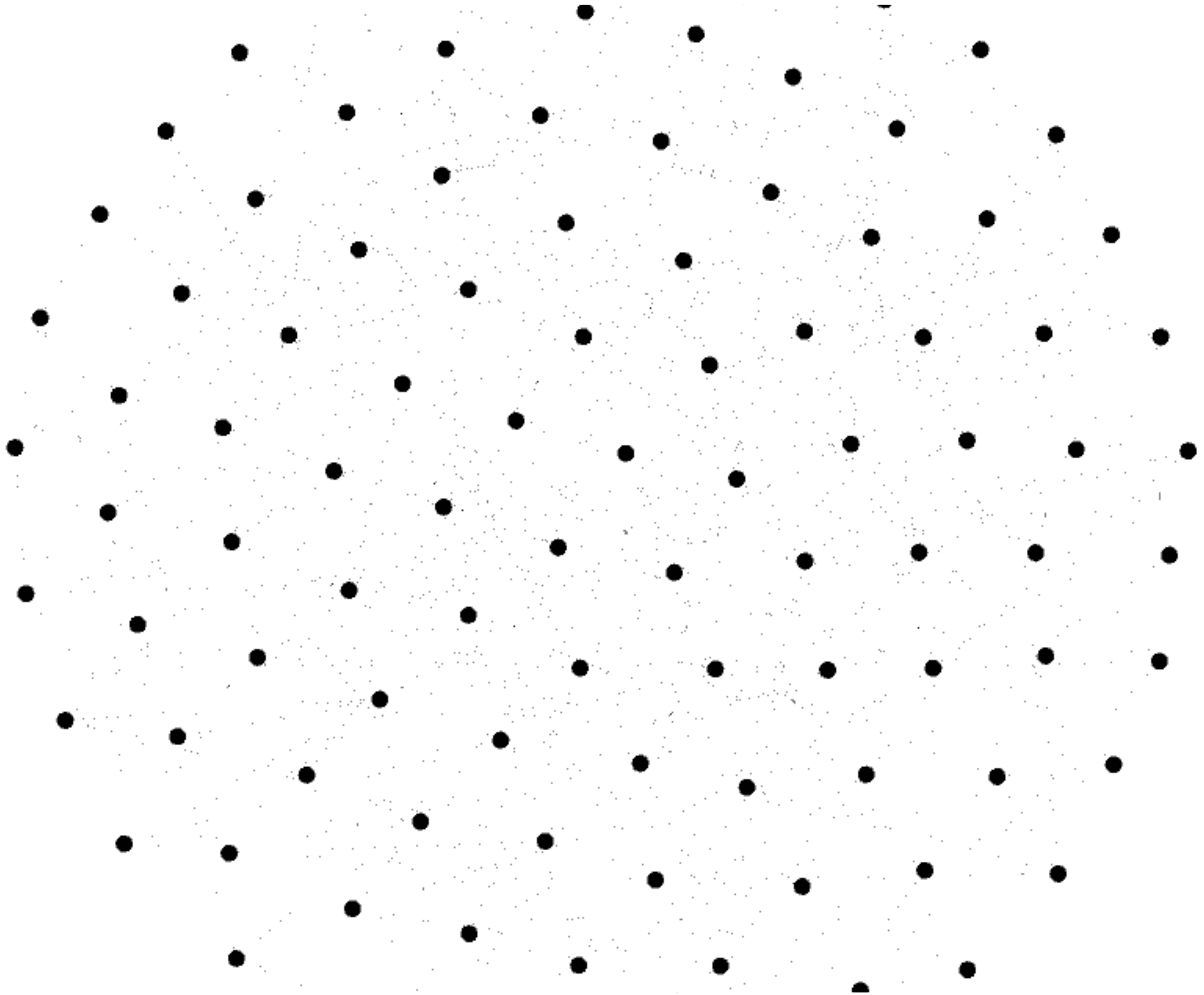
T=10



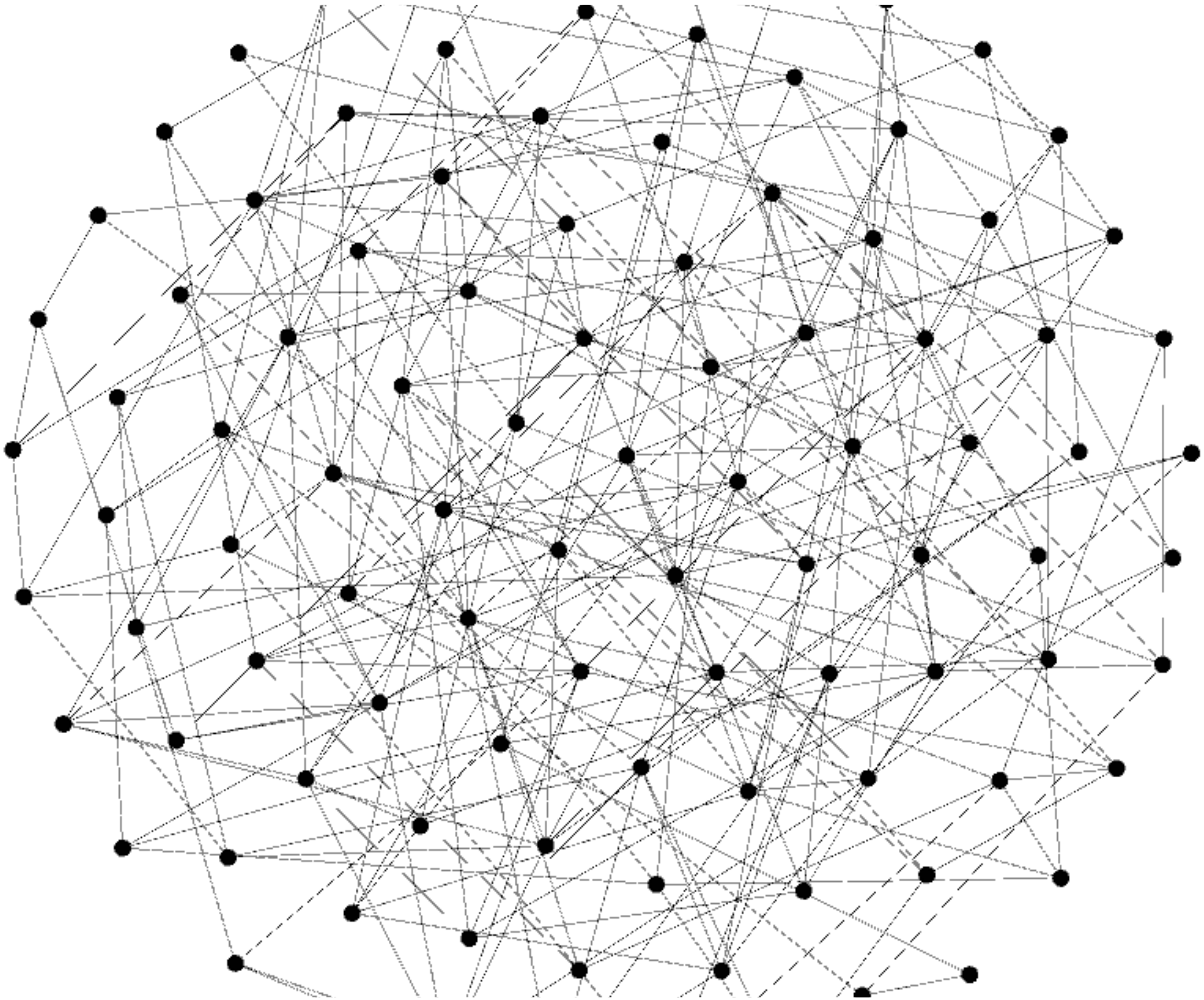
T=100



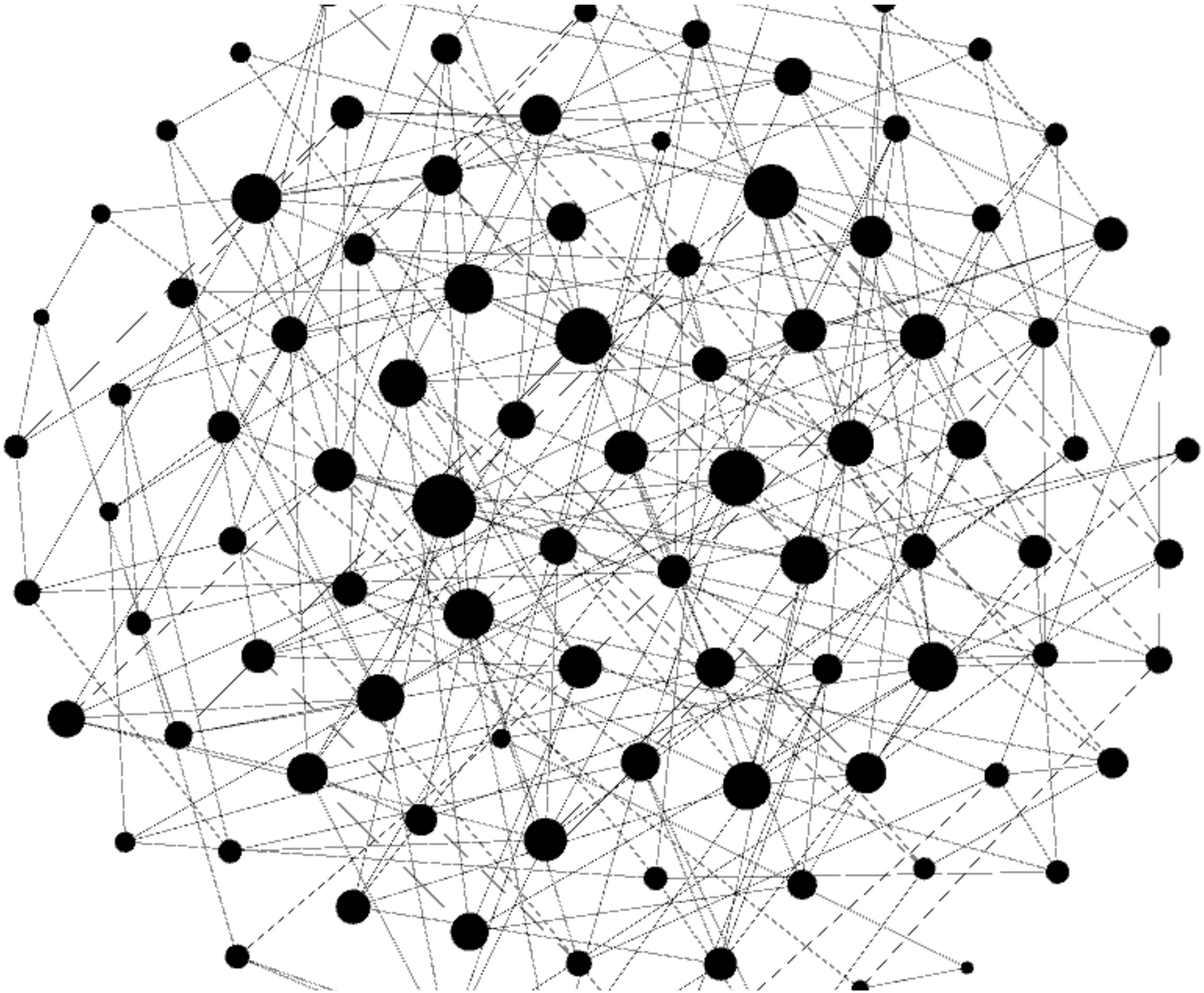
Nodes



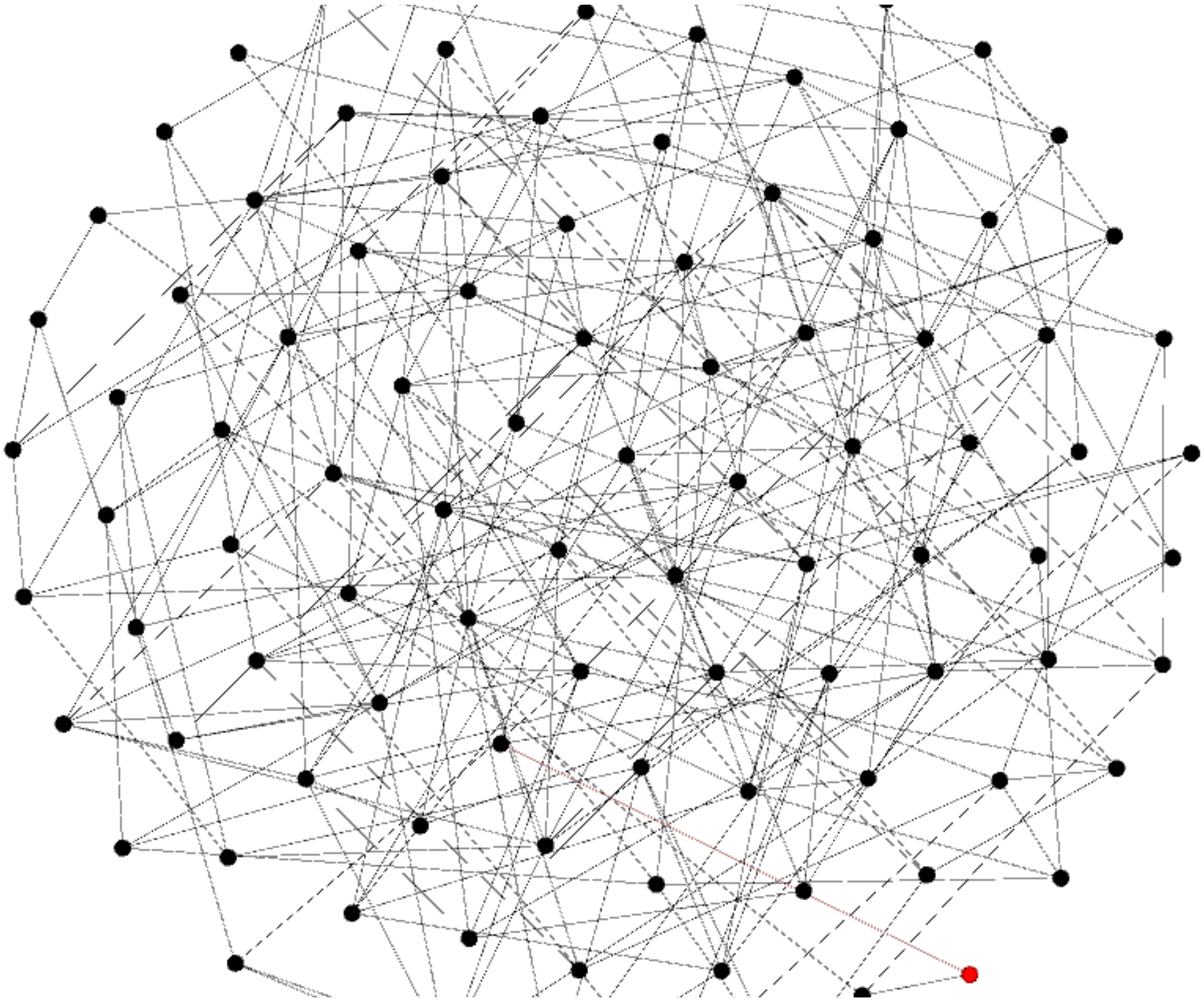
P=.06



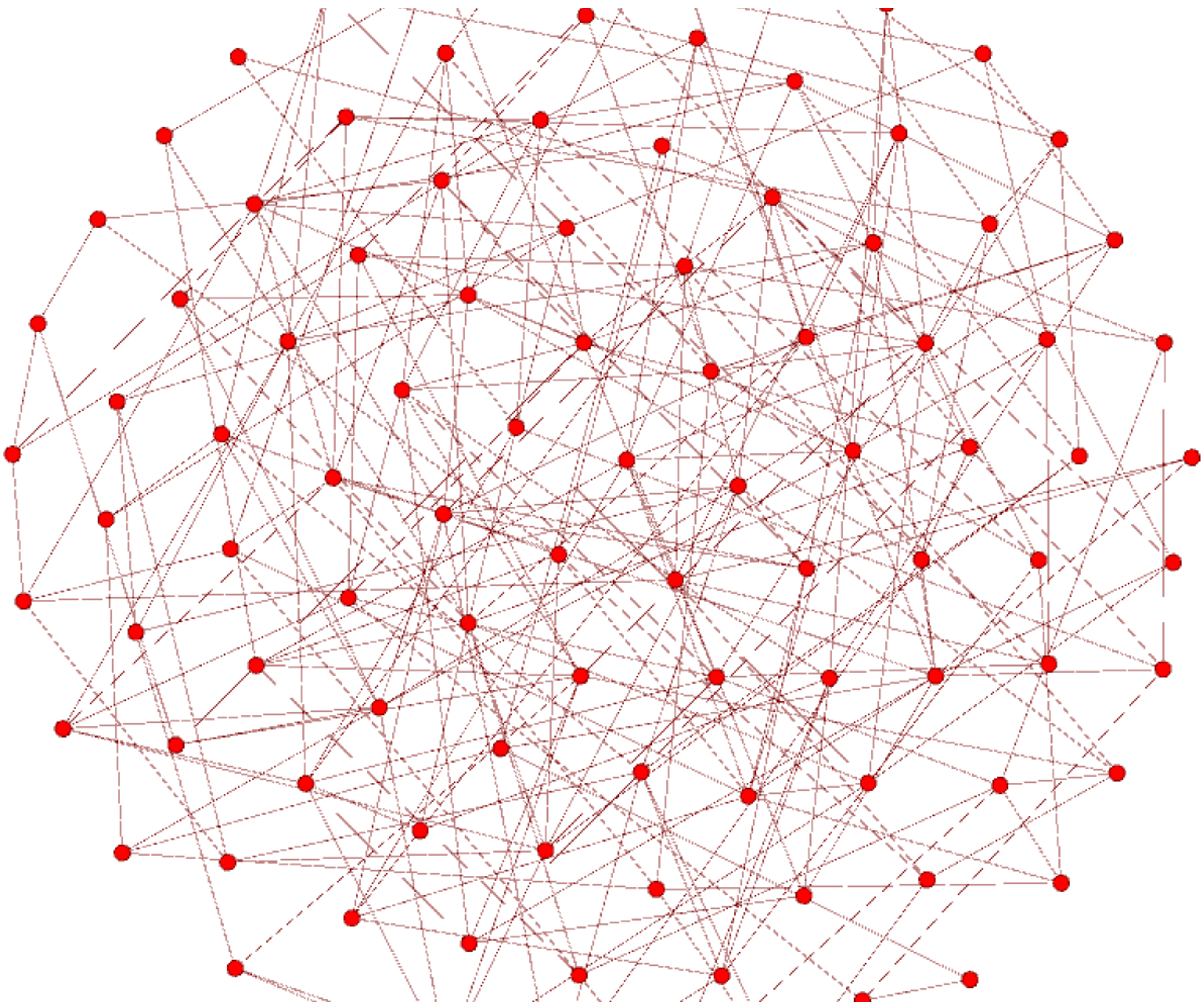
Centrality



Seeding



T=10



Reflection

- Interesting threshold for phase transition of random ER graph:

$$p_{\text{th}} = \frac{\log n}{n}$$

- Disease model is very simple
  - What if people recover?
  - What if they can be immunized?
  - How can we address such questions?

Network Effect: Do We Live in a Small World?

- Early 20th century Hungarian poet and writer Frigyes Karinthy first came up with the idea that we live in "small world". He suggested, in a play, that any two people among the one and a half billion inhabitants of the earth then were linked through at most five acquaintances.
- The sociologist Stanley Milgram made this famous in his study "The Small World Problem" (1967)—though this study is now largely discredited.
- He asked certain residents of Wichita and Omaha to contact and send a folder to a target person by sending it to an acquaintance, who would then do likewise etc., until the target person was reached. This would allow Milgram to measure how many "intermediate nodes" would be necessary to link the original sender and the target.
- 42 of the 160 letters supposedly made it to their target, with a median number of intermediates equal to 5.5.

Network Effect: Do We Live in a Small World?

- Hence was born the idea of six degrees of separation.
- Can you think why Milgram's procedure could give misleading results? Or why we may not wish to take these results on faith?
- There are similar studies for other types of networks.
- For example, Albert, Jeong, and Barabasi (1999) "Diameter of the World Wide Web" estimated that in 1998 it took on average 11 clicks to go from one random website to another (at the time there were 800 million websites, now 1 billion).
- What do these kind of "small world" results imply?

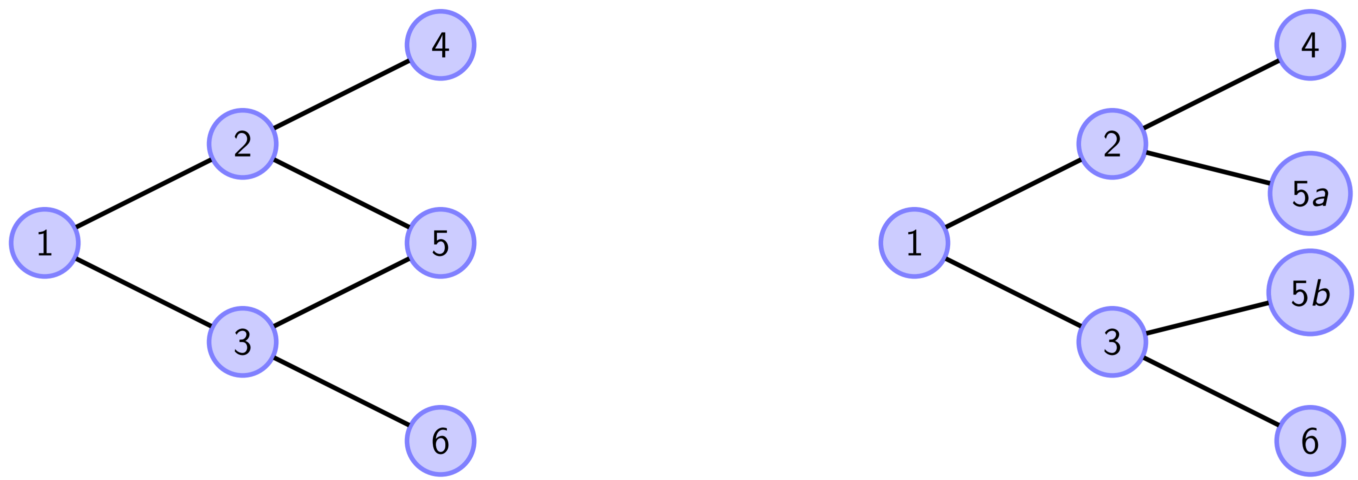
Network Effect: Interpreting Small Worlds

- Suppose that each node has λ neighbors (e.g., each website has links to λ other websites).
- Each of my λ neighbors will then have λ neighbors themselves.
- Suppose (unrealistically) that my neighbors don't have any neighbors in common (i.e., the λ websites that are linked to my website are not linked among themselves). Then in two steps, I can reach λ^2 other nodes.
- Repeating the same reasoning (and maintaining the same unrealistic assumption), in d steps I can reach λ^d other nodes.
- Now imagine that this network has $n = \lambda^d$ nodes.
- This implies that the "degrees of separation" (average distance) is

$$d = \frac{\ln n}{\ln \lambda}.$$

Network Effect: Interpreting Small Worlds

- But our unrealistic assumption rules out "social clusters", common in most networks including social networks



- Interestingly, Poisson (Erdos-Renyi) random graphs, we will see that average distance can be approximated for large n by $d = \ln n / \ln \lambda$ (where λ is the expected degree of a node).
- This is because relations shown in the figure are relatively rare in such graphs.

Network Effect: Interpreting Small Worlds

- This last result in fact can be interpreted as stating that Poisson (Erdos-Renyi) random graphs, though mathematically convenient, will not be good approximations to social networks.
- Per the Milgram conjecture, of six degrees of separation, each person should have had approximately
  - in 1960s, 38 friends: $(\exp[\ln(3,039,451,023)/6] \simeq 38.05)$
  - now, 44 friends: $(\exp[\ln(7,000,000,000)/6] \simeq 43.74$
- In reality, most people connect to their "geographic neighors"
  - And have *few* "special links" to people in remote part of the world
  - through political representatives, cousins in different city, undergraduate room-mates, etc.
- Models of "small world networks" try to capture this pattern (albeit not always perfectly).

Network Effect: Viral Advertisement

- How do people learn about new products?
  - Example: the Japanese graphic novel.
  - "Cult following" for movies or records.
- How does a new technology spread?
  - More important examples: the diffusion of new technologies and agriculture. Famous example: hybrid corn in the United States in the early 20th century. Spreading with a clear special pattern. Word-of-mouth from the early adopters important.
  - Similar patterns seen in prescription of new medication by doctors in the Midwest in the 1960s.
- How do people form their political, social and religious opinions?
  - Imitate family, friends and neighbors? Wisdom of the crowds?
  - More sophisticated information aggregation by talking and observing friends and news sources?
  - Does the social network matter?

The Central Question

- Networks are "enablers"
  - The structure of the network can play important role.
  - And, so does the dynamics over the network.

- Questions:
  - What are the commonalities between different networks and effects?
  - What are common analytic tools?
  - For example, diffusion of new technologies and spread of epidemics have certain commonalities
    - Does this mean that they obey the same logic?
    - Should we have a single theory to explain both?

- Key take-away: how to use understanding of networks, effects to better engineer them?

Structure: Graph Representation and Measures

- Network structure plays very important role in inducing effect. For example,
  - Communication network with single bottleneck can lead to network-wide failure / outage.
  - Or, social interaction network with multiple connections can make it difficult to prevent spread of disease.
- Graphs provide concise representation of network structure
- Different graph properties can lead to explanation of different effect
  - And, hence suggest what sorts of graphs are good to "engineer" to lead to desired effect.
  - For example, what network structure in Power grid will avoid cascaded failures?
  - Or, what structure of malware spread will avoid easy detection?
- Graph measures will play central role in capturing different aspects of network structure

The Rest of the Course

- **Part 1:** Network structures and dynamics – no strategy [S]
  - **Lectures 2 and 3**: Network representations, measures, metrics
  - **Lecture 4**: Linear dynamic systems
  - **Lecture 5**: Review of Markov chains, Markov chains on graphs
  - **Lecture 6 and 7**: Spread of information, Epidemics and distributed computation
  - **Lecture 8**: Centrality measures
  - **Lectures 9 and 10**: Random graph models
  - **Lectures 11 and 12**: Generative network models and other network models

The Rest of the Course

- **Part 2:** Network structures and dynamics – with strategy [T]
  - **Lectures 13 and 14**: Introduction to game theory
  - **Lecture 15**: Games in networks - I
  - **Lectures 16**: Games in networks - II
  - **Lectures 17**: Networked markets
  - **Lecture 18**: Strategic network formation
  - **Lectures 19 and 20**: Network effects, diffusion, and contagion
  - **Lectures 21 and 22**: Social learning, herding, and informational cascades
  - **Lectures 23 and 24**: Project presentations