- Why are some goods auctioned while others are sold by fixed prices?

- Why are some auction platforms (eBay) more successful than others, and is there a tendency towards concentration?

- How has the business of auctioning items online changed over the past 20 years?

- We will not talk about auction mechanisms (First Place Sealed Bid, Dutch, etc.) and auction theory yet –we'll talk about those topics when we get to sponsored search auctions.

Auction history

Auctions have been around for a long time.

- Ancient Rome: auctions were used to sell plunder from army conquests, household effects of wealthy people, slaves, wives, commodities, and even the entire empire.

  - 193 AD: Roman Empire was put up for auction by the Praetorian Guard. Marcus Didius Severus Julianus won (25,000 sesterces/soldier) and served as emperor for two months. Is it credible for the Praetorian Guard to sell the emperorship given that they have all of the power? It turns out that they executed Didius Julianus after 66 days.
  - "Auction" comes from the Latin *auctio* meaning "increase"
  - The highest bidder was "emptor." The more accurate translation of "*caveat emptor*" may be "winner's curse" not "buyer beware."

- Holland & tulips

  - traders from the Ottoman Empire brought tulip bulbs back to Holland in the 1600s
  - novel product, so demand unknown, potentially ↑. takes a long time (7-12 years) for cultivators to go from seed to tradable bulb, so supply fixed in the short run
  - sellers had to adapt to quickly changing demand conditions, so they invented the "Dutch Auction" (start at very high price and lower until someone buys –had advantage that there were fewer bids, so sales consummated faster than traditional auctions)
  - forward contracts & options were also pioneered before and during the Dutch Tulip Panic –if you didn't know, a famous commodity bubble ensued, followed by a crash of tulip prices

- Christie's opened in 1766: the first auction was two chamber pots, a pair of sheets, two pillow cases, and four irons.

- Sotheby's opened in 1744: the first auction was the sale of the library of Sir John Stanley.

- Both Christie's and Sotheby's are still very active to this day in fine art, jewelry, decorative arts, and collectibles auctions.

- Today, the government routinely conducts auctions for surplus goods, land leases (oil & gas drilling), electromagnetic spectrum, etc.

- It also runs procurement auctions for items or services it wants to buy, such as fighter jets, a new bridge, a security detail in Afghanistan, etc.

Online auctions

- Before 1993 (when Mosaic, the first web browser was released), online auctions took place on text-based Internet newsgroups and email discussion lists.

    - the interfaces and degree of automation were nothing like today.

    - auctions were for specialized items that the newsgroup would be interested in.

    - bids were submitted via email and sellers would post daily updates of high bids.

    - the ending date would sometimes be announced, but sometimes not.

- May 1995: Onsale launched as a merchant site selling its own goods (mostly refurbished electronics)

    - Onsale became a listing service in 1997, started selling fixed price in 1998, and merged with Egghead in 1999

- Sept 1995: eBay launched as a listing site. eBay was only listing others' goods (which could be anything), never taking possession of goods or receiving payments (only charging a listing fee).

- these two sites offered automated bids, search engines to locate goods, and clickable categories.

- eBay revenues (not volume of sales, which is much larger):

1996	$372,000
1997	$41,370,000
1998	$81,129,000
...	
2005	$4,552,401,000
...	
2011	$11,651,654,000

Why auction?

If you have a good to sell, you can:
1) post a price and offer it to the first buyer willing to pay that price.
2) auction the good off.

Considerations:

- transaction costs are significantly higher for auctions

    - the seller has to monitor and wait to sell

    - the buyer has to monitor and wait to purchase. the buyer also might lose and have to bid on a similar item in another future auction

- seller receives, for free, information on the value of the good (in fact, the whole distribution of buyers' values, even those who do not end up transacting)

So what do we infer about the choice of sales mechanisms given these arguments?

Some goods are more likely to be auctioned:

- unique goods

- expensive goods (transaction costs might not scale with price but foregone surplus from uncertainty might)

- goods whose characteristics are costly to assess

- goods where buyers know more than sellers

- goods with heterogeneity in buyer valuation

Examples:

- keywords on Google: there are so many words and Google couldn't possibly figure out how to price each one.

- radio spectrum: Verizon, T-Mobile, AT&T know much more about what it's worth than the government does

- houses: unique, expensive, heterogenous valuation, unknown participants

- art

- leases for tracts of land for drilling

- right to build sixth generation jet fighters for US military: bid negative amounts and the bid that is smallest in absolute value wins

- senate seats (Rod Blagojevich)

- wholesale used cars: cars are costly to transport and dealers like a particular mix of used cars on their lot, which leads to heterogeneity in valuation

- beef cattle

Simple model

- N consumers whose valuations V_i of a product are distributed uniformly on a unit interval.

- the seller can offer 1 unit of the product (marginal cost = 0) posted at price or auction it off.

- the seller knows the distribution of valuations but not individual realizations from that distribution.

- fixed price

  price at P, sell if any $V_i \geq P$

  $$E(\Pi(P)) = P \underbrace{\text{Prob}\{V_i \geq P \text{ for at least one } i\}}_{\substack{\text{1 - CDF of Nth order statistic} \\ \text{from uniform PDF evaluated at } P}}$$

  CDF for $u[0,1]$ is $f(x) = x$, $x \in (0,1)$

  so, $\text{Prob}\{V_i \geq P \text{ for at least one } i\} = 1 - P\{V_i < P \text{ for all } V_i\}$
  $$= 1 - P^N$$

  to maximize, we take the first-order condition:

  $$\frac{\partial E\Pi}{\partial P} = 1 - (N+1)P^N$$

  $$P^* = \sqrt[N]{\frac{1}{N+1}}$$

$$\text{and } E\Pi(P^*) = \sqrt[N]{\frac{1}{N+1}}(1 - \frac{1}{N+1})$$

$$= \frac{N}{N+1}\sqrt[N]{\frac{1}{N+1}}$$

N	P	$E\Pi$
1	$\frac{1}{2}$	$\frac{1}{4}$
2	$\sqrt{\frac{1}{3}} \approx 0.58$	$\frac{2}{3}\sqrt{\frac{1}{3}} \approx 0.38$
3	$\sqrt[3]{\frac{1}{4}} \approx 0.63$	$\frac{3}{4}\sqrt[3]{\frac{1}{4}} \approx 0.47$
...		
9	$\sqrt[9]{\frac{1}{10}} \approx 0.77$	$\frac{9}{10}\sqrt[9]{\frac{1}{10}} \approx 0.70$

- auction

    - price of product gradually increases
    - consumers stay in bidding for product until $P > V_i$
    - when only one consumer is left, he gets product –note that he gets it at V_{N-1}, the second highest valuation
    - so to compete expected profit, we need a distribution of $(N-1)$th order statistics[1] from uniform

$$E(\Pi(N)) = \int_0^1 N(N-1)(1-x)x^{N-2}x\,dx$$

$$= N(N-1)\int_0^1 (x^{N-1} - x^N)\,dx$$

$$= \frac{N(N-1)}{N} - \frac{N(N-1)}{N+1}$$

$$= \frac{N(N-1)}{N(N+1)}$$

$$= \frac{N-1}{N+1}$$

N	P	$E\Pi$
1		0
2		$\frac{1}{3}$
3		$\frac{1}{2}$
...		
9		0.8

    - so for $N > 3$, auction is better for the seller
    - more formally, auction better for seller if:

$$N - 1 > N\sqrt[N]{\frac{1}{N+1}}$$

$$1 - \frac{1}{N} > \sqrt[N]{\frac{1}{N+1}}$$

$$(1 - \frac{1}{N})^N > \frac{1}{N+1}$$

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[1] See notes on "Order Statistics"

- for large N, $\approx \frac{1}{e}$, so auction is better for large N
- N.B.: auction doesn't require knowledge of the distribution of valuations. Fixed price is very sensitive to characteristics of the distribution
  * if $V\ u[0,2]$ or $V\ u[0,\frac{1}{2}]$, fixed price does terribly
- this model did not have transaction costs

Order Statistics

$X_1, ..., X_n^x$ continuous, independent, identically distributed, each with PDF $f_X(x)$

- "independent, identically distributed" is abbreviated as "iid" and also called a "random sample"

  $Y_n = \max\{X_1, ..., X_n\}$ (Y_n is the largest value in the sample)

  CDF of $Y - n$: $G_{Y_n}(y_n) = P(Y_n \leq y_n) = P(X_1 \leq y_n, X_2 \leq y_n, ..., X_n \leq y_n)$
  $\hookrightarrow$ definition of function
  $= P(X_1 \leq y_n)P(X_2 \leq y_n)...P(X_n \leq y_n)$
  $\hookrightarrow$ because $x_1, ..., x_n$ are independent
  $= F_X(y_n)F_X(y_n)...F_X(y_n)$
  $\hookrightarrow$ definition of CDF, x's identical distribution
  $= [F_X(y_n)]^n$

  PDF of Y_n: $\boxed{g_{Y_n}(y_n) = \dfrac{dG_{Y_n}(y_n)}{dy_n} = n[F_X(y_n)]^{n-1}f_X(y_n)}$

  similarly, $Y = \min\{X_1, ...X_n\}$ (Y_1 is the smallest value in sample)

  $\boxed{g_{Y_1}(y_1) = n[1 - F_X(y_1)]^{n-1}f_X(y_1)}$

  e.g. $X_i \sim U[0,1], i = 1, 2, ...5 \to f_X(x) = 1, F_X(x) = x, x \in [0,1]$
  $Y_5 = \max\{X_1, ..., X_5\}$
  $g_{Y_5}(y) = 5y^4(1)$ for $y \in [0,1]$
  $= 5y^4$

What's the PDF of these guys?
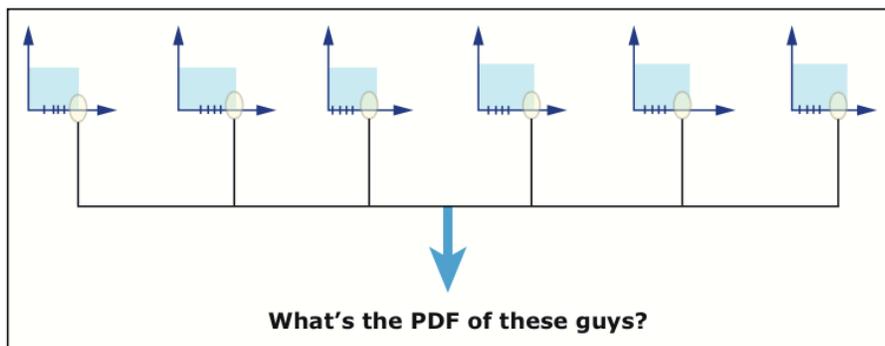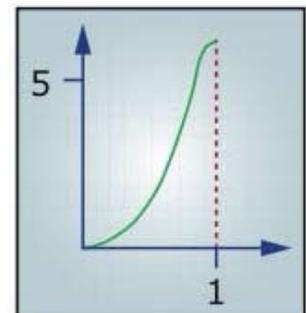
$X_1, ..., X_n$ iid $f_X(x)$

$Y_n = \max\{X_1, ..., X_n\}$

$G_{Y_n}(y_n) = [F_X(y_n)]^n$

$g_{Y_n}(y_n) = n[F_X(y_n)]^{n-1} f_X(Y_n)$

e.g. $X_i \approx \varepsilon(\lambda)$ for $i = 1, ..., 10$ $f_x(x) = \lambda e^{-\lambda x}, x > 0$

$\quad Y_{10} = \max\{X_1, ..., X_{10}\}$ $F_x(x) = 1 - e^{-\lambda x}, x > 0$

$\quad g_{Y_{10}} = 10(1 - e^{-\lambda y})^9 \lambda e^{-\lambda y}$ for $y > 0$ $0, x \leq 0$

$F_X(x) = 1 - e^{-Xx}, x > 0$

$\qquad 0, x \leq 0$
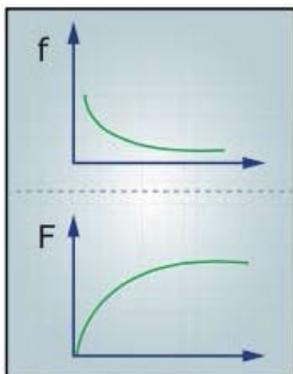
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