Problem Set #2

Due September 25, 2013

1. Suppose you are the manufacturer of surfboards which are sold in two separate markets: California and Hawaii. You have factories in both locations, and each can produce an unlimited number of surfboards at a constant marginal cost of $10 per surboard. Over the last fourteen weeks you've conducted an experiment by varying your prices each week. Your sales at various prices were:

Price	Q in Calif.	Q in Hawaii
10	130	31
11	106	27
12	105	31
14	100	24
15	60	24
16	70	25
17	65	18
18	60	23
20	48	21
22	28	14
24	12	18
25	2	14
26	1	10
30	0	9

(a) Use an OLS regression to estimate linear demand curves for each market.

(b) Given these estimated demand curves what prices would you set in each market? How would you change these prices if antitrust laws required that you set a common price across both markets?

(c) How would profits and consumer surplus be affected by the shift to uniform pricing?

(d) Suppose retailers can ship surfboards between California and Hawaii for $4 per board. Would this disturb your discriminatory pricing strategy, and if so what would your response be? How is this problem similar to and different from a standard 2nd degree price discrimination model?

2. The most common textbook version of the monopoly pricing problem has a monopolist with $c = 0$ choose p to maximize $pD(p)$ with $D(p) = 1 - p$. One way to motivate why the monopolist uses linear pricing (rather than some nonlinear function of the quantity demanded) is to assume that the aggregate demand curve comes from a continuum of consumers with unit demands. Specifically, one assumes that the types θ are uniform on $[0, 1]$, and that a type θ consumer receives utility $\theta - p$ if she buys one unit and utility 0 if she does not buy.

One often hears it said that no nonlinear pricing is possible in this model. All the monopolist can do is to set the price p at which it will sell one unit.

In fact, there is a way to implement nonlinear pricing in this model. The monopolist could sell lottery tickets that provide a probability q of getting the good for any $q \in [0, 1]$. These lottery tickets are like higher and lower quality versions of the good and could be used to price discriminate. As in the continuum-of-types price discrimination model, any nonlinear tariff $T(q)$ is implicitly a

choice to have a type θ consumer purchase quality $q(\theta)$ at price $p(\theta) \equiv T(q(\theta))$. We typically solve these problems by viewing $q(\theta)$ and $p(\theta)$ as choice variables chosen subject to the consumer IR and IC constraints. The type θ consumer's equilibrium expected utility is $u(\theta) = \theta q(\theta) - p(\theta)$.

(a) What are the full set of IR and IC constraints for this model?

(b) The binding IC constraints in a problem like this are usually that the type θ consumer must be exactly indifferent to imitating the type $\theta - d\theta$ consumer. Use this IC constraint to find $\frac{du}{d\theta}$. Integrate this expression to find $u(\theta)$.

(c) Using $\pi = \int_0^1 \theta q(\theta) - u(\theta) d\theta$ find the optimal nonlinear policy. (Hint: Unlike in the general problem, you don't need to integrate by parts. You can just change the order of integration.)

(d) What does the answer to part (c) tell us about the unit demand justification of linear pricing? How is the outcome of this problem different from the typical outcome of the nonlinear pricing problem as I described in class. What aspect of this model seems likely to be responsible for the differences?

3. Consider a model with consumers uniformly distributed on the interval $[0, 1]$. Two suppliers selling the same good are located at points a and $1 - b$ with $0 \le a, b \le \frac{1}{2}$. Their production costs per unit are c_1 and c_2, respectively. Consumers buy zero or one unit of the good. They receive zero utility if they don't buy the good and utility $v - p - tx^2$ if they buy the good from a firm at a distance of x from their location. Assume that the firms choose prices simultaneously, and that their objective is to maximize profits.

(a) Find the Nash equilibrium prices and profits in this model assuming that v is sufficiently large so that the equilibrium involves all consumers purchasing the good. How large can firm 1's cost disadvantage be if it does make positive profits in equilibrium?

(b) Suppose that before choosing prices the firms play a first period game where they simultaneously choose where to locate. Assume that the firms costs are equal, $c_1 = c_2 = c$. Show that in equilibrium the firms are maximally differentiated.

4. Consider the model of vertical differentiation discussed in class (and in section 7.5.1 of Tirole). Suppose that the firms' costs are higher than I assumed so that the equilibrium prices end up being such that some consumers do not buy the product. Write down the equations for demand when prices are such that the highest value consumers buy from firm H, some buy from firm L and some do not buy at all. Assuming that the best responses are always given by the first order conditions obtained by maximizing relative to these demands find the best response functions and solve for the Nash equilibrium. For what values of c do the equations you've written really give the Nash equilibrium of the game?

5. Read at least the first 10 pages of Chu, Leslie, and Sorensen's 2011 AER paper "Bundle-Size Pricing as an Approximation to Mixed Bundling." Suppose that you wanted to write a theoretical paper that complemented their results. Briefly describe two sets models you might try to solve as part of such a paper. Discuss whether the models are likely to be tractable and why people might or might not find them to be valuable additions to the literature. If you're more interested in empirical work, you can instead read section III, think about whether there are other applications for which a similar analyis might be interesting, and discuss a potential application commenting on how you might estimate joint distributions of consumer valuations.

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