Formula Sheet Exam 3

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Herman Bennett

Let $X_1, ..., X_n$ be a random sample of size n from a $N(\mu, \sigma^2)$ population. Then,

a. $\bar{X}$ and S^2 are independent random variables. (1)

b. $\bar{X}$ has a $N(\mu, \sigma^2/n)$ distribution. (2)

c. $\dfrac{(n-1)S^2}{\sigma^2}$ has a $\chi^2_{(n-1)}$ distribution. (3)

Let $X_1, ..., X_n$ be *iid* random variables with $E(X_i) = \mu$ (finite) and $\text{Var}(X_i) = \sigma^2$ (finite). Define $\bar{X}_n = \frac{1}{n}\sum_{i=1}^{n} X_i$. Then,

$$\lim_{n\to\infty} P(|\bar{X}_n - \mu| < \varepsilon) = 1 \quad \text{For every number } \varepsilon > 0.$$

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$$\lim_{n\to\infty} P\left(\frac{\sqrt{n}(\bar{X}_n - \mu)}{\sigma} < x\right) = \int_{-\infty}^{x} \frac{1}{\sqrt{2\pi}} e^{-x^2/2} = \Phi(x) \tag{4}$$

Let X be a RV such that $P(X \geq 0) = 1$. The for any number $t > 0$,

$$P(X \geq t) \leq \frac{E(X)}{t} \tag{5}$$

Let X be a RV for which $Var(X)$. Then for any number $t > 0$,

$$P(|X - E(X)| \geq t) \leq \frac{Var(X)}{t^2} \tag{6}$$

$$MSE(\hat{\theta}) = E[(\hat{\theta} - \theta)^2] = Var(\hat{\theta}) + (bias(\hat{\theta}))^2 \tag{7}$$

$$L(\theta|\mathbf{x}) = L(\theta_1, ..., \theta_k | x_1, ..., x_n) = f(x_1, ..., x_n | \theta_1, ..., \theta_k) \tag{8}$$

Let $X \sim N(0,1)$ and $Z \sim \chi^2_n$ be independent RVs. Then, the RV H is distritbuted t-student with n degrees of freedom.

$$H = \frac{X}{\sqrt{Z/n}} \sim t_{(n)} \tag{9}$$

Let $X \sim \chi^2_n$ and $Z \sim \chi^2_m$ be independent RVs. Then, the RV G is distritbuted F with n and m degrees of freedom.

$$G = \frac{X/n}{Z/m} \sim F_{(n,m)} \tag{10}$$

$$\pi(\theta|\delta) = P\Big(\text{rejecting } H_0 \Big|\ \theta \in \Omega\Big) = P(\mathbf{X} \in C|\theta) \qquad \text{for all } \theta \in \Omega. \tag{11}$$

$$W = \frac{\sup_{\theta \in \Omega_1} L(\theta_1, ..., \theta_k | x_1, ..., x_n)}{\sup_{\theta \in \Omega_0} L(\theta_1, ..., \theta_k | x_1, ..., x_n)} = \frac{\sup_{\theta \in \Omega_1} f(\mathbf{x}|\theta \in \Omega_1)}{\sup_{\theta \in \Omega_0} f(\mathbf{x}|\theta \in \Omega_0)}. \tag{12}$$

$$T = \frac{\sup_{\theta \in \Omega_0} L(\theta_1, ..., \theta_k | x_1, ..., x_n)}{\sup_{\theta \in \Omega} L(\theta_1, ..., \theta_k | x_1, ..., x_n)} = \frac{\sup_{\theta \in \Omega_0} f(\mathbf{x}|\theta \in \Omega_0)}{\sup_{\theta \in \Omega} f(\mathbf{x}|\theta \in \Omega)} \tag{13}$$

$$-2lnT \ \overset{n \to \infty}{\sim} \ \chi^2_{(r)} \ ; \tag{14}$$

where r is the # of free parameters in Ω minus the # of free parameters in Ω_0.