Recitation 6

Fundamentalness

This section is largely based on a review paper by Alessi, Barigozzi, and Capasso (2008). Part of the motivation for studying factor models is the concern that VARs might be nonfundamental, i.e. the variables included in the VAR might not span the space of structural shocks. Nonfundamentalness can lead to misleading results. Anna talked about the price puzzle where a VAR using just inflation and the federal funds rate erroneously finds that tight monetary policy increases inflation. Another important debate related to fundamentalness is the Lippi and Reichlin (1993) critique of Blanchard and Quah (1989). Here, nonfundamentalness arises because the MA represetation of an economic model is not invertible, rather than because we have not included enough variables. (Another point of view is that the economic model implies that there are inherently non-observable variables, such as agents' higher order expectations, that force the VAR to be nonfundamental.) After log-linearizing around the steady state, most macro models lead to an MA representation:

$$x_t = C(L)u_t$$

If $det(C(z))$ has no roots on or inside the unit circle, then the model is fundamental and we can invert $C(L)$ to get a one-sided VAR representation,

$$D(L)x_t = u_t$$

If $det(C(z))$ has no roots on the unit circle, but does have roots inside, then it is fundamental and inverting $C(L)$ gives a two-sided $D(L)$. Lippi and Reichlin (1993) pointed out that a small modification of the motivating model of Blanchard and Quah, namely adding extra dynamics due to learning by doing, leads to a nonfundamental VAR representation. This is problematic because if we do not know that the structural shocks have a fundamental representations, then the reduced form shocks are related to the structural shocks by

$$e_t = H(L)u_t$$

where $H(L)$ is a Blaschke filter, i.e. $H(e^{i\theta})'H(e^{-i\theta}) = I$. There are far too many such Blaschke filters to derive any identifying restrictions on them.

Nonfundamental representations are also common for models with rational expectations, as first pointed out by Hansen and Sargent (1980). Nonfundamentalness almost always happens in rational expectation models with asymmetric information. I think Beaudry and Portier try to explain their findings using such a model (certainly people have used the findings of Beaudry and Portier to motivate theory papers about such models). If that's true, their entire exercise is suspect.

Forni, Giannone, Lippi, and Reichlin (2007) point out that nonfundamentalness is much less likely in a factor model. The condition for fundamentalness is still that $C(z)$ has rank equal to the number of shocks, q, on and inside the unit circle. However, now instead of $C(z)$ being $q \times q$, $C(z)$ is $n \times q$ in a factor model, where $n >> q$. This makes rank deficiency far less probable. In fact, I think that Forni, Giannone, Lippi, and Reichlin (2007) prove that fundamentalness is a generic property, or maybe something like as n gets large, under some conditions, the probability of fundamentalness approaches 1.

Testable FAVAR restrictions

Stock and Watson (2005) discuss testing some of the many restrictions imposed by a FAVAR. Recall from lecture 10 that in VAR form, a FAVAR can be written as:

$$
\left[\begin{array}{c} F_t \\ X_t \end{array} \right] = \left[\begin{array}{cc} \Phi(L) & 0 \\ \Lambda\Phi(L) & D(L) \end{array} \right] \left[\begin{array}{c} F_{t-1} \\ X_{t-1} \end{array} \right] + \varepsilon_t \tag{1}
$$
$$
\varepsilon_t = \left[\begin{array}{c} I \\ \Lambda \end{array} \right] G\eta_t + \left[\begin{array}{c} 0 \\ v_t \end{array} \right]
$$

Some of the restrictions of this model are used in estimation, others can be used for testing. Stock and Watson list the following restrictions:

1. Factor structure of $\Sigma_{\tilde{x}} = \Lambda\Sigma_F\Lambda' + \Sigma_v$ – used to estimate number of static factors

2. Reduced rank of ε_t – used to estimate number of dynamic factors

3. Factor structure of $\varepsilon_{x_t} = \Lambda G\eta_t + v_t$ – another way to estimate number of dynamic factors

4. x_t does not predict F_t given lagged F_t – a testable restriction

5. x_j does not predict x_i given lagged F_t – a testable restriction

6. x_j does not predict x_i given current F_t – testable

7. If order of $\Phi(L)$ is greater than 1, there are restrictions implied by the fact that both $\Phi(L)$ and $\Lambda\Phi(L)$ appear in (1)

Stock and Watson test restrictions 4, 5, & 6 by treating the factors as observed and doing standard regression based tests. They find rejections, but the finite (and perhaps asymptotic) behavior of these tests are not yet known.

Applications of Factor Models

Bai and Ng (J. of Econometrics 2006) looks at the relationship between observed and latent factors. In empirical work, it is convenient to work with observed variables as though they are the factors. This makes the model easier to work with and interpret. This paper is about how to test whether a set of observed variables spans the space spanned by the latent factors. It works out a few test statistics and their limiting distribution, then applies the tests to two examples. The intuition underlying the tests is as follows: let F_t be the true, unobserved factors and let G_t be the observed factors. If an element G_{it} is an exact factor, then it is in the span of F_t and can be written as:

$$
G_{it} = \sum_{k=1}^{q} b_q F_{kt}
$$

so we can form a test by regressing G_{it} on $\hat{F}_t$ and testing whether the residuals are zero.

Bai and Ng apply their test to two examples. One is from arbitrage pricing theory. They find that the three observable factors proposed by Fama and French (market excess return, small minus big, and high minus low) are good proxies for the latent factors, but do not span the space spanned by the factors.

The second example is more macroeconomic. They test whether annual consumption growth, inflation, growth of industrial production, term permium, risk permium, and unemployment span the space of latent factors. One could think of this as a way of addressing fundamentalness of VARs. They find that these variables do not span the space of factors. They find that the risk premium and industrial production have the strongest correlations with the factors. They also find some evidence that the relationship between the latent factors and observed variables was different before and after 1982.

Forni, Hallin, Lippi, and Reichlin (ReStat 2000) is mostly about estimating factor models using the spectrum, but as an example, estimates a coincident index for the EU. The idea is to summarize the state of the economy by a single index. They formalize this idea in the following procedure:

- Estimate a FAVAR using many variables from many countries in the EU
- Define the common component of each country's GDP as the projection of GDP onto the factors
- Define the coincident index as the weighted average of each country's common component of GDP.

They perform this procedure, show that the coincident index is correlated with many variables, but don't say much else.

Bernanke, Boivin, and Eliasz (QJE 2005) use FAVAR to examine the effect of monetary policy. Specifically, they look at whether FAVAR can solve the price puzzle caused by non-fundamentalness. Here is their methodology:

- Run a FAVAR on a large dataset of macro variables, including the federal funds rate – identify factors $\hat{C}(F_t, Y_t)$
- Identify monetary policy shocks – estimated factors, $\hat{C}_t(F_t, Y_t)$, are linear combos of monetary shocks, Y_t, and other factors, F_t, need to separate them
  - This requires an identification restriction, they assume that the federal funds rate responds immediately to monetary policy shocks, but there is a set of slow-moving variables that do not respond to monetary policy shocks within a period (a month).
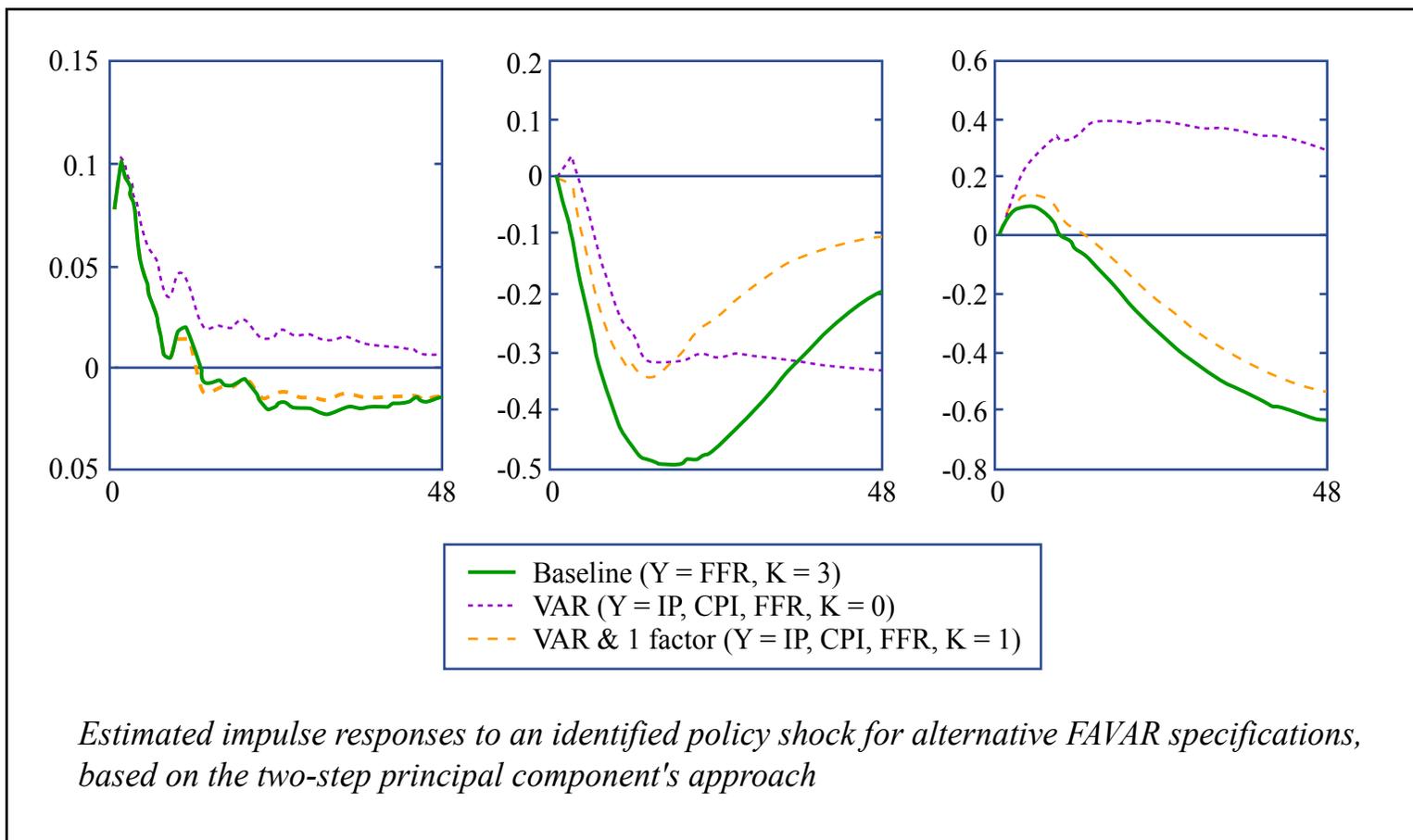 - estimate factors just using slow variables, $\hat{C}^*(F_t)$, and estimate

  $$\hat{C}_t(F_t, Y_t) = b_c \hat{C}^*(F_t) + b_R R_t + e_t$$

  where R_t is the federal funds rate. Set $F_t = \hat{C}_t(F_t, Y_t) - \hat{b}_R R_t$.
  - For each variable of interest, Y_t, estimate a VAR in Y_t, F_t, and R_t and impose short run restriction to identify monetary policy shock, plot impulse response

Figure 1 shows the results.

Figure 1: BBE Figure 1

Estimated impulse responses to an identified policy shock for alternative FAVAR specifications, based on the two-step principal component's approach

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