Introduction to Political Economy 14.770
Problem Set 5

Due date: November 27, 2017.

Question 1:

This question builds on Esteban and Ray (2001). Consider an economy where there are two villages, village 1 and village 2, with a population N_1 and N_2 respectively. The regional government is deciding where to build a road that will go through one of this villages. Having the road built in village j has a gross benefit

$$\begin{cases} (1-\lambda)P + \lambda \frac{M}{N_i}, \text{ if } i = j \\ 0 \text{ otherwise} \end{cases}$$

where $P > 0$ is a symmetric private benefit from having the road in your town and $M > 0$ is a public benefit to the town. The likelihood of having the road going through the town is a function of the relative effort that the village exerts. In particular, letting a_{ij} denote the effort that individual i in village j exerts, the probability of getting the road through the town is

$$\pi_j = \frac{A_j}{A_1 + A_2} = \frac{\sum_{i=1}^{N_j} a_{ij}}{\sum_{i=1}^{N_1} a_{ij} + \sum_{i=1}^{N_2} a_{ij}}$$

The individual private cost of exerting effort is $v(a) = \frac{a^\beta}{\beta}$, where $\beta \geq 1$. Assume that individuals are risk neutral so that there expected utility is

$$\pi_j \left[(1-\lambda)P + \lambda \frac{M}{N_j} \right] - \frac{a_{ij}^\beta}{\beta}$$

1. Solve for the symmetric Nash Equilibrium of the game above.
2. How do the equilibrium effort levels (individual and aggregate) vary with λ, P, M, and N_i? Does the Olsonian conjecture hold in this case?

Question 2:

Consider the following extension to the Gentzkow and Shapiro (2006) model we saw in class, which adds with imperfect signals for the high types.

There are two type of newspapers: λ fraction are high quality newspapers (denoted by h) and the remaining $1-\lambda$ fraction normal newspapers (denoted by n). The high quality newspapers receive a signal $s_h \in \{r, l\}$ of the true state of the world, R, L, and the signal is accurate with probability $\pi_h > 1/2$. The normal newspapers receive a signal $s_n \in \{r, l\}$ of the true state of the world and the signal is accurate with probability π, where $\pi_h \geq \pi > 1/2$.

Consumers want to learn the true state of the world a best as they can. They have a prior belief $\theta \in (1/2, \pi)$ of the true state of the world being R. They also get informative feedback with probability μ.

Denote $X \in \{R, L, 0\}$ the feedback that consumers get. Define $\sigma_j(\widehat{j})$ to be the probability that a normal firm reports $\widehat{j}$, given that it gets signal j.

Normal firms want to maximize the posterior beliefs $\lambda(\widehat{j}, X)$ that agents have after getting a report $\widehat{j}$.

1. Suppose that a consumer gets a report $\widehat{r}$. What is the likelihood ratio of this coming form a high firm $(\Pr(\widehat{r}|h)/\Pr(\widehat{r}|n))$? Is this increasing in θ? How does it change with π_h?
2. Calculate the Posterior belief $\lambda(\widehat{r}, 0)\,(\Pr(h|\widehat{r}))$. How does it change with θ and with π_h?
3. What happens to bias in the case when there is no feedback? How does bias changes with π_h? What happens in the limit case when $\pi_h \to \pi$? (Note: Focus on the equilibrium which minimizes the slanting when firms receive signal $\widehat{r}$).
4. What happens to bias in the case when there is full feedback? How does bias changes with π_h? What happens in the limit case when $\pi_h \to \pi$?
5. Suppose now we are in the original Gentzkow-Shapiro environment with $\pi_h = 1$. How does the equilibrium of the game change with λ? In particular, describe what happens when $\lambda = 0$.
6. In the segmented equilibrium of Section 5 of the paper (the extension with heterogeneous priors), is it the case that signals of l are more informative about underlying states than signals of r? If so, prove it.

References

Gentzkow, Matthew and Jesse M. Shapiro (2006). “Media Bias and Reputation”, *Journal of Political Economy* 114(2): 280-316.

Ray, Debraj and Esteban, Joan Maria (2001). “Collective Action and the Group Size Paradox”, *American Political Science Review* 95(3): 663-672.