14.770-Fall 2017
Recitation 2 Notes

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Today:

- A brief review of the second and third lectures.
- Going over Feddersen and Pesendorfer (APSR, 1998) once more.

A Helicopter Tour of the Class

Lecture 2 mostly builds on the first lecture, introducing two key ideas you should remember:

1. **Single-Crossing Preferences.** A restriction on preferences much in the same vein as single-peakedness. Just like single-peakedness, it ensures that MVT and Donwsian Convergence holds.
   - Unlike single-peakedness, this is a property of the whole preference profile, not an individual's preference.
   - Neither of them implies the other!
   - Easier than single-peakedness to check in some applications, e.g. redistributive taxation.
2. **Probabilistic Voting.** The "smoothed out" version of electoral competition model – so that we get continuous best responses.
   - Turns out this is also a model which predicts why parties tend to cater towards swing voters.
   - Useful, but sometimes you need to place extreme restrictions, e.g. uniform distributions, to make it tractable.
   - If interested, Persson and Tabellini's Section 3.4 is a good resource to check.

Lecture 3, on the other hand, shifts gears a little bit.

- Lectures 1 and 2 take the assumption that "everybody has individual, heterogeneous preferences (**private values**)" as given.
  - Sometimes a reasonable assumption (redistributive taxation?)...
  - But sometimes, a policy has a "common interest" element (trade liberalization?)...
  - In these cases, there is no conflict of preferences, but perhaps a conflict of *information.*
  - So, let's take one extreme and consider a model where everybody has the same preferences (**common values**), but different information.

  - Real life is probably somewhere in between private preferences and common values, but it's really intractable.[1]

- Somewhat interestingly, these models share a lot of features with the models on **common value auctions**.
  - In a common value auction, bidders place their bids conditional on winning. In a common value policy choice, voters cast their votes conditional on being pivotal. In both cases, these events (winning or being pivotal) contain useful information.
  - Not surprising that Pesendorfer also wrote a group of highly influential papers on auction theory!
  - Fun fact: "Swing Voter's Curse" is a pun on "Winner's Curse" – a well-known concept in common value auctions.
- In today's recitation, we'll cover an example where things may go in counterintuitive directions, because people realize that being pivotal is valuable information.
- That's all for the theoretical analysis of voting (for now)! Next few lectures will focus on empirics – testing the theories laid out by the theoretical models we covered so far.

The Jury Problem (Feddersen-Pesendorfer 1998)

- Folk wisdom about jury verdicts, which require unanimous agreement on conviction:

  > "Trial by jury is not an instrument of getting at the truth; it is a process designed to make it as sure as possible that no innocent man is convicted."

- This seems to be the correct heuristic when people vote sincerely (everybody brings their partial information to the table, we aggregate, and the law of large numbers kicks in).
- Not so much when people act strategically – i.e. when they realize being pivotal contains some information.
- Intuitively: if you are a juror, you realize that the only scenario when your vote will matter is: *when all other eleven people vote for conviction.* That's a very strong set of signals to overturn!

The Model

What we covered in the lecture:

- State of the world $\theta \in \{I, G\}$
  - $Pr\{\theta = G\} = \pi \in (0, 1)$
- n jurors, each receive signal $s \in \{i, g\}$
  - Each signal iid conditional on state
  - $Pr\{s = g|\theta = G\} = p$, $p > 0.5$
  - $Pr\{s = i|\theta = I\} = q$, $q > 0.5$
- Each juror would like to convict if and only if

$$Pr\{\theta = G|\text{information}\} \geq z$$

  where $z \in [0, 1]$ is the *threshold of reasonable doubt.*

[1] Auction theory has spent a lot of time figuring out the correct way to model this midpoint (Google "affiliated values" if interested), and believe me it's not easy.

Define $\beta(k,n)$ as the posterior probability that the defendant is guilty conditional on observing k guilty signals out of n:

$$\beta(k,n) = \frac{\pi p^k (1-p)^{n-k}}{\pi p^k (1-p)^{n-k} + (1-\pi)(1-q)^k q^{n-k}}$$

The following assumption ensures that the model is interesting:

Assumption 1. *There is a k^* with $n \geq k^* \geq 1$, such that*

$$\beta(k^*-1,n) < z < \beta(k^*,n)$$

In layman's terms, this assumption says that "the collective information of all jurors matter".

Sincere Voting

We are interested in whether the folk wisdom holds; in other words:

> "What is the probability that $\theta = G$ conditional on conviction?"

Let's first answer this question by assuming that the jurors simply tell what their signals are without being strategic. In this case, the posterior probability conditional on conviction (i.e. n guilty signals) is:

$$\beta(n,n) = \frac{\pi p^n}{\pi p^n + (1-\pi)(1-q)^n}$$

Since $p, q > 0.5$, this converges to 1 as $n \to \infty$. In other words, the unanimity rule ensures that no innocent man gets convicted! (Under sincere voting, of course.)

Strategic Voting

Do the reasoning above hold under strategic voting, too? That is, is sincere voting an equilibrium?

- If $k^* = n$, yes. But this is a very demanding condition!
- If $k^* < n$, no. This is because then we have:

  $$\beta(n-1,n) > z \tag{1}$$

  That is, conditional on a juror being pivotal, she believes that the defendant is guilty even when her private signal says otherwise, so she votes for conviction. (Consistent with the intuition we provided earlier!)

For the rest of the analysis, we'll assume that Equation 1 holds.

Analysis of Equilibrium

The power of a game theoretical model is: it can go beyond being cynical about some other predictions, and make its own! So, let's use this machinery to analyze what happens.

Some notes:

- Let's look for a **symmetric Bayesian Nash Equilibrium** of the game induced by this setup.
- There's always an equilibrium where everyone votes acquit regardless of their signals. This is not an interesting one, and perhaps not very realistic.

- Is there an equilibrium where everyone votes convict regardless of the signal? Not if

$$\frac{\pi(1-p)}{\pi(1-p)+(1-\pi)q} < z \tag{2}$$

and this is an easy condition to satisfy.

- A symmetric strategy profile is given by

$$(\sigma(i), \sigma(g)) \in [0,1] \times [0,1]$$

where $\sigma(s)$ is the probability of voting to convict after observing signal $s \in \{i, g\}$.

- Define the probabilities of conviction vote by a juror in both states:

$$\gamma_G = p\sigma(g) + (1-p)\sigma(i)$$
$$\gamma_I = (1-q)\sigma(g) + q\sigma(i)$$

We're looking for a **responsive equilibrium** where $\gamma_G \neq \gamma_I$. That is, a juror's vote must be informative of its signal (to some extent, if not fully).

Start by realizing that we cannot have a responsive equilibrium with $\sigma(g) < 1$.

1. You can check that, if a juror who receives $s = g$ weakly prefers to acquit, then a juror who receives $s = i$ strictly prefers to acquit.
2. In other words, we need to have $\sigma(i) = 0$.
3. But remember, being pivotal means $n-1$ other conviction votes. Since only those who receive $s = g$ votes to convict, this means $n-1$ other guilty signals.
4. As long as Equation 1 holds, this cannot be an equilibrium, for the same reason why sincere voting cannot be an equilibrium.

So, we pinned down $\sigma(g) = 1$. What about $\sigma(i)$?

1. It cannot be 0 (as long as Equation 1 holds) and it cannot be 1 (as long as Equation 2 holds) either!
2. For $\sigma(i) \in (0,1)$, we need an indifference condition:

$$\frac{\pi(\gamma_G)^{n-1}(1-p)}{\pi(\gamma_G)^{n-1}(1-p)+(1-\pi)(\gamma_I)^{n-1}q} = z$$

Or, more explicitly:

$$\frac{\pi(p+(1-p)\sigma(i))^{n-1}(1-p)}{\pi(p+(1-p)\sigma(i))^{n-1}(1-p)+(1-\pi)((1-q)+q\sigma(i))^{n-1}q} = z$$

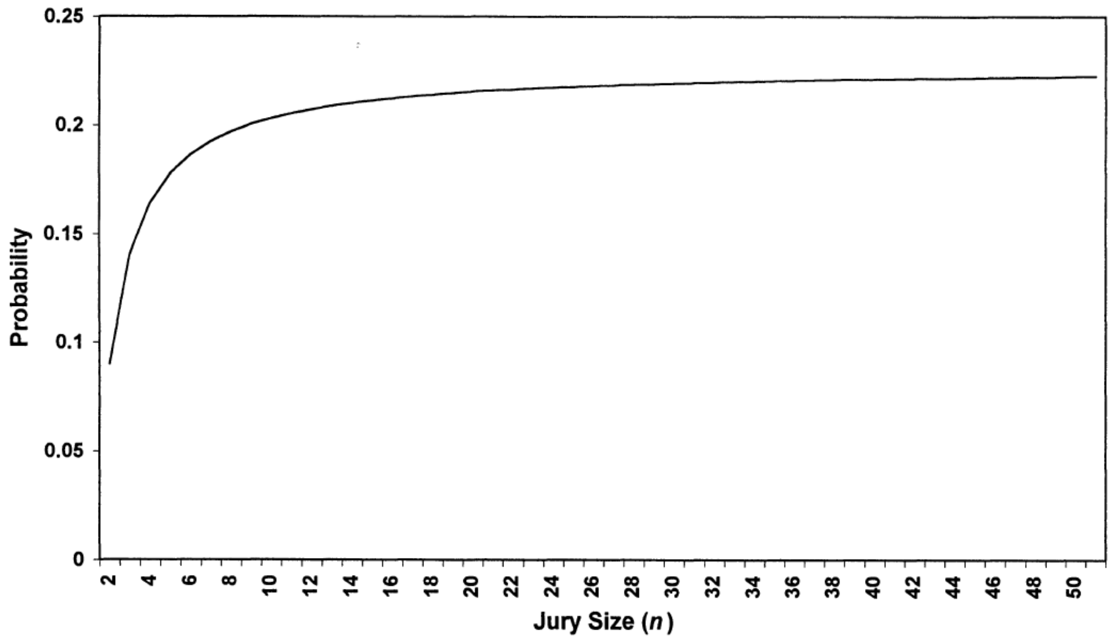
One can then back out $\sigma(i)$ from this equation. As long as Equations 1 and 2 are satisfied, we have $\sigma(i) \in [0,1]$.

Feddersen and Pesendorfer analyze the case where $\pi = 0.5$ and $p = q$. In this case,

$$\sigma(i) = \frac{\left(\frac{(1-z)(1-p)}{zp}\right)^{1/(n-1)} p - (1-p)}{p - \left(\frac{(1-z)(1-p)}{zp}\right)^{1/(n-1)} (1-p)}$$

This is a nice, closed form solution, and one can calculate the probability that an innocent defendant gets convicted. Spoiler: it does not converge to zero. The following figure plots it for $\pi = 0.5$, $p = q = 0.7$ and $z = 0.5$.

FIGURE 1. The Probability an Innocent Defendant Is Convicted as a Function of Jury Size



SAD. :(

Non-Unanimous Rules

If even unanimous verdicts sometimes convict innocent defendants, what is a rule which doesn't? Based on the analysis so far, you should be able to guess the answer:

"Literally any other rule which requires only a fraction $\alpha \in (0,1)$ of conviction votes."

Why? I'm not going to give the whole answer here, but only some intuition.

Fix some $\alpha \in (0,1)$. In a responsive symmetric Bayesian Nash Equilibrium, a juror who receives $s = i$ must again be indifferent when she is pivotal. This requires:

$$\frac{\pi(\gamma_G)^{\alpha n}(1-\gamma_G)^{(1-\alpha)n}(1-p)}{\pi(\gamma_G)^{\alpha n}(1-\gamma_G)^{(1-\alpha)n}(1-p)+(1-\pi)(\gamma_I)^{\alpha n}(1-\gamma_I)^{(1-\alpha)n}q} = z$$

The only way this doesn't blow up or vanish as $n \to \infty$ is:

$$\frac{(\gamma_G)^{\alpha}(1-\gamma_G)^{1-\alpha}}{(\gamma_I)^{\alpha}(1-\gamma_I)^{1-\alpha}} = 1$$

This, along with the fact that $\gamma_G > \gamma_I$, implies that we must have:

$$\gamma_G > \alpha > \gamma_I$$

But then, when $\theta = G$, a fraction $\gamma_G > \alpha$ votes are in favor of conviction, so the juror convicts with probability 1! Similar argument goes for $\theta = I$. Note that the above argument doesn't work with $\alpha = 1$ – in that case, both γ_G and γ_I would need to be close to one.

Finally, note that the assumption of large n is crucial for above reasoning to work – so, rather than dealing with particulars of the α, maybe one should just increase the size of the jury.