14.771 Development Economics: Microeconomic Issues and Policy Models
Fall 2008

14:771: Recitation Handout #10
Some Contract Theory - Moral Hazard

Overview

Two key subjects in contract theory are moral hazard and adverse selection. Both of these will come up in the following weeks as we cover land and credit markets, but I'll focus on moral hazard today. In both moral hazard and adverse selection models, there is some private information that we would like to be able to contract on, but cannot. In moral hazard, we have problems of hidden action. In adverse selection, we have problems of hidden type. Given our constraints, we then try and find the constrained optimal contract. This recitation is intended to give you a brief overview so it's easier to follow lectures - you'll learn lots more about these topics in 14.124.

Moral Hazard

Basic Framework

Moral hazard comes up in principal-agent problems. The idea is that a principal (say, a landlord) offers an agent (say, a sharecropper) a contract. The agent's job is to take a costly action that influences expected output. The problem is that we cannot contract on this action - instead we have to contract on output. This *hidden action* is at the crux of moral hazard - we'll see that it often means that effort levels will not be at first best levels. *What defines a first-best level of effort?*

Here is a simple moral hazard problem: A landlord wishes to pay a sharecropper to farm his land. The harvest can either be high or low: $y \in \{y_H, y_L\}$. The sharecropper can influence the probability of a good harvest by exerting effort, a, on the plot. Specifically:

$$\begin{aligned} \Pr(y = y_H \mid a) &= p(a) \\ p(0) &= 0 \\ p(\infty) &= 1 \end{aligned}$$

Payoffs are given by:

$$\begin{aligned} \text{Landlord} &: \quad V(y - w) \\ \text{Sharecropper} &: \quad u(w) - c(a) \end{aligned}$$

where w is the wage paid to the agent and $c(a)$ is the cost of effort, which is convex: $c'(a) > 0$ $c''(a) \geq 0$. We'll also assume that landlords and sharecroppers are both either risk averse or risk neutral: $V'(.) > 0 \geq V''(.)$ and $u'(.) > 0 \geq u''(.)$. Also, assume that sharecroppers have an outside option given by $\bar{U}$.

In the first best, we would be able to contract on output and effort:

$$\arg\max_{a,\, w_L,\, w_H} p(a)\, V(y_H - w_H) + (1 - p(a))\, V(y_L - w_L) \text{ s.t.}$$
$$p(a)\, u(w_H) + (1 - p(a))\, u(w_L) - c(a) \geq \bar{U} \quad \text{[Participation Constraint; LM=}\lambda\text{]}$$

We get the following FOCs:

$$\begin{aligned}
[w_L] \quad &: \quad (1 - p(a))\, V'(y_L - w_L) = \lambda (1 - p(a))\, u'(w_L) \\
&\quad \frac{V'(y_L - w_L)}{u'(w_L)} = \lambda \\
[w_H] \quad &: \quad p(a)\, V'(y_H - w_H) = \lambda p(a)\, u'(w_H) \\
&\quad \frac{V'(y_H - w_H)}{u'(w_H)} = \lambda \\
&\Rightarrow \quad \frac{V'(y_L - w_L)}{u'(w_L)} = \frac{V'(y_H - w_H)}{u'(w_H)}
\end{aligned}$$

what does this imply when the landlord is risk neutral? What about when the tenant is risk neutral?

Now, what happens when we cannot contract on effort? Then we have to add another constraint to our problem:

$$\arg\max_{w_L,\, w_H} p(a)\, V(y_H - w_H) + (1 - p(a))\, V(y_L - w_L) \text{ s.t.}$$
$$p(a)\, u(w_H) + (1 - p(a))\, u(w_L) - c(a) \geq \bar{U} \quad \text{[Participation Constraint; LM=}\lambda\text{]}$$
$$a \in \arg\max_{\hat{a}} p(\hat{a})\, u(w_H) + (1 - p(\hat{a}))\, u(w_L) - c(\hat{a}) \quad \text{[Individual Rationality Cont; LM=}\mu\text{]}$$

So, how can we solve this? Ideally, we solve out for the agent's choice of effort and plug that into the problem. However, in the case with general utility functions like this, we cannot derive a function $a^*(w_L, w_H)$. The most common approach used in this case is the *first order approach* - we take the FOC from the agent's problem and plug it in as a constraint. Note, however, that this is ONLY valid when the FOC is sufficient to characterize the optimal choice of a. This is something that people sometimes forget about, but it's nice to have in the back of your mind. The FOC on the agent's problem is:

$$p'(a)\,[u(w_H) - u(w_L)] - c'(a) = 0$$

Rewriting the problem we get:

$$\arg\max_{w_L,\, w_H} p(a)\, V(y_H - w_H) + (1 - p(a))\, V(y_L - w_L) \text{ s.t.}$$
$$p(a)\, u(w_H) + (1 - p(a))\, u(w_L) - c(a) \geq \bar{U} \quad \text{[Participation Constraint; LM=}\lambda\text{]}$$
$$p'(a)\,[u(w_H) - u(w_L)] - c'(a) = 0 \quad \text{[IR Constraint; LM=}\mu\text{]}$$

Now our FOCs are:

$$\begin{aligned}
[w_H] \quad &: \quad -p(a)\, V'(y_H - w_H) + \lambda p(a)\, u'(w_H) + \mu p'(a)\, u'(w_H) = 0 \\
&\Rightarrow \quad \frac{V'(y_H - w_H)}{u'(w_H)} = \lambda + \mu \frac{p'(a)}{p(a)} \\
[w_L] \quad &: \quad -(1 - p(a))\, V'(y_L - w_L) + \lambda (1 - p(a))\, u'(w_L) - \mu p'(a)\, u'(w_L) = 0 \\
&\Rightarrow \quad \frac{V'(y_L - w_L)}{u'(w_L)} = \lambda - \mu \frac{p'(a)}{1 - p(a)}
\end{aligned}$$

compared to what we have before we see that we set w_H higher than the first best and w_L lower than the first best - this is to give extra incentive to the agent to exert more effort. *Can you think of a way we could get around the moral hazard problem when the sharecropper is risk neutral?*

Sometimes we'll need to add even more constraints to an optimal contracting problem. A very common one is a *limited liability constraint.* Basically, this sets a lower bound on the payoff given to the agent: $w_i \geq v$. This can further hurt our ability to write optimal contracts.

A Practice Problem - A Model of Sharecropping

Assume a simple production function: $y = e + \theta$ where y is output, e is effort put in by the cultivator, and θ is a normally distributed random shock with zero mean and variance σ^2. Also assume that all agents are risk averse with a mean-variance expected utility function of income (y):

$$E\left(U\left(y\right)\right) = E\left(y\right) - \frac{r\left(w_i\right)}{2} Var\left(y\right) \;\; i = T, L$$

where $r\left(w_i\right)$ is the coefficient of risk-aversion with $r'\left(w_i\right) < 0$. (This formulation of expected utility is equivalent to exponential instantaneous utility in the presence of normally distributed income shocks: $u\left(y\right) = -\exp\left(-r\left(w_i\right) y\right)$, $y\tilde{}Normal$.

Consider two agents: Agent L is a landlord who owns a piece of land, has monetary wealth w_L and had no labor, and Agent T is the tenant who owns no land, has monetary wealth w_T and one unit of labor. Also assume that land, labor and goods markets are perfectly competitive, but insurance markets are nonexistent. This means that if the landowner wants, he could buy the tenant's at the market wage rate m, and if the labor-owner wanted, he could buy the landlords land at the market rental rate p.

Note that there is a difference between labor and effort. The cost of exerting effort is $\frac{ce^2}{2}$ and the competitive wage rate in the labor market is net of the effort cost. Assume the monitoring is costless, so e is observable and contractible. Also assume that contracts are linear, so the tenants income y_T is written: $y_T = sy - R$ where s is the share of total output going to the tenant (with $0 \leq s \leq 1$) and R is the fixed rent. (Assume that $-w_L \leq R \leq w_T$).

1. Write equations for the expected utility of the tenant and the landlord (both as a function of e, s and R). Then solve for the socially optimal level of effort (e) and the optimal share of output that goes to the tenant (s).

2. Next assume that the landowner can make a take-it-or-leave it offer to the tenant. What would be the resulting levels of e, s, R and S? How does the riskiness of production affect the fixed rent component, the level of effort and the share of output that goes to the tenant? Why? How does the productivity of labor and the wage rate (m) affect the same three variablees? Finally, how does the risk aversion of the tenant and landlord affect these results? What if the landlord is so rich that he is risk-netural (i.e. $r\left(w_L\right) = 0$)? Or if the tenant is risk neutral (i.e. $r\left(w_T\right) = 0$) but the landlord is risk averse?

3. What would happen if e is no longer contractible? (For this part, you may assume $\omega_L = \omega_T = \omega$).