LECTURE 4:

Growth, TFP, Domestic and International Capital Flows with Other Frictions in Financial Intermediation: Costly State Verification, Adverse Selection, and Moral Hazard

Cross-country in steady state, and an example of unbalanced growth

Greenwood, Sanchez, and Wang (2013), "Quantifying the Impact of Financial Development on Economic Development"

A model which is meant to capture Levine's review of the first lecture, that is, a particular function of financial intermediation, and technological progress in that intermediation, incorporated into a growth model.

Address cross-country interest rates spreads and a resource-using costly state verification with diminishing returns and exogenous technological progress.

Uganda could more than double its output if it would adopt best practice in financial sector (maximum technology available world-wide). However, this is still only 29% of the gap between its potential and actual output).

In the model, improvements in financial intermediation account for 29% of U.S. growth.

The framework also is capable of mimicking the striking decline in the Taiwanese interest-rate spread. At the same time, it predicts a significant rise in its capital-to-output ratio. It is estimated that dramatic improvements in Taiwans financial sector accounted for 45% of the country's economic growth.

Greenwood, Sanchez, and Wang (2013), "Quantifying the Impact of Financial Development on Economic Development"

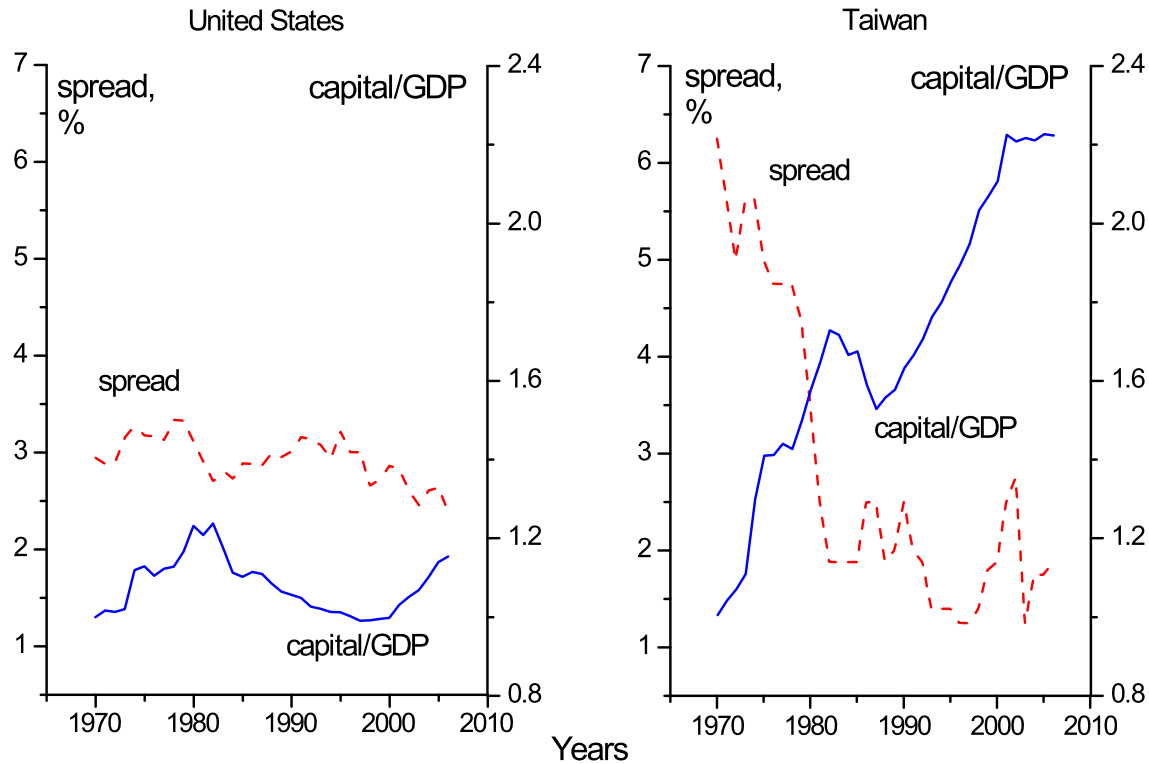

Fig. 1. Interest-rate spreads and capital-to-GDP ratios for the United States and Taiwan, 1970–2005. Data sources for all figures are discussed in Appendix A.

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Greenwood, Sanchez, and Wang (2013), "Quantifying the Impact of Financial Development on Economic Development"

Levine (2005), King and Levine (1993): the upshot is that financial development has a causal effect on economic development; specifically, it leads to higher rates of growth in income and productivity.

We investigate this impact quantitatively, using a costly state verification model. The source of inspiration for the framework is the classic work by Townsend (1979) and Williamson (1986).

Novel twists:

1. Firms monitor cash flows; however, here the efficiency of this activity depends on both the amount of resources devoted to it and the productivity of the monitoring technology used in the financial sector.
2. Firms have ex-ante differences in the structure of returns that they offer.

Greenwood, Sanchez, and Wang (2013), "Quantifying the Impact of Financial Development on Economic Development"

A financial theory of firm size emerges:

- At any point in time, firms offering high expected returns are underfunded (relative to a world without informational frictions), whereas others yielding low expected returns are overfunded. This results from diminishing returns in information production.
- As the efficiency of the financial sector rises (relative to the rest of the economy), funds are redirected away from less productive firms in the economy toward more productive ones.
- As the interest-rate spread declines and the cost of borrowing falls, capital deepening occurs in the economy.

Greenwood, Sanchez, and Wang (2013), "Quantifying the Impact of Financial Development on Economic Development"

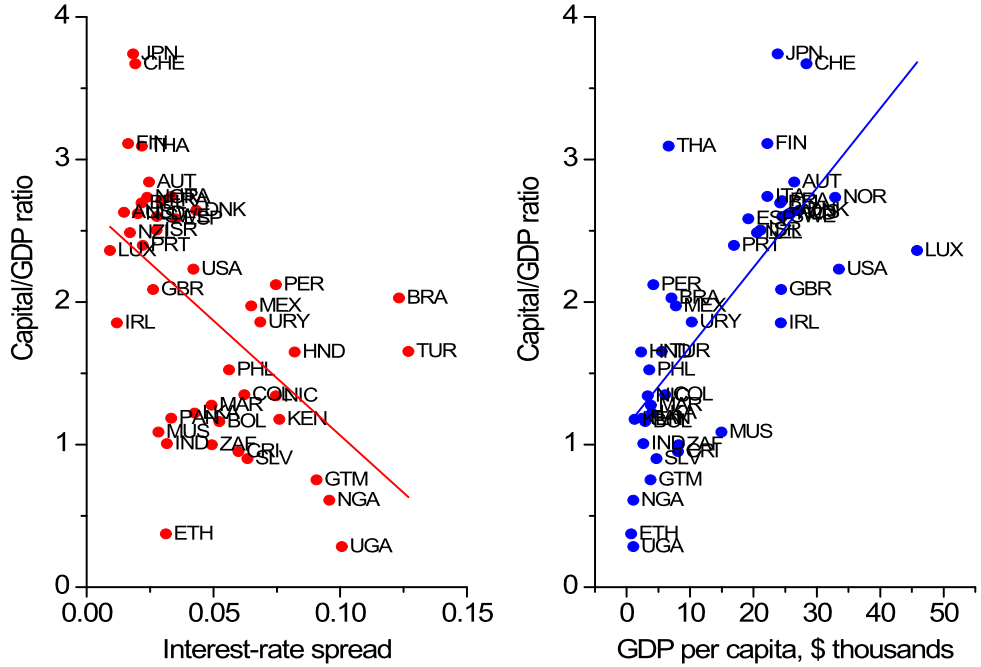

Fig. 2. The cross-country relationship among interest-rate spreads, capital-to-GDP ratios and GDPs per capita. The three letter country codes are taken from the International Organization for Standardization, ISO 3166-1 alpha-3.

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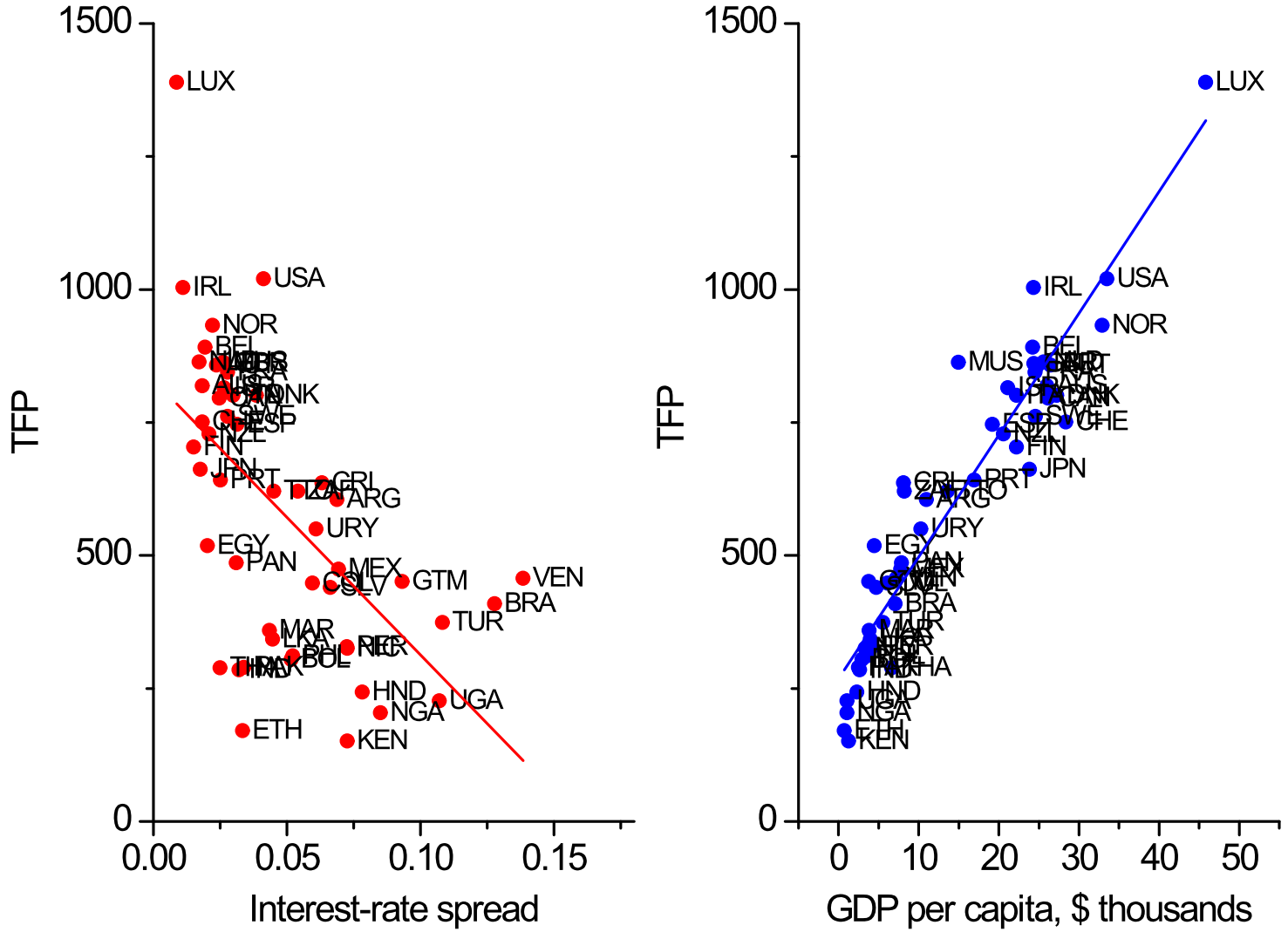

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Fig. 3. The cross-country relationship among interest-rate spreads, TFPs and GDPs per capita.

Greenwood, Sanchez, and Wang (2013), "Quantifying the Impact of Financial Development on Economic Development"

Firms:

- Firms hire capital, k, and labor, l, to produce output, o, in line with the constant-returns-to-scale production function $o = x\theta k^{\alpha} l^{1-\alpha}$.
- The productivity level of a firms production process is represented by $x\theta$, where x is aggregate and θ is idiosyncratic. The idiosyncratic level of productivity is a random variable. The realized value of θ is drawn from the two-point set $\tau = \{\theta_1, \theta_2\}$, with $\theta_1 < \theta_2$. The set τ is the firms type and differs across firms.

Financial intermediaries:

- Intermediation is competitive.
- Intermediaries raise funds from consumers and lend them to firms.
- Even though an intermediary knows a firms type, τ, it cannot observe the state of a firms business (θ , o, and l) either costlessly or perfectly.

Greenwood, Sanchez, and Wang (2013), "Quantifying the Impact of Financial Development on Economic Development"

Let $P_{ij}(l_{mj}, k, z)$ denote the probability that the firm is caught cheating conditional on the following:

1. The true realization of productivity is θ_i
2. The firm makes a report of θ_j
3. The intermediary allocates l_{mj} units of labor to monitor the claim
4. The size of the loan is k (which represents the scale of the project)
5. The level of productivity in the monitoring activity is z

The function $P_{ij}(l_{mj}, k, z)$ is increasing in l_{mj} and z and decreasing in k.

The steady state for the model provides a mapping between productivity in the production (x) and financial sectors (z) on the one hand, and output (o) and interest-rate spreads (s) on the other. This mapping can be inverted to infer x and z using observations on o and s, given a vector of parameter values, p. Take the parameter vector p that was calibrated/estimated for the U.S. economy and use the Taiwanese data on per-capita GDPs and interest-rate spreads for the years 1974 and 2004 to obtain the imputed Taiwanese technology vector.

Martin and Taddei (2012), "International Capital Flows and Credit Market Imperfections: a Tale of Two Frictions"

Excessive capital flows and boom-bust cycles (at least in theory, not quantitative/calibrated).

In recent years, global imbalances large and persistent capital flows from Asia to the United States and other developed economies have spurred renewed interest in the macroeconomic effects of financial frictions. Financial frictions have also been invoked to explain the run-up to the financial crisis of 2007-08 and the unfolding of events during the crisis itself.

Martin and Taddei (2012), "International Capital Flows and Credit Market Imperfections: a Tale of Two Frictions"

Instead of limiting the amount of resources that can be channeled towards productive investment, financial frictions are portrayed in the literature as the source of an excessive supply of assets that has channeled too many resources towards unproductive investment. (We covered such papers earlier, as on China.)

We need to acknowledge that there are different types of frictions. On the one hand, underprovision of assets and limited investment are typically attributed to limited pledgeability. On the other hand, overprovision of assets is typically attributed to some form of asymmetric information regarding the quality of borrowers, which fuels investment by unproductive or inefficient individuals.

Existing macroeconomic models focus mostly on limited pledgeability while neglecting adverse selection (see previous lecture).

Martin and Taddei (2012), "International Capital Flows and Credit Market Imperfections: a Tale of Two Frictions"

We have a standard growth model in which credit markets intermediate resources between savers and investors in capital accumulation. Individuals are endowed with some resources and an investment project for producing capital, and they must decide whether: (i) to undertake their project and become entrepreneurs, in which case they demand funds from credit markets, or; (ii) to forego their project and become savers, in which case they supply their resources to credit markets.

To give adverse selection a central role in credit markets, we also assume that an individual's productivity is private information and thus unobservable by lenders. This induces cross-subsidization between high- and low-productivity entrepreneurs.

All borrowers must pay the same contractual interest rate in equilibrium. This implies that high-productivity entrepreneurs, who repay often, effectively face a higher cost of funds than low-productivity entrepreneurs, who repay only seldom. It is this feature that gives rise to adverse selection by providing some low-productivity individuals, who would be savers in the absence of cross-subsidization, with incentives to become entrepreneurs.

Martin and Taddei (2012), "International Capital Flows and Credit Market Imperfections: a Tale of Two Frictions"

Macroeconomic implications of adverse selection:

1. It leads to an increase in the economy's equilibrium interest rate, while boosting equilibrium borrowing and investment.
2. By fostering inefficient entrepreneurship, it generates a negative wedge between the marginal return to investment and the equilibrium interest rate.

Through (1), adverse selection induces the economy to attract more capital flows and boosts net capital inflows from the international financial market, relative to the full-information economy. By (2), since the true marginal return to investment lies below the world interest rate, these capital inflows can lead to a fall in aggregate consumption.

Moll, Townsend, and Zhorin (2012), "Entrepreneurship, Inequality, and Growth with Information Constrained Factor Markets"

There is evidence that even within a given economy, obstacles to trade may vary depending on location. In a companion paper, Karaivanov and Townsend (2012) estimate the financial/information regime in place for households including those running businesses using Townsend Thai data from rural areas (villages) and from urban areas (towns and cities). They find differences across these locations. For example, a moral hazard constrained financial regime fits best in urban areas and a more limited savings regime in rural areas. More generally, there seems to be (related) regional variation.

A number of recent papers argue that financial frictions arising from limited commitment problems can explain large cross-country income differences. We argue that different micro financial underpinnings have potentially very different implications at both the macro and the micro level. To this end, we develop a general equilibrium framework that encompasses different regimes of frictions, and compare the implications of two concrete frictions: limited commitment and moral hazard.

Moll, Townsend, and Zhorin (2012), "Entrepreneurship, Inequality, and Growth with Information Constrained Factor Markets"

1. Aggregate TFP in the two regimes is depressed but for completely different reasons:
   - Under *limited commitment* this results from a misallocation of capital across firms with given productivities.
   - Under *moral hazard*, TFP is endogenously lower at the firm level because entrepreneurs exert suboptimal effort.
2. Occupational choice, the firm productivity and size distributions, and income and wealth inequality also differ markedly.
3. Individual transitions are much faster in the limited commitment regime than under the moral hazard, resulting for example in more dispersed wealth growth rates:
   - In the *limited commitment* regime binding borrowing constraints and high marginal products of capital provide an incentive for entrepreneurs to attempt to save themselves out of these constraints.
   - Under *moral hazard* individual wealth or promised utility moves slowly as output-dependent penalties and awards are spread into the future.
4. There are implications as well for regional and sectoral capital flows.

Moll, Townsend, and Zhorin (2012), "Entrepreneurship, Inequality, and Growth with Information Constrained Factor Markets"

In particular, the most realistic financial regime for the given economy, which varies regionally and in urban vs. rural stratifications of the data, is a not a simple convex combination of the two extremes. The bottom line is that the behavior of macro aggregates depends on micro financial underpinnings.

Finance and Development:

Limited Commitment vs. Moral Hazard

Benjamin Moll
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Motivation

- Micro evidence: even within given economy, obstacles to trade may vary depending on location.
- For example, **?** using Townsend Thai data: moral hazard constrained financial regime fits best in urban areas and a more limited savings regimes in rural areas.
- More generally, regional variation: **??**

What We Do

- Ask: What difference do the micro financial foundations make for the macro economy? Will argue: a big one.
- Develop a general equilibrium model of entrepreneurship and financial frictions that is general enough to encompass:

(1) financial frictions stemming from limited commitment.

(2) financial frictions stemming from private information (moral hazard).

(3) Mixtures of different regimes in different regions.

What We Do

- Study aggregates: GDP, TFP, capital accumulation, wages and interest rates...
- ...but also micro moments: prod. distribution, size distribution of firms, dispersion in MPKs.
- Show: all of these look potentially very different, depending on the underlying financial regime.

Implications for Literature

- Large literature studies role of financial market imperfections in development.
- Most existing studies: limited commitment.
- Much fewer: moral hazard (**???**)
- We should use micro data to choose between the myriad of alternative forms of introducing a financial friction into our models.

Common Theoretical Framework

- Households: wealth, a, entrepreneurial ability, z. Markov process $\mu(z'|z)$.
- Continuum of households of measure one, indexed by $i \in [0, 1]$
- Preferences over consumption and effort:

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t u(c_{it}, e_{it}).$$

- Occupational choice: entrepreneur ($x = 1$) or worker ($x = 0$).

Entrepreneurs and Workers

- **Entrepreneurs,** $x = 1$**:** technologies

$$y = f(z, \varepsilon, k, l) = z\varepsilon k^{\alpha} l^{\gamma}, \quad \alpha + \gamma < 1$$

- $\varepsilon \equiv$ idiosyncratic production risk, with distribution $p(\varepsilon|e)$.
- **Workers,** $x = 0$**:** supply ε efficiency units of labor, with distribution $p(\varepsilon|e)$.
- Note: Depending on $x = 0$ or $x = 1$, ε is either firm productivity or worker's efficiency units. Allow for differential responsiveness to e through appropriate scaling.

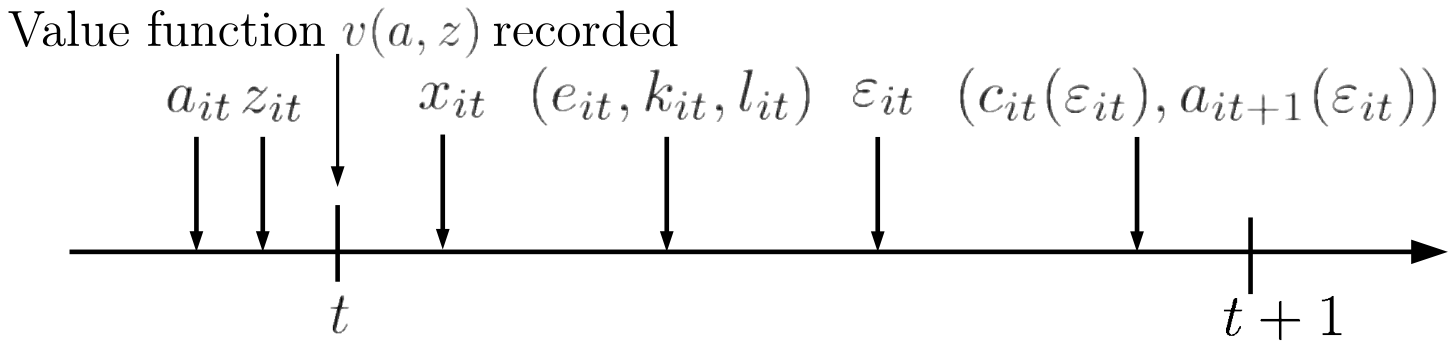
Risk-Sharing

- Households contract with risk-neutral intermediaries to form "risk-sharing syndicates": intermediaries bear some of HH risk.
- "Risk-sharing syndicates" take (w, r) as given.
- Assume: can only insure against production risk, ε, but not against talent, z.
- Optimal contract:

(1) assigns occupation, x, effort, e, capital, k, and labor, l. After ε is drawn, assigns consumption and savings $c(\varepsilon)$ and $a'(\varepsilon)$.

(2) leaves zero profits to intermediary $\Leftrightarrow$ maximizes individual's utility.

Timing



Courtesy of Benjamin Moll, Robert M. Townsend, and Victor Zhorin. Used with permission.

Optimal Contract: Bellman Equation

$$v(a,z) = \max_{e,x,k,l,c(\varepsilon),a'(\varepsilon)} \sum_{\varepsilon} p(\varepsilon|e)\left\{u[c(\varepsilon),e] + \beta \mathbb{E} v[a'(\varepsilon),z']\right\} \quad \text{s.t.}$$

$$\sum_{\varepsilon} p(\varepsilon|e)\left\{c(\varepsilon) + a'(\varepsilon)\right\}$$

$$\leq \sum_{\varepsilon} p(\varepsilon|e)\left\{x[z\varepsilon k^{\alpha} l^{\gamma} - wl - (r+\delta)k] + (1-x)w\varepsilon]\right\} + (1+r)a$$

and s.t. regime-specific constraints

▸ Capital Accumulation

Private Information

- effort, e, unobserved $\Rightarrow$ moral hazard problem.
- Note: moral hazard for **both** entrepreneurs and workers.
- IC constraint:

$$\sum_{\varepsilon} p(\varepsilon|e)\left\{u[c(\varepsilon),e]+\beta\mathbb{E}v[a'(\varepsilon),z']\right\}$$

$$\geq \sum_{\varepsilon} p(\varepsilon|\hat{e})\left\{u[c(\varepsilon),\hat{e}]+\beta\mathbb{E}v[a'(\varepsilon),z']\right\} \quad \forall e,\hat{e},x$$

- ▸ Lotteries ▸ Connection to Optimal Dynamic Contract

Formulation with Lotteries ▸ Return

- Notation: control variables $d = (c, \varepsilon, e, x)$.
- Lotteries: $\pi(d, a'|a, z) = \pi(c, \varepsilon, e, x, a'|a, z)$

$$v(a,z) = \max_{\pi(d,a'|a,z)} \sum_{D,A} \pi(d,a'|a,z)\{u(c,e) + \beta \mathbb{E} v(a',z')\} \quad \text{s.t.}$$

$$\sum_{D,A} \pi(d,a'|a,z)\{a' + c\}$$

$$= \sum_{D,A} \pi(d,a'|a,z)\{x\Pi(\varepsilon,e,z;w,r) + (1-x)w\varepsilon\}(1+r)a.$$

$$\sum_{(D\setminus E),A} \pi(d,a'|a,z)\{u(c,e) + \beta \mathbb{E} v(a',z')\}$$

$$\geq \sum_{(D\setminus E),A} \pi(d,a'|a,z)\frac{p(\varepsilon|\hat{e})}{p(\varepsilon|e)}\{u(c,\hat{e}) + \beta \mathbb{E} v(a',z')\} \quad \forall e, \hat{e}, x$$

$$\sum_{C,A} \pi(d,a'|a,z) = p(\varepsilon|e) \sum_{C,\varepsilon,A} \pi(d,a'|a,z), \quad \forall \varepsilon, e, x$$

Limited Commitment

- effort, e, observed $\Rightarrow$ perfect insurance against production risk, ε.
- But collateral constraint:

$$k \leq \lambda a, \quad \lambda \geq 1.$$

Factor Demands

- Denote optimal occupational choice and factor demands by

$$x(a,z), \quad l(a,z;w,r), \quad k(a,z;w,r)$$

- and individual (average) labor supply:

$$n(a,z;w,r) \equiv [1-x(a,z)] \sum_{\varepsilon} p[\varepsilon|e(a,z)]\varepsilon.$$

Steady State Equilibrium

- Prices r and w, and corresponding quantities such that:

(i) Taking as given r and w, quantities are determined by optimal contract

(ii) Markets clear

$$\int l(a,z;w,r)dG(a,z) = \int n(a,z;w,r)dG(a,z)$$
$$\int k(a,z;w,r)dG(a,z) = \int a dG(a,z).$$

Parameterization

- Preferences

$$u(c,e) = U(c) - V(e), \quad U(c) = \frac{c^{1-\sigma}}{1-\sigma}, \quad V(e) = \frac{\chi}{1+\varphi} e^{1+\varphi}$$

- Recall production function $\varepsilon z k^{\alpha} l^{\gamma}$.
- Parameters:

$$\alpha = 0.3, \quad \gamma = 0.4, \quad \delta = 0.06$$

$$\beta = 1.05^{-1}, \quad \sigma = 1.5, \quad \chi = .5, \quad \varphi = .2$$

$$\varepsilon \in \{2,4\}, \quad e \in \{0,1\}, \quad p(\varepsilon|e) = \begin{bmatrix} 0.8 & 0.2 \\ 0.2 & 0.8 \end{bmatrix}$$

- Parameters same (range) as those estimated from micro data by **?**

Limited Commitment vs. Moral Hazard

- Savings behavior very different in two regimes.
- Limited commitment: borrowing constrained.

$$U'(c_{it}) = \beta \mathbb{E}_{z,t} \left[U'(c_{it+1})(1+r) + \mu_{it+1}\lambda \right]$$

$$U'(c_{it}) > \beta(1+r)\mathbb{E}_{z,t} U'(c_{it+1})$$

Limited Commitment vs. Moral Hazard

- Moral hazard: inverse Euler equation (**???**).

$$U'(c_{it}) = \beta(1+r)\mathbb{E}_{z,t}\left(\mathbb{E}_{\varepsilon,t}\frac{1}{U'(c_{it+1})}\right)^{-1}$$

- Jensen $\Rightarrow$ savings constrained

$$U'(c_{it}) < \beta(1+r)\mathbb{E}_{z,t}\mathbb{E}_{\varepsilon,t}U'(c_{it+1}).$$

- Note: presence of uninsurable ability z.
- Difference in savings reflected in equilibrium r among others.

Limited Commitment vs. Moral Hazard

Table: Comparison of Different Regimes

	Limited Commitment	Moral Hazard
GDP (% of FB)	0.783	0.870
TFP (% of FB)	0.866	0.902
Capital-Output Ratio (%of FB)	0.824	1.031
Labor Supply (% of FB)	1.078	0.990
Welfare (% of FB)	0.594	0.888
Wage (%of FB)	0.725	0.876
Interest Rate	-0.041	0.010
% Entrepreneurs	0.227	0.143
External Finance/GDP	1.468	3.295

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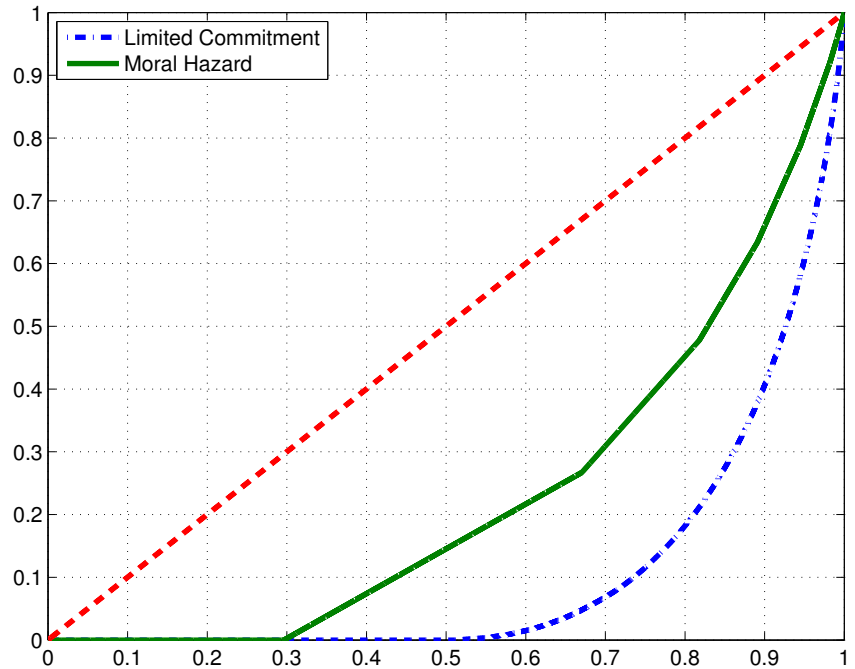
Limited Commitment vs. Moral Hazard

Figure: Wealth Lorenz Curves



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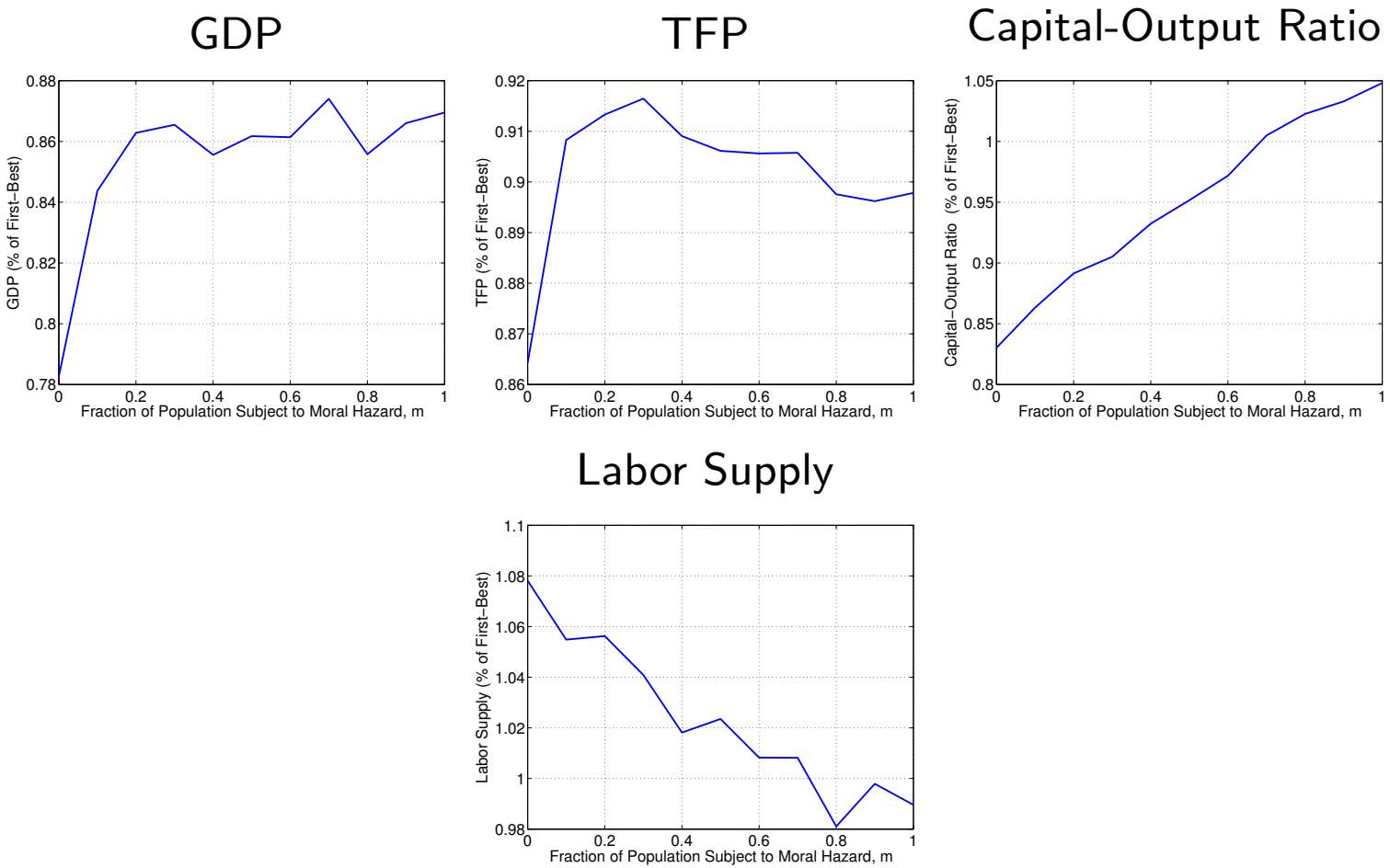
It can be seen that wealth inequality in higher in the limited commitment regime. This is a direct consequence of the bigger dispersion in marginal products of capital.

Mixtures of Moral Hazard and Limited Commitment

- Combine the two regimes in one economy. 50% of pop. subject to moral hazard, 50% to limited commitment.
- Motivation: no reason why economy as a whole should be subject to only one friction.
- Estimated "on the ground" by **?** and **?**: for Thailand, MH fits better in and around Bangkok and LC better in Northeast (see also **?**)
- Also: factor prices different in two regimes $\Rightarrow$ potentially interesting GE effects.

Mixtures of Moral Hazard and Limited Commitment

Figure: Aggregate Impact of Importance of Moral Hazard vs. Limited Commitment, m



Mixtures of Moral Hazard and Limited Commitment

Table: Comparison of LC and MH Sectors in Mixed Regime

	Mixed Regime, m=0.5	LC sector	MH sector
	(a) National and Sectoral Aggregates		
GDP (% of FB)	0.862	0.665	1.059
TFP (% of FB)	0.906	0.963	0.905
Capital-Output Ratio (%of FB)	0.952	0.600	1.172
Labor Supply (% of FB)	1.024	0.789	1.258
Welfare (% of FB)	0.831	0.787	0.876
Wage (%of FB)	0.841	0.841	0.841
Interest Rate	0.003	0.003	0.003
% Entrepreneurs	0.176	0.175	0.176
External Finance/Sectoral GDP	2.774	1.030	3.868
	(b) Importance of Sectors in Aggregate Economy		
% Sector Contributes to GDP		0.386	0.614
% of Labor Employed in Sector		0.385	0.615
% of Capital Used in Sector		0.243	0.757
% of Labor Supplied by Sector		0.531	0.469
% of Capital Supplied by Sector		0.466	0.534

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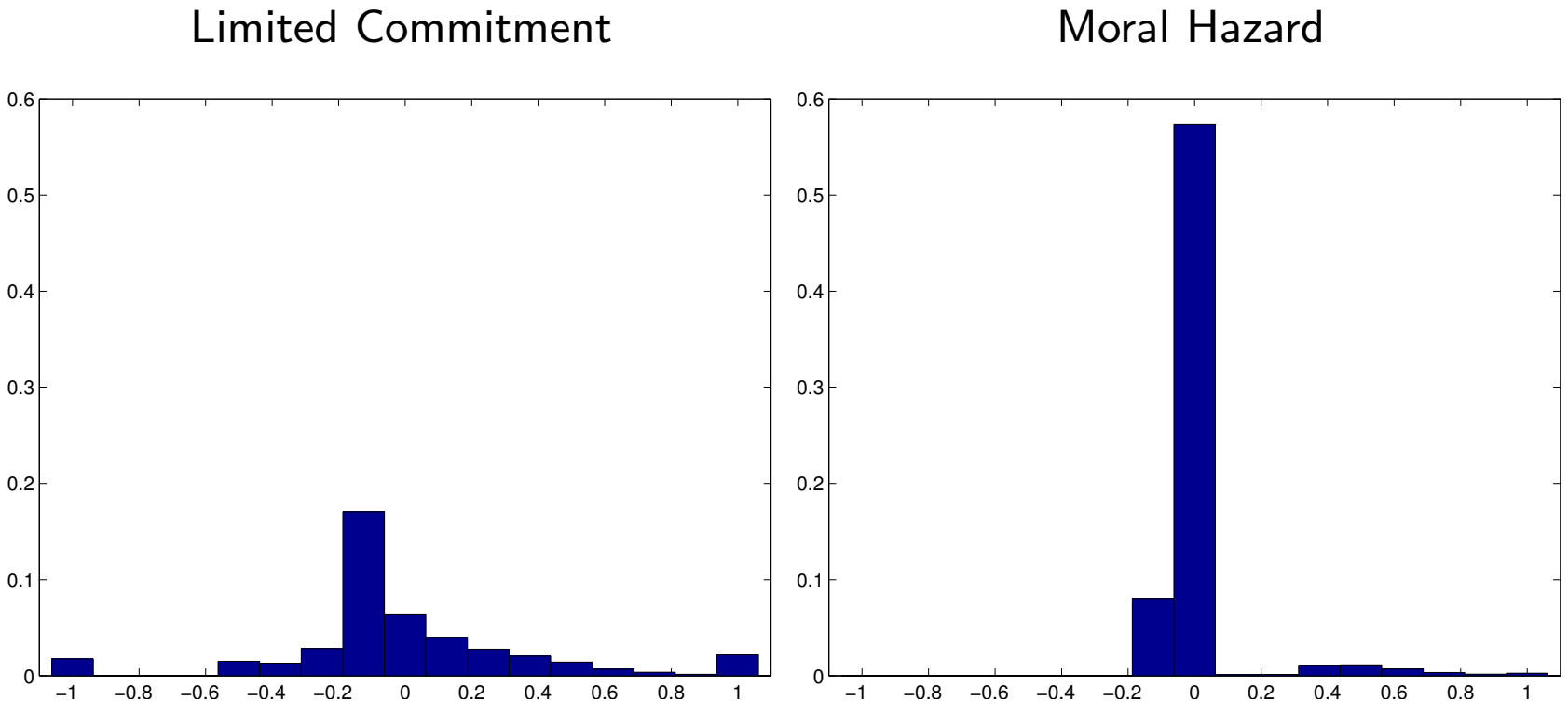
Individual Transitions

- Speed of individual transitions is also very different.
- Examine eigenvalue of transition matrix $\Pr(a', z'|a, z)$ that governs speed of convergence.
- Limited commitment: eig. $= 0.9396 \Rightarrow$ half life $= 11.1$ years.
- Moral hazard: eig. $= 0.9823 \Rightarrow$ half life $= 38.8$ years.
- The slower speed of individual transitions under MH can also be seen in next figure which shows the distribution of individual wealth growth rates

Distribution of Wealth Growth Rates



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- Note: these are of course numerical examples rather than general proofs.

A Transition Experiment

- Start economy in steady state with 100% of pop. subject to limited commitment.
- At time $t = 10$, friction changes: entire pop. now subject to moral hazard.
- Possible interpretation: big migration from area where limited commitment is prevalent to one with moral hazard.

Transition Dynamics

- Similar to before but w_t, r_t vary over time. Bellman:

$$V_t(a,z) = \max_{e,x,k,l,c(\varepsilon),a'(\varepsilon)} \sum_{\varepsilon} p(\varepsilon|e)\left\{u[c(\varepsilon),e] + \beta \mathbb{E}_z V_{t+1}[a'(\varepsilon),z']\right\} \quad \text{s.t.}$$

$$\sum_{\varepsilon} p(\varepsilon|e)\left\{c(\varepsilon) + a'(\varepsilon)\right\}$$

$$\leq \sum_{\varepsilon} p(\varepsilon|e)\left\{x[z\varepsilon f(k,l) - w_t l - (r_t+\delta)k] + (1-x)w_t\varepsilon]\right\} + (1+r_t)a$$

and s.t. regime-specific constraints

- Market clearing analogous to before.

Algorithm

- Adaptation of Buera and Shin (forthcoming)
- Begin with initial guesses $\{(w_t^0, r_t^0)\}_{t=1}^T$. Then for $j = 0, 1, 2, ...$ we follow

(1) Set $V_T^j(a,z) = V_\infty^j(a,z)$. Given $V_T^j(a,z)$, find $V_{T-1}^j(a,z)$ and so on.

(2) Compute factor demands and supplies $\{k_t^j(a,z), l_t^j(a,z), n_t^j(a,z)\}_{t=0}^T$

(3) Compute excess demand $ED_t^j(\{(w_t^j, r_t^j)\}_{t=1}^T), t = 1, ..., T$.

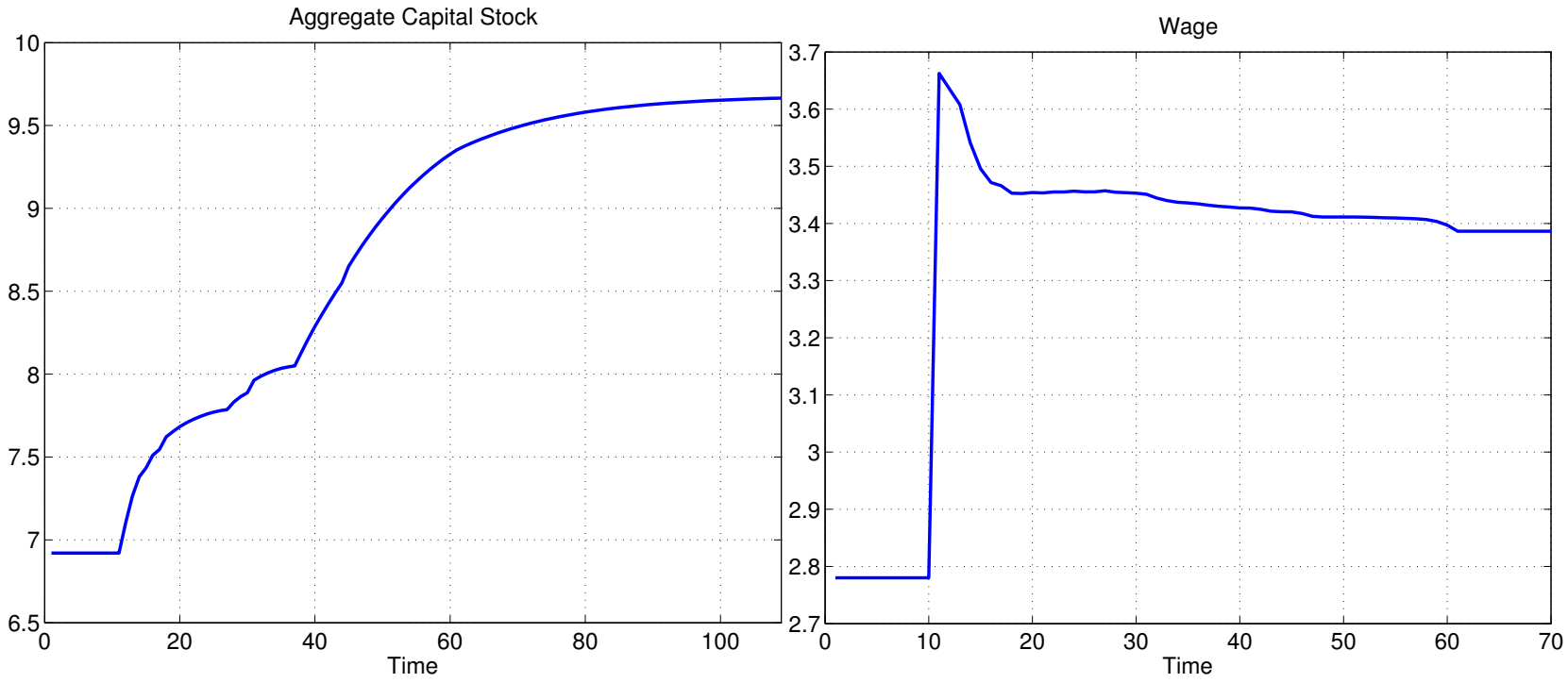
(4) Construct $\{(w_t^{j+1}, r_t^{j+1})\}_{t=1}^T$: find $(\hat{w}_t^j, \hat{r}_t^j)$ that sets $ED_t^j = 0$ and set

$$(w_t^{j+1}, r_t^{j+1}) = \eta(w_t^j, r_t^j) + (1-\eta)(\hat{w}_t^j, \hat{r}_t^j), \quad \eta \in (0,1)$$

- Repeat (1)-(4) until $ED_t^j = 0$ for all t.

Transition

- So far: only small open economy, fixed r. But results encouraging.



Conclusion

- Details of financial sector matter for the macro economy.
- Needed: more research that makes use of micro data and takes seriously the micro financial underpinnings of macro models.
- Join what have been largely two distinct literatures – macro development and micro development – into a coherent whole:
- Macro development needs to take into account the contracts we see on the ground.
- Micro development needs to take into account GE effects of interventions.

Formulation with Lotteries ▸ Return

- Capital and labor only enter the budget constraint $\Rightarrow$ can reduce dimensionality of problem.

$$\max_{k,l} \sum_{Q} p(q|e)\{zqk^{\alpha}l^{\gamma} - wl - (r+\delta)k\}$$

- FOC:

$$\alpha z \sum_{Q} p(q|e) q k^{\alpha-1} l^{\gamma} = r + \delta, \quad \gamma z \sum_{Q} p(q|e) q k^{\alpha} l^{\gamma-1} = w$$

- Solutions: $k(e, z; w, r), l(e, z; w, r)$.
- Realized (not expected) profits:

$$\Pi(q, z, e; w, r) = zqk(e, z; w, r)^{\alpha} l(e, z; w, r)^{\gamma} - wl(e, z; w, r) - (r+\delta)k(e, z; w, r)$$

Formulation with Lotteries (cont'd)

Return

- Notation: control variables $d = (c, q, e, x)$.
- Lotteries: $\pi(d, a'|a, z) = \pi(c, q, e, x, a'|a, z)$

$$v(a,z) = \max_{\pi(d,a'|a,z)} \sum_{D,A} \pi(d,a'|a,z)\{u(c,e) + \beta \mathbb{E} v(a',z')\} \quad \text{s.t.}$$

$$\sum_{D,A} \pi(d,a'|a,z)\{a' + c\}$$

$$= \sum_{D,A} \pi(d,a'|a,z)\{x\Pi(q,e,z;w,r) + (1-x)wq\}(1+r)a.$$

$$\sum_{(D\setminus E),A} \pi(d,a'|a,z)\{u(c,e) + \beta \mathbb{E} v(a',z')\}$$

$$\geq \sum_{(D\setminus E),A} \pi(d,a'|a,z)\frac{p(q|\hat{e})}{p(q|e)}\{u(c,\hat{e}) + \beta \mathbb{E} v(a',z')\} \quad \forall e, \hat{e}, x$$

$$\sum_{C,A} \pi(d,a'|a,z) = p(q|e) \sum_{C,Q,A} \pi(d,a'|a,z), \quad \forall q, e, x$$

Connection to Optimal Dynamic Contract

Return

- Two sources of uncertainty: productivity, z, and prod. risk, ε.
- Argue: our formulation has optimal ε-insurance, but no z-insurance.
- Consider two cases:

(1) special case with no z-shocks, and only ε-shocks: our formulation equivalent to optimal dynamic contract $\Rightarrow$ optimal insurance arrangement regarding ε shocks.

(2) general case: uninsurable z-shocks added on top. No equivalence.

Equivalence with only $\varepsilon-$ but no z-Shocks

- Standard formulation of optimal dynamic contract

$$\Pi(W) = \max_{e,x,k,l,c(\varepsilon),W'(\varepsilon)} \sum_{\varepsilon} p(\varepsilon|e) \quad \tau(\varepsilon) + (1+r)^{-1}\Pi[W'(\varepsilon)] \qquad \text{s.t.}$$

$$\tau(\varepsilon) + c(\varepsilon) = x[\varepsilon f(k,l) - wl - (r+\delta)k] + (1-x)w\varepsilon$$

$$\sum_{\varepsilon} p(\varepsilon|e)\{u[c(\varepsilon),e] + \beta W'(\varepsilon)\} \geq \sum_{\varepsilon} p(\varepsilon|\hat{e})\{u[c(\varepsilon),\hat{e}] + \beta W'(\varepsilon)\} \ \ \forall e,\hat{e},x$$

$$\sum_{\varepsilon} p(\varepsilon|e)\{u[c(\varepsilon),e] + \beta W'(\varepsilon)\} = W.$$

Equivalence with only $\varepsilon-$ but no z-Shocks

Proposition

Suppose the Pareto frontier $\Pi(W)$ is decreasing at all values of promised utility, W, that are used as continuation values at some point in time. Then the following dynamic program is equivalent to the optimal dynamic contract on the last slide:

$$v(a) = \max_{e,x,k,l,c(\varepsilon),a'(\varepsilon)} \sum_{\varepsilon} p(\varepsilon|e)\left\{u[c(\varepsilon),e] + \beta v[a'(\varepsilon)]\right\} \quad s.t.$$

$$\sum_{\varepsilon} p(\varepsilon|e)\left\{u[c(\varepsilon),e] + \beta v[a'(\varepsilon)]\right\} \geq \sum_{\varepsilon} p(\varepsilon|\hat{e})\left\{u[c(\varepsilon),\hat{e}] + \beta v[a'(\varepsilon)]\right\} \ \forall e,\hat{e},x$$

$$\sum_{\varepsilon} p(\varepsilon|e)\left\{c(\varepsilon) + a'(\varepsilon)\right\}$$

$$= \sum_{\varepsilon} p(\varepsilon|e)\left\{x[\varepsilon f(k,l) - wl - (r+\delta)k] + (1-x)w\varepsilon\right\} + (1+r)a$$

Equivalence with only $\varepsilon-$ but no z-Shocks

Proof: The proof has two steps.

Step 1: write down dual to standard formulation. Because the Pareto frontier $\Pi(W)$ is decreasing at the W under consideration, we can write the promise-keeping constraint with a (weak) inequality rather than an inequality. This does not change the allocation chosen under the optimal contract. The dual is then to maximize

$$V(\pi) = \max_{e,x,k,l,c(\varepsilon),\pi'(\varepsilon)} \sum_{\varepsilon} p(\varepsilon|e)\left\{u[c(\varepsilon),e] + \beta V[\pi'(\varepsilon)]\right\} \quad \text{s.t.}$$

$$\sum_{\varepsilon} p(\varepsilon|e)\left\{u[c(\varepsilon),e] + \beta V[\pi'(\varepsilon)]\right\} \geq \sum_{\varepsilon} p(\varepsilon|\hat{e})\left\{u[c(\varepsilon),\hat{e}] + \beta V[\pi'(\varepsilon)]\right\} \ \forall e,\hat{e},$$

$$\sum_{\varepsilon} p(\varepsilon|e) \quad \tau(\varepsilon) + (1+r)^{-1}\pi'(\varepsilon) \quad \geq \pi.$$

$$\tau(\varepsilon) = x[\varepsilon f(k,l) - wl - (r+\delta)k] + (1-x)w\varepsilon - c(\varepsilon)$$

Equivalence with only $\varepsilon-$ but no z-Shocks

Step 2: express dual in terms of asset position rather than profits. Let

$$\pi = -a(1+r), \quad \pi'(\varepsilon) = -a'(\varepsilon)(1+r)$$

and rewrite the dual using this change of variables. Finally, define $v(a) = V[-(1+r)a]$.□

- The change of variables just uses the present-value budget constraint to express the problem in terms of assets rather than the PDV of intermediary profits.

General Case with Both $\varepsilon-$ and z-Shocks

- Standard formulation of optimal dynamic contract

$$\Pi(W,z) = \max_{e,x,k,I,c(\varepsilon),W'(\varepsilon)} \sum_{\varepsilon} p(\varepsilon|e) \;\; \tau(\varepsilon) + (1+r)^{-1}\mathbb{E}_z\Pi[W'(\varepsilon),z'] \qquad \text{s.t.}$$

$$\tau(\varepsilon) + c(\varepsilon) = x[z\varepsilon f(k,I) - wI - (r+\delta)k] + (1-x)w\varepsilon$$

$$\sum_{\varepsilon} p(\varepsilon|e)\left\{u[c(\varepsilon),e] + \beta W'(\varepsilon)\right\} \geq \sum_{\varepsilon} p(\varepsilon|\hat{e})\left\{u[c(\varepsilon),\hat{e}] + \beta W'(\varepsilon)\right\} \;\; \forall e,\hat{e},x$$

$$\sum_{\varepsilon} p(\varepsilon|e)\left\{u[c(\varepsilon),e] + \beta W'(\varepsilon)\right\} = W.$$

  - Compare this to our formulation
  - Optimal contract: utility $W(\varepsilon)$ cannot depend on $z' \Rightarrow$ principal absorbs all gains or losses from z shocks.
  - Our formulation: agent's utility varies with z' and its wealth does not. Since agent wealth equals principal's utility (profit) this means that the principal's welfare is independent of z'.

Why Are MPKs Equalized? ▸ Return

- Suppose more general production tech:
- Output $y \sim g(y|e,k)$, cdf $G(y|e,k)$.
- Make argument with simplified version of model:

$$V(w,k) = \max_{e,c(y),k'(y),w'(y)} \int g(y|e,k)\{y - \tau(y) + (1/R)V(w'(y),k'(y))\}\,dy \quad s$$

$$c(y) + k'(y) = \tau(y) + (1-\delta)k$$

$$\int g(y|e,k)\{U[c(y),e] + \beta w'(y)\}\,dy = w$$

$$\int g(y|e,k)\{U[c(y),e] + \beta w'(y)\}\,dy \geq \int g(y|\hat{e},k)\{U[c(y),\hat{e}] + \beta w'(y)\}\,dy,$$

Why Are MPKs Equalized? ▸ Return

- **Assumption 1** There exist functions P and f such that

$$G(y|e,k) = P\left(\frac{y}{f(k)}\middle|e\right)$$

- E.g.: y is log-normally distributed

$$G(y|e,k) = \Phi\left(\frac{\log y - \mu(e,k)}{\sigma(e,k)}\right)$$

- Sufficient condition for Assumption 1:

$$\mu(e,k) = \mu_e(e) + \mu_k(k), \quad \sigma(e,k) = \sigma_e(e)$$

- Follows from

$$G(y|e,k) = \Phi\left(\frac{\log y - \mu_e(e) - \mu_k(k)}{\sigma_e(e)}\right)$$

$$= \Phi\left(\frac{\log(y/f(k)) - \mu_e(e)}{\sigma_e(e)}\right) = P\left(\frac{y}{f(k)}\middle|e\right), \quad f(k) \equiv \exp(\mu_k(k))$$

Why Are MPKs Equalized? ▸ Return

Claim 1: Under Assumption 1, expected output can be written as

$$yg(y|e,k)dy = \quad qp(q|e)dq \quad f(k) \tag{1}$$

Proof: Define $p(x|e) \equiv \partial G(x|e)/\partial x$. Then

$$g(y|e,k)dy = p \quad \frac{y}{f(k)} \; e \quad \frac{1}{f(k)} dy$$

or using the change of variables $q = y/f(k)$

$$g(y|e,k)dy = p\,(q|e)\,dq$$

Similarly, we obtain (1).□

Why Are MPKs Equalized? ▸ Return

Claim 2: Under Assumption 1, expected marginal products of capital are equalized across agents and equal $R - 1 + \delta$,

$$\frac{\partial}{\partial k} \int yg(y|e,k)dy = \int qp(q|e)dq \quad f'(k) = R-1+\delta, \quad \text{all } (w,k)$$

Proof:

$$V(w,k) = \max_{e,c(q),k'(q),w'(q)} \int p(q|e)\left\{qf(k) - \tau(q) + (1/R)V(w'(q),k'(q))\right\} dq$$

$$c(q) + k'(q) = \tau(q) + (1-\delta)k$$

$$\int p(q|e)\left\{U[c(q),e] + \beta w'(q)\right\} dq = w$$

$$\int p(q|e)\left\{U[c(q),e] + \beta w'(q)\right\} dq \geq \int p(q|\hat{e})\left\{U[c(q),\hat{e}] + \beta w'(q)\right\} dq, \quad \forall\, e,$$

FOCs $\Rightarrow$ MPKs equalized.

Capital Accumulation

▸ Return

- Representative capital producing firm solves

$$V_0 = \max \sum_{t=0}^{\infty} \frac{D_t}{(1+r)^t} \quad \text{s.t.}$$

$$B_{t+1} + I_t + D_t = R_t K_t + (1+r_t)B_t, \quad K_{t+1} = I_t + (1-\delta)K_t$$

- $\Rightarrow$ no arbitrage: $R_t = r_t + \delta$.
- Bond market clearing

$$B_t + \quad adG_t(a,z) = 0, \quad \text{all } t$$

- Can show:

$$V_t = (1+r)(K_t + B_t), \quad \text{all } t$$

- Zero profits + bond market clearing $\Rightarrow$

$$K_t = \quad adG_t(a,z), \quad \text{all } t.$$

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