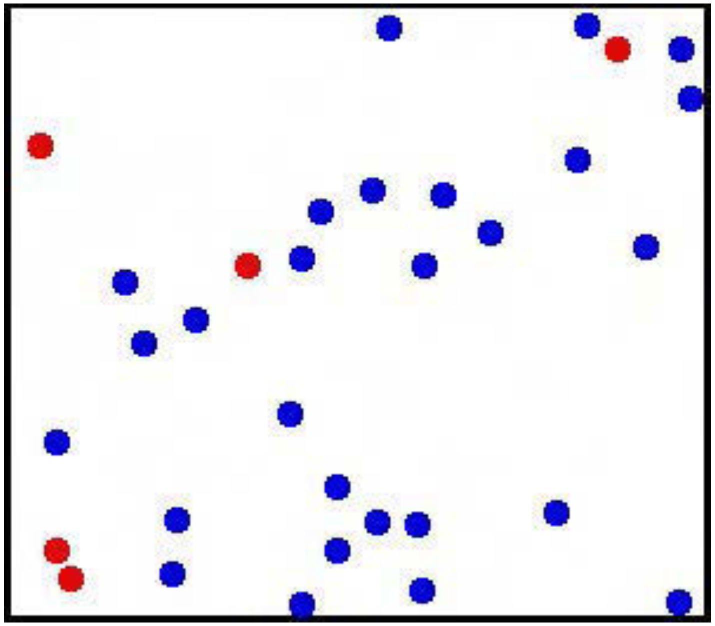
Lecture 5: Random Walks

Relevant Reading

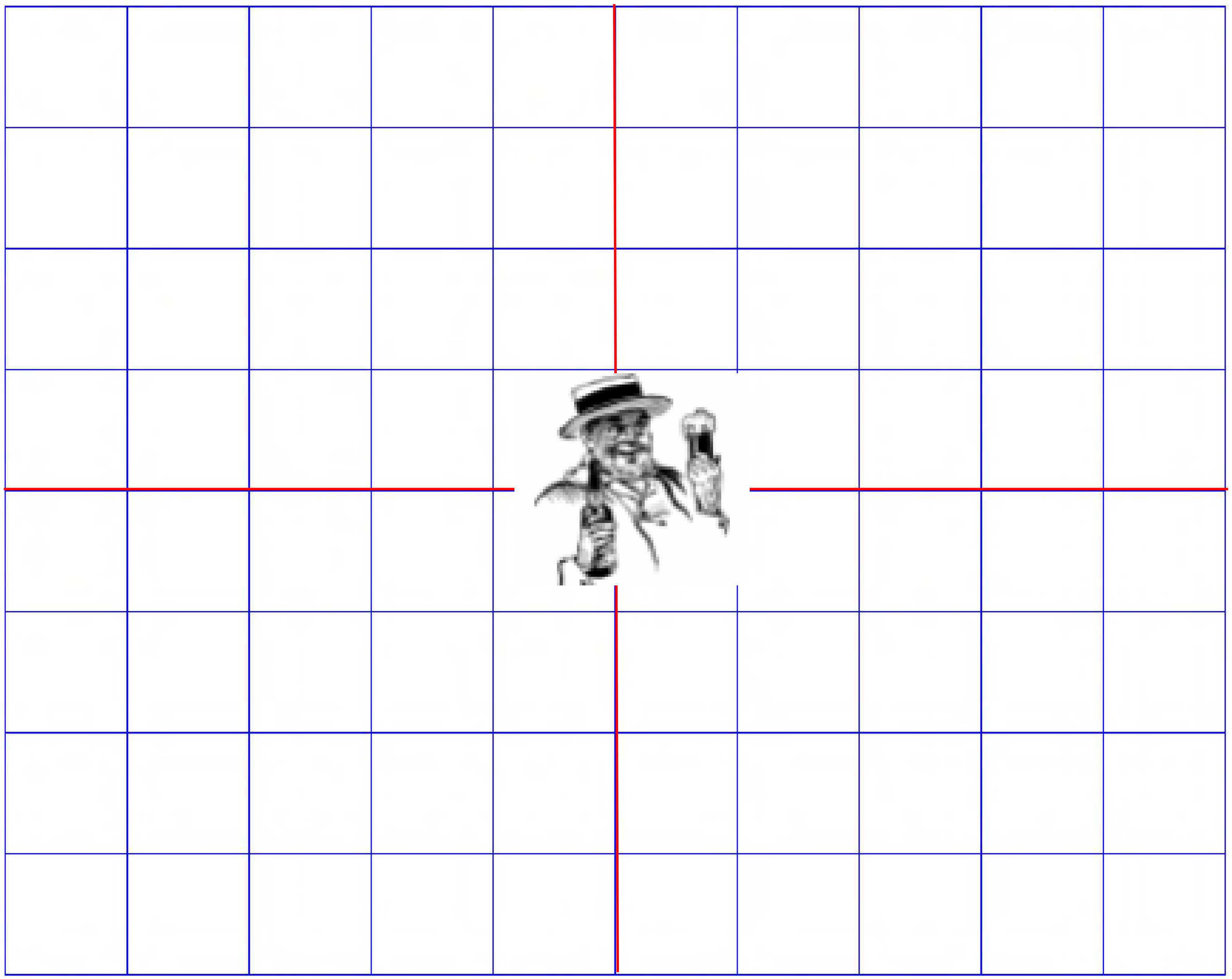
- Chapter 11
- Chapter 14

Why Random Walks?

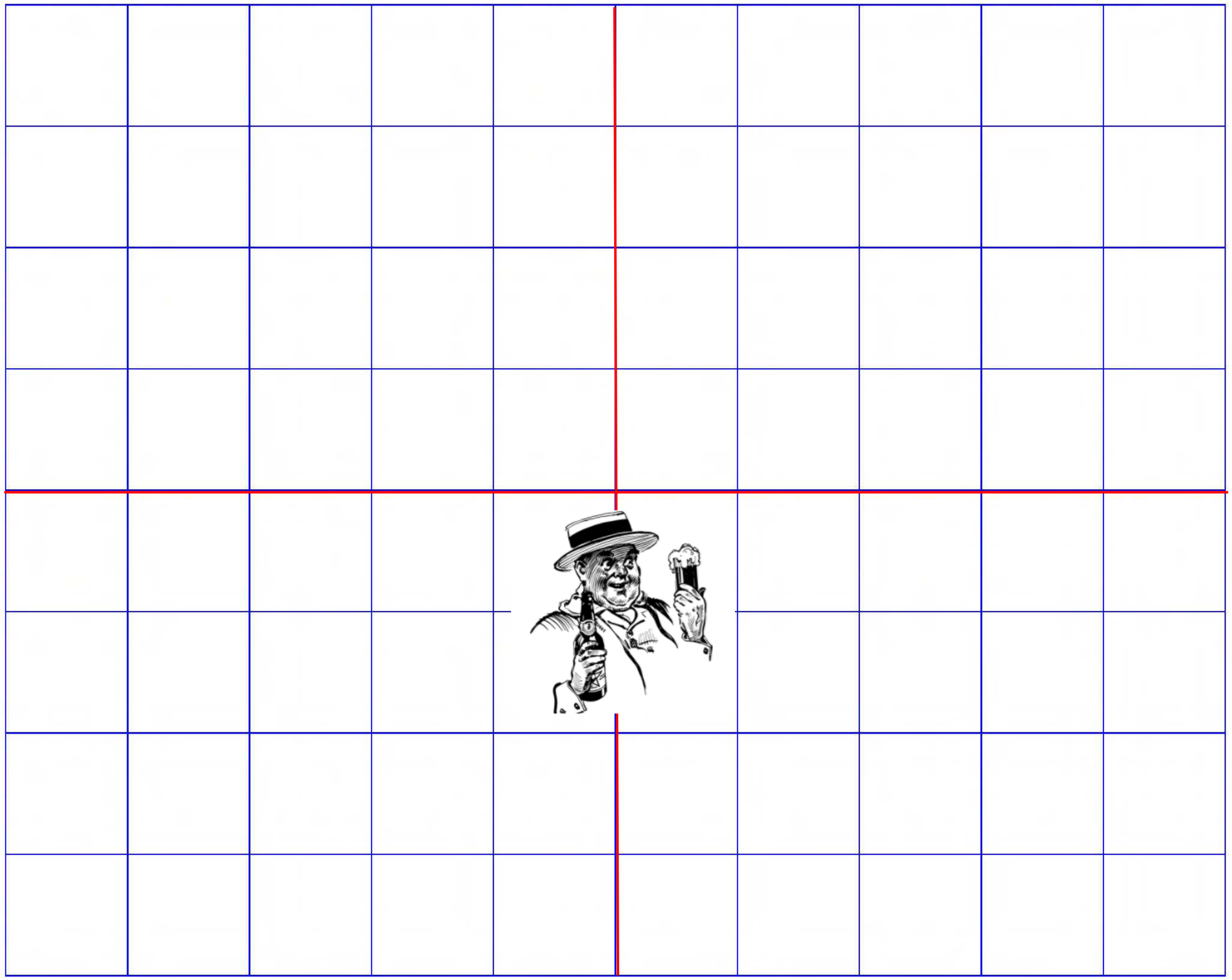
- Random walks are important in many domains
  - Understanding the stock market (maybe)
  - Modeling diffusion processes
  - Etc.
- Good illustration of how to use simulations to understand things
- Excuse to cover some important programming topics
  - Practice with classes
  - Practice with plotting



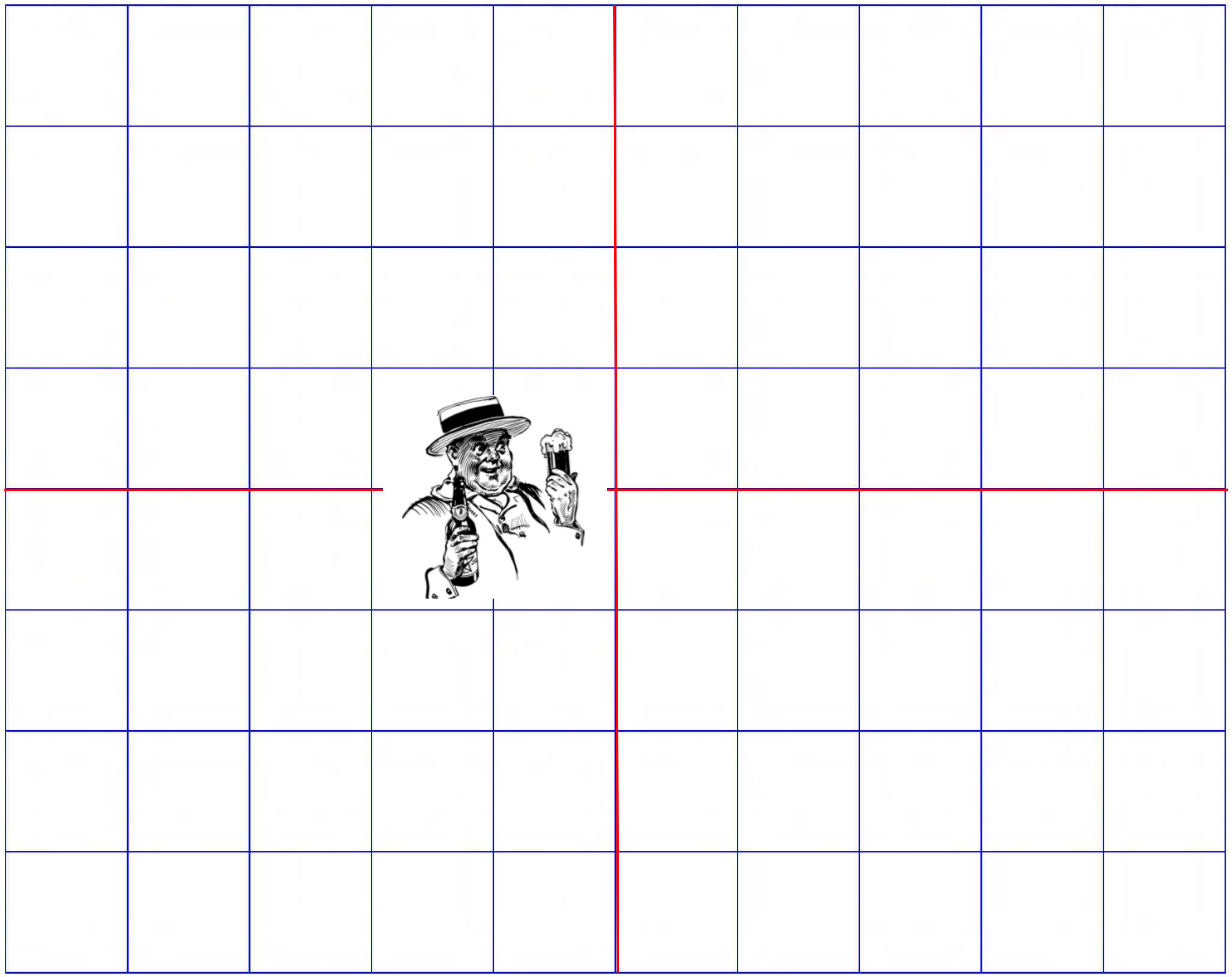
Drunkard's Walk



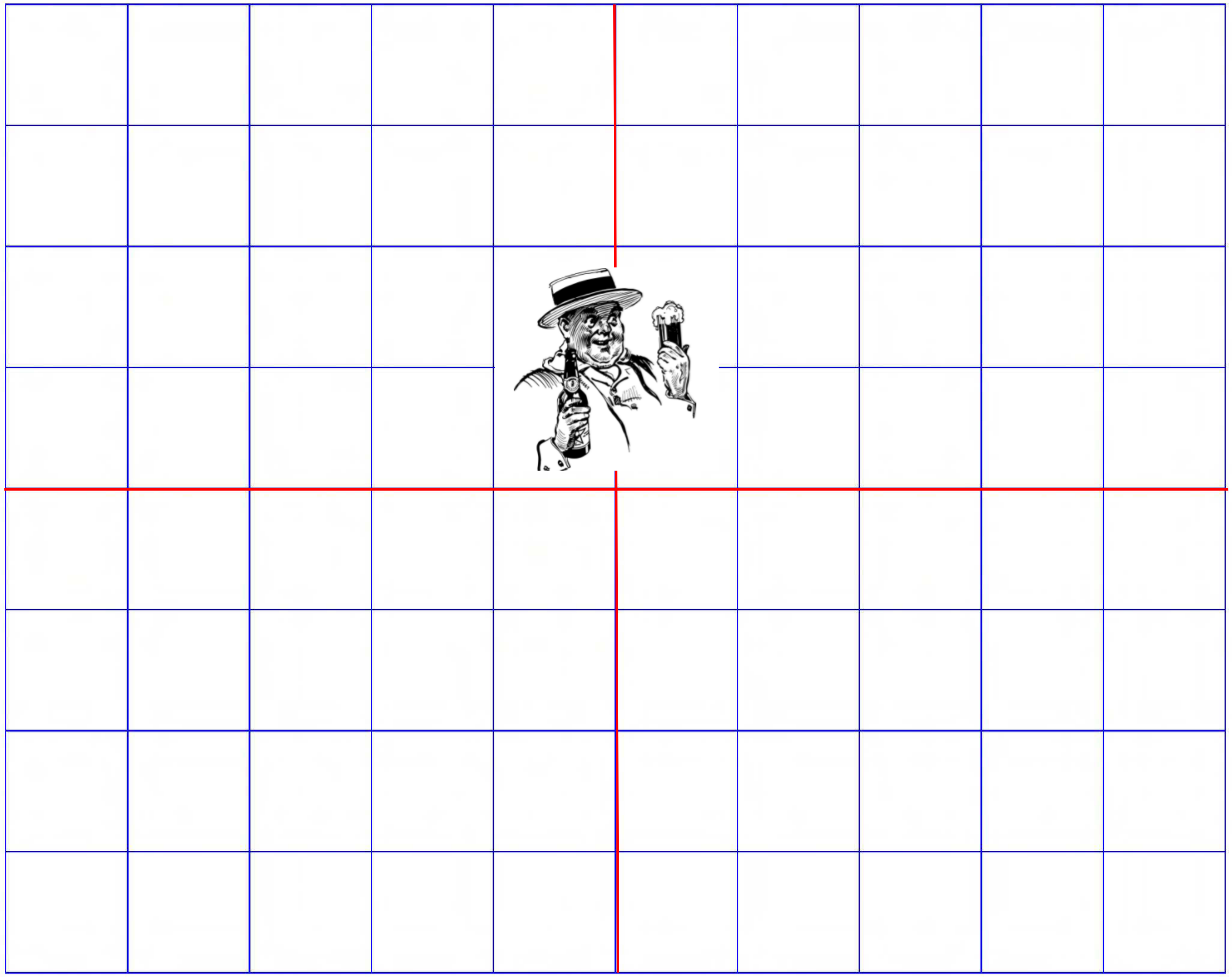
One Possible First Step



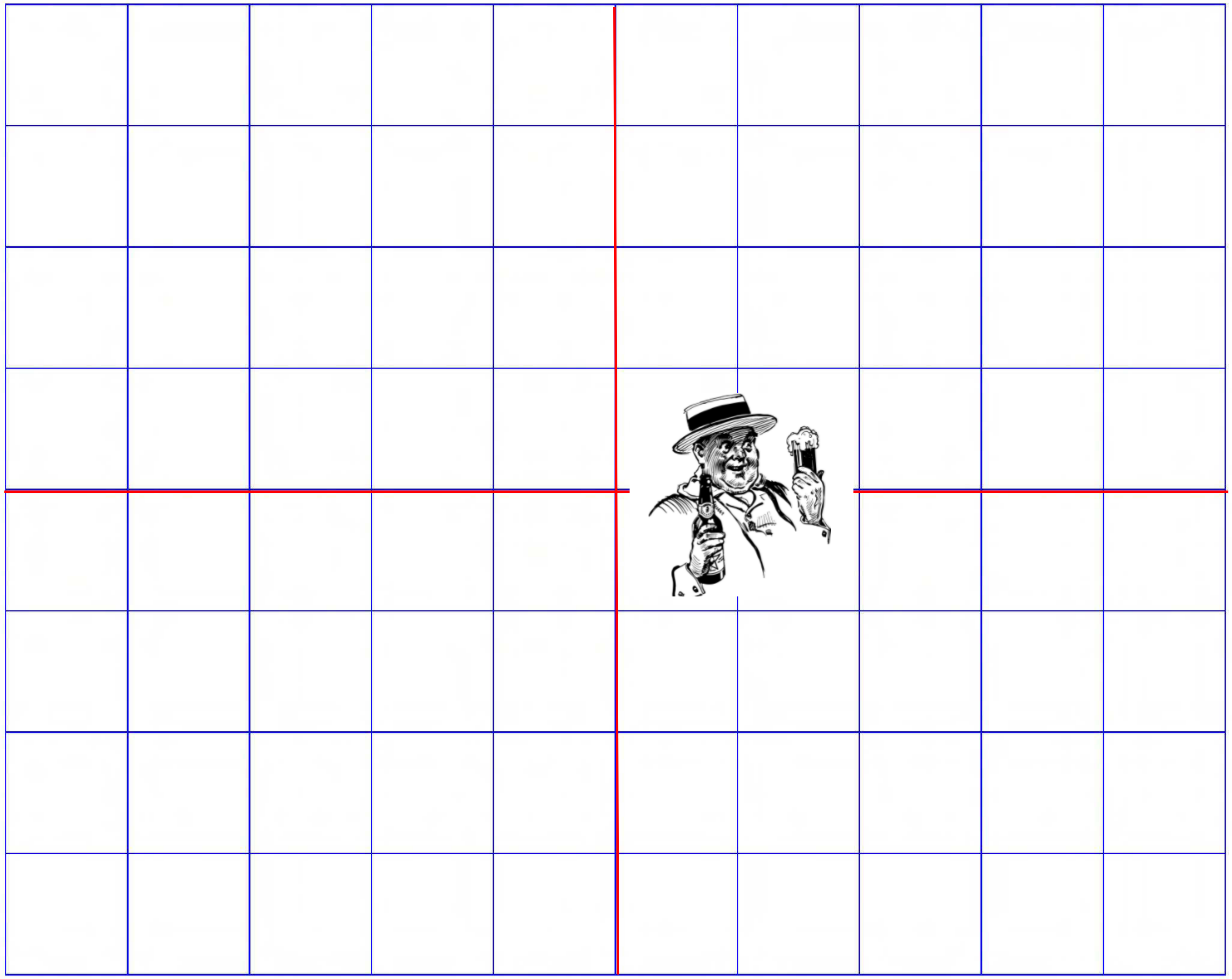
Another Possible First Step



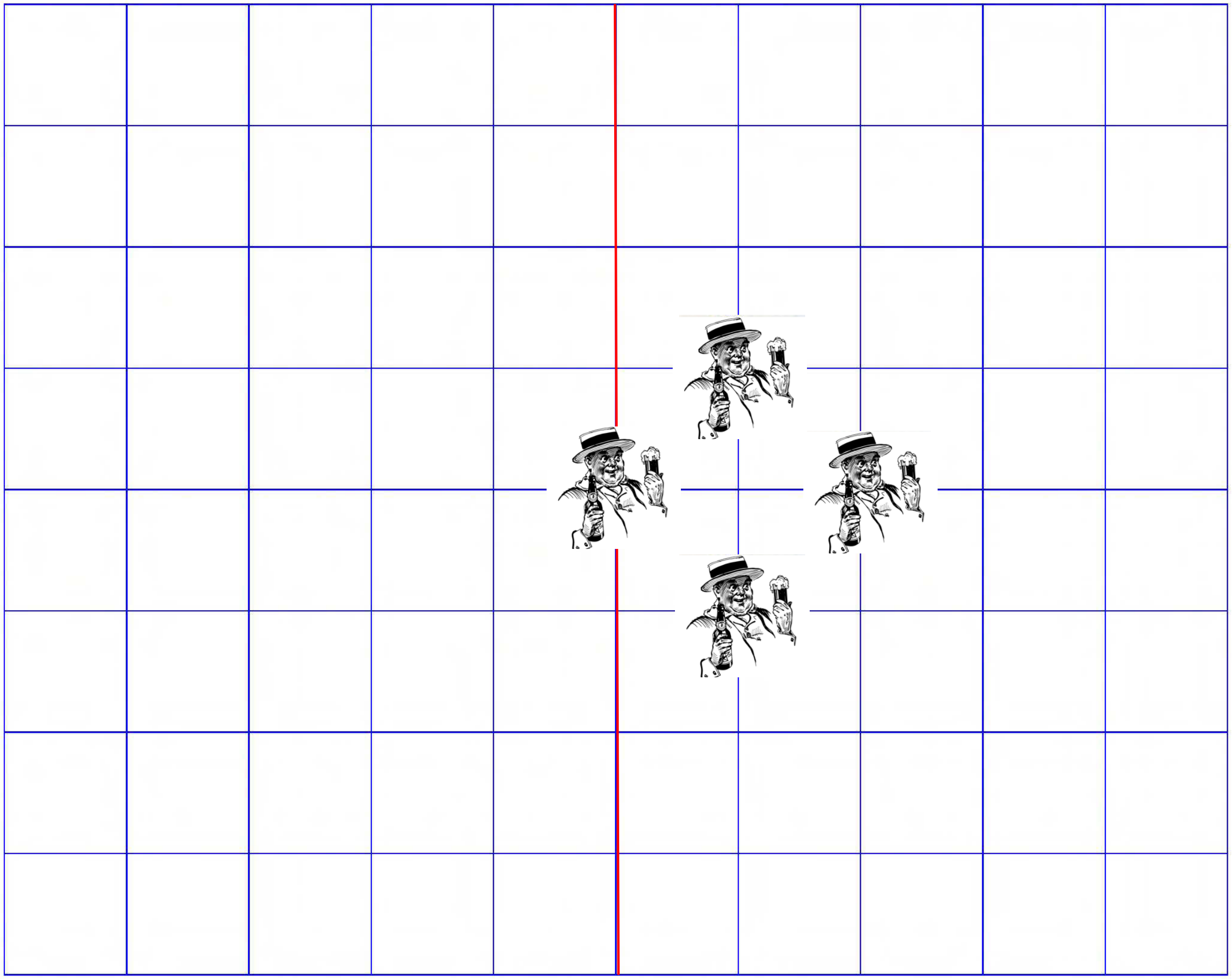
Yet Another Possible First Step



Last Possible First Step



Possible Distances After Two Steps



Expected Distance After 100,000 Steps?

- Need a different approach to problem
- Will use simulation

Structure of Simulation

- Simulate one walks of k steps
- Simulate n such walks
- Report average distance from origin

First, Some Useful Abstractions

- Location—a place
- Field—a collection of places and drunks
- Drunk—somebody who wanders from place to place in a field

Class Location, part 1

Immutable type

```
class Location(object):
    def __init__(self, x, y):
        """x and y are floats"""
        self.x = x
        self.y = y

    def move(self, deltaX, deltaY):
        """deltaX and deltaY are floats"""
        return Location(self.x + deltaX,
                        self.y + deltaY)
    def getX(self):
        return self.x
    def getY(self):
        return self.y
```

Class Location, continued

```
def distFrom(self, other):
        xDist = self.x - other.getX()
        yDist = self.y - other.getY()
        return (xDist**2 + yDist**2)**0.5

def __str__(self):
        return '<' + str(self.x) + ', '\
               + str(self.y) + '>'
```

Class Drunk

```
class Drunk(object):
    def __init__(self, name = None):
        """Assumes name is a str"""
        self.name = name

    def __str__(self):
        if self != None:
            return self.name
        return 'Anonymous'
```

Not intended to be useful on its own

A base class to be inherited

Two Subclasses of Drunk

- The “usual” drunk, who wanders around at random
- The “masochistic” drunk, who tries to move northward

Two Kinds of Drunks

```
import random

class UsualDrunk(Drunk):
    def takeStep(self):
        stepChoices = [(0,1), (0,-1), (1, 0), (-1, 0)]
        return random.choice(stepChoices)

class MasochistDrunk(Drunk):
    def takeStep(self):
        stepChoices = [(0.0,1.1), (0.0,-0.9),
                       (1.0, 0.0), (-1.0, 0.0)]
        return random.choice(stepChoices)
```

Immutable or not?

Class Field, part 1

```
class Field(object):
    def __init__(self):
        self.drunks = {}

    def addDrunk(self, drunk, loc):
        if drunk in self.drunks:
            raise ValueError('Duplicate drunk')
        else:
            self.drunks[drunk] = loc

    def getLoc(self, drunk):
        if drunk not in self.drunks:
            raise ValueError('Drunk not in field')
        return self.drunks[drunk]
```

Class Field, continued

```
def moveDrunk(self, drunk):
    if drunk not in self.drunks:
        raise ValueError('Drunk not in field')
    xDist, yDist = drunk.takeStep()
    #use move method of Location to get new location
    self.drunks[drunk] =\
        self.drunks[drunk].move(xDist, yDist)
```

Immutable or not?

Simulating a Single Walk

```
def walk(f, d, numSteps):
    """Assumes: f a Field, d a Drunk in f, and
       numSteps an int >= 0.
       Moves d numSteps times; returns the distance
       between the final location and the location
       at the start of the  walk."""
    start = f.getLoc(d)
    for s in range(numSteps):
        f.moveDrunk(d)
    return start.distFrom(f.getLoc(d))
```

Simulating Multiple Walks

```
def simWalks(numSteps, numTrials, dClass):
    """Assumes numSteps an int >= 0, numTrials an
         int > 0, dClass a subclass of Drunk
       Simulates numTrials walks of numSteps steps
         each. Returns a list of the final distances
         for each trial"""
    Homer = dClass()
    origin = Location(0, 0)
    distances = []
    for t in range(numTrials):
        f = Field()
        f.addDrunk(Homer, origin)
        distances.append(round(walk(f, Homer,
                                    numTrials), 1))
    return distances
```

Putting It All Together

```
def drunkTest(walkLengths, numTrials, dClass):
    """Assumes walkLengths a sequence of ints >= 0
         numTrials an int > 0,
         dClass a subclass of Drunk
       For each number of steps in walkLengths,
         runs simWalks with numTrials walks and
         prints results"""
    for numSteps in walkLengths:
        distances = simWalks(numSteps, numTrials,
                             dClass)
        print(dClass.__name__, 'random walk of',
              numSteps, 'steps')
        print(' Mean =',
              round(sum(distances)/len(distances), 4))
        print(' Max =', max(distances),
              'Min =', min(distances))
```

Let's Try It

```
drunkTest((10, 100, 1000, 10000), 100,
           UsualDrunk)

UsualDrunk random walk of 10 steps
 Mean = 8.634
 Max = 21.6 Min = 1.4
UsualDrunk random walk of 100 steps
 Mean = 8.57
 Max = 22.0 Min = 0.0
UsualDrunk random walk of 1000 steps
 Mean = 9.206
 Max = 21.6 Min = 1.4
UsualDrunk random walk of 10000 steps
 Mean = 8.727
 Max = 23.5 Min = 1.4
```

Plausible?

Let's Try a Sanity Check

- Try on cases where we think we know the answer
  - A very important precaution!

Sanity Check

```
drunkTest((0, 1, 2) 100, UsualDrunk)

UsualDrunk random walk of 0 steps
 Mean = 8.634
 Max = 21.6 Min = 1.4
UsualDrunk random walk of 1 steps
 Mean = 8.57
 Max = 22.0 Min = 0.0
UsualDrunk random walk of 2 steps
 Mean = 9.206
 Max = 21.6 Min = 1.4
```

```
distances.append(round(walk(f, Homer,
                            numTrials), 1))
```

Let's Try It

```
drunkTest((10, 100, 1000, 10000), 100,
          UsualDrunk)

UsualDrunk random walk of 10 steps
 Mean = 2.863
 Max = 7.2 Min = 0.0
UsualDrunk random walk of 100 steps
 Mean = 8.296
 Max = 21.6 Min = 1.4
UsualDrunk random walk of 1000 steps
 Mean = 27.297
 Max = 66.3 Min = 4.2
UsualDrunk random walk of 10000 steps
 Mean = 89.241
 Max = 226.5 Min = 10.0
```

And the Masochistic Drunk?

```
random.seed(0)
simAll((UsualDrunk, MasochistDrunk),
       (1000, 10000), 100)
```

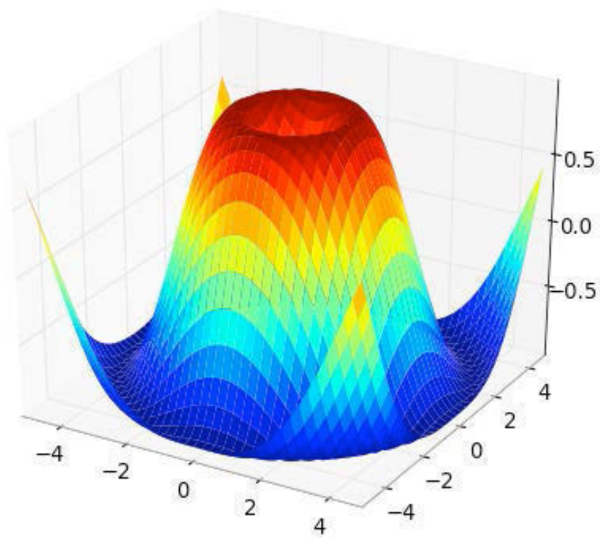
```
UsualDrunk random walk of 1000 steps
 Mean = 26.828
 Max = 66.3 Min = 4.2
UsualDrunk random walk of 10000 steps
 Mean = 90.073
 Max = 210.6 Min = 7.2
MasochistDrunk random walk of 1000 steps
 Mean = 58.425
 Max = 133.3 Min = 6.7
MasochistDrunk random walk of 10000 steps
 Mean = 515.575
 Max = 694.6 Min = 377.7
```

Visualizing the Trend

- Simulate walks of multiple lengths for each kind of drunk
- Plot distance at end of each length walk for each kind of drunk

Pylab

- NumPy adds vectors, matrices, and many high-level mathematical functions
- SciPy adds mathematical classes and functions useful to scientists
- MatPlotLib adds an object-oriented API for plotting
- PyLab combines the other libraries to provide a MATLAB®- like interface



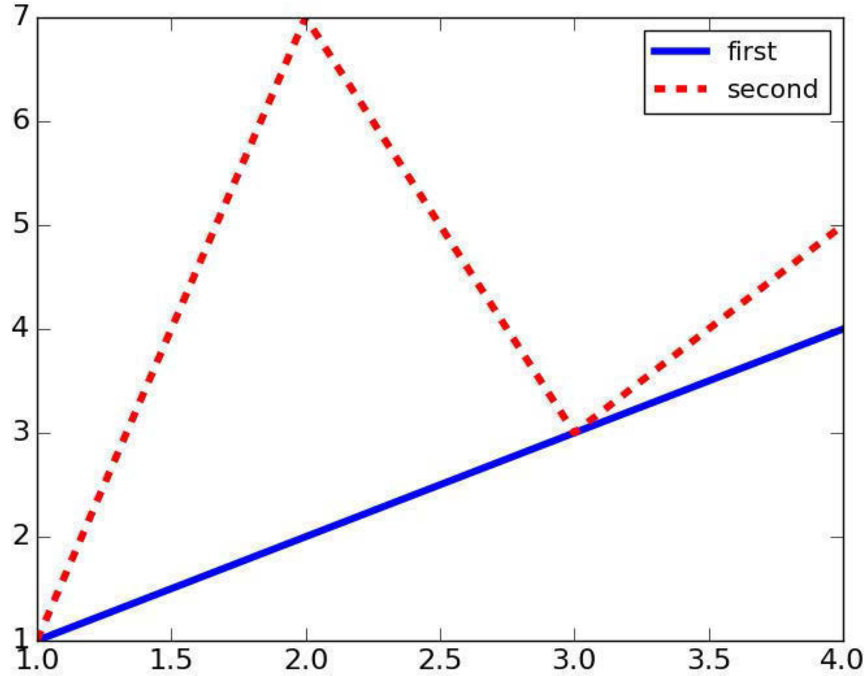
plot

- The first two arguments to `pylab.plot` must be sequences of the same length.
- First argument gives x-coordinates.
- Second argument gives y-coordinates.
- Many optional arguments
- Points plotted in order. In default style, as each point is plotted, a line is drawn connecting it to the previous point.

Example

```
import pylab

xVals = [1, 2, 3, 4]
yVals1 = [1, 2, 3, 4]
pylab.plot(xVals, yVals1, 'b-', label = 'first')
yVals2 = [1, 7, 3, 5]
pylab.plot(xVals, yVals2, 'r--', label = 'second')
pylab.legend()
```

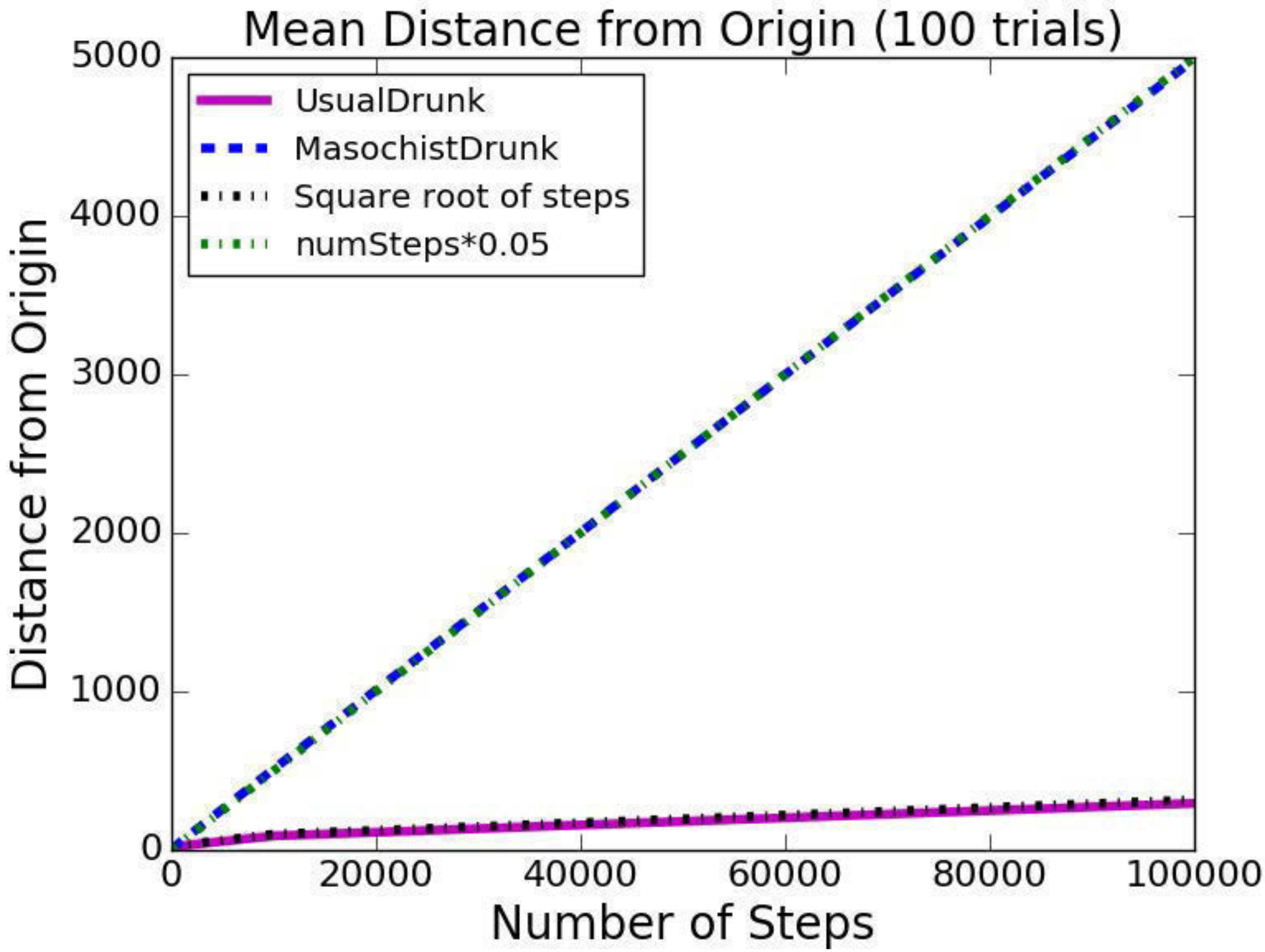


Details and Many More Examples

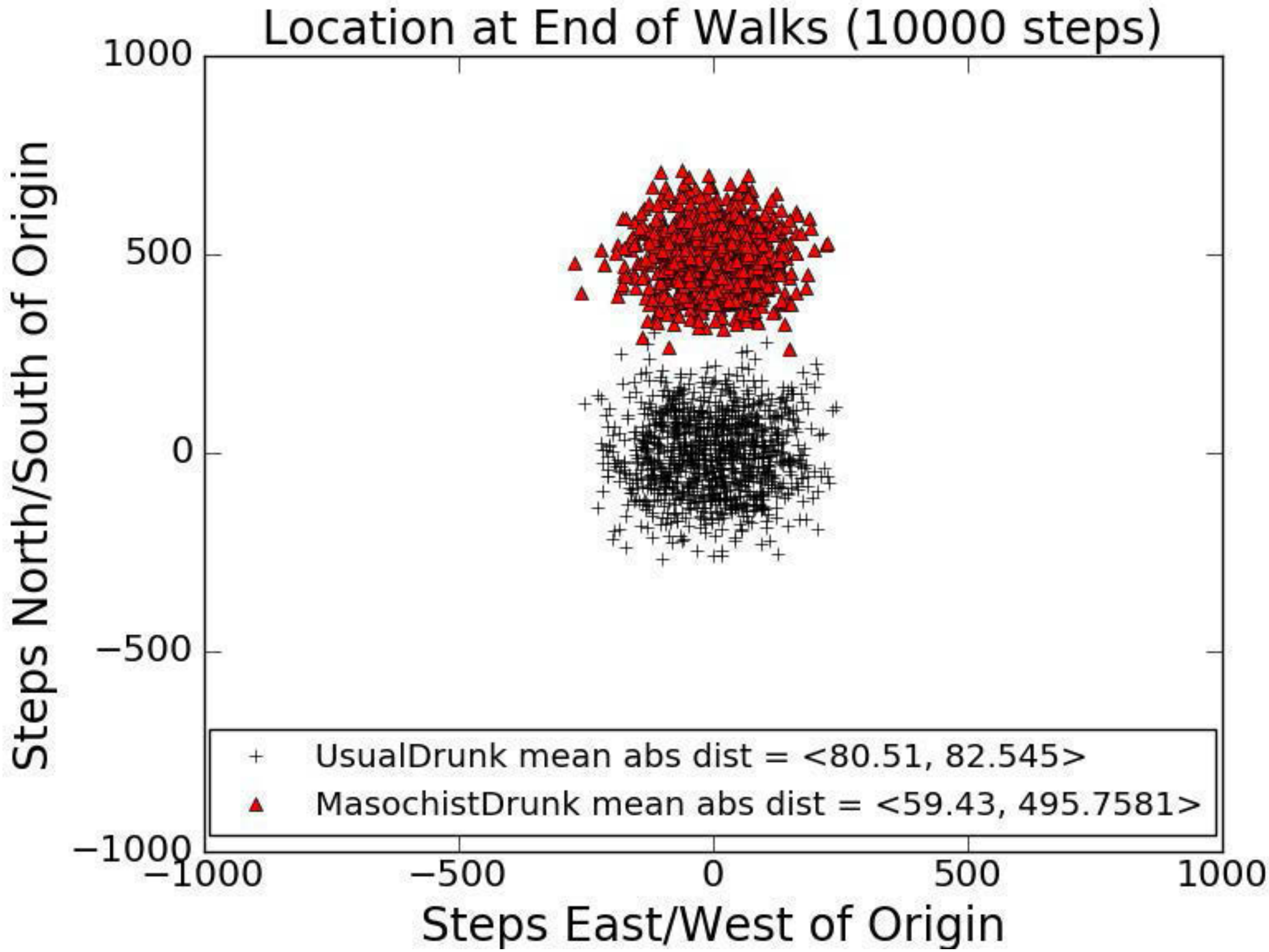
- Assigned reading
- Video of Prof. Grimson’s lecture from 6.00x.1
- Code for this lecture
- matplotlib.org/api/pyplot_summary.html
- www.scipy.org/Plotting_Tutorial

You should learn how to produce the plots that I will show you

Distance Trends



Ending Locations



Fields with Wormholes

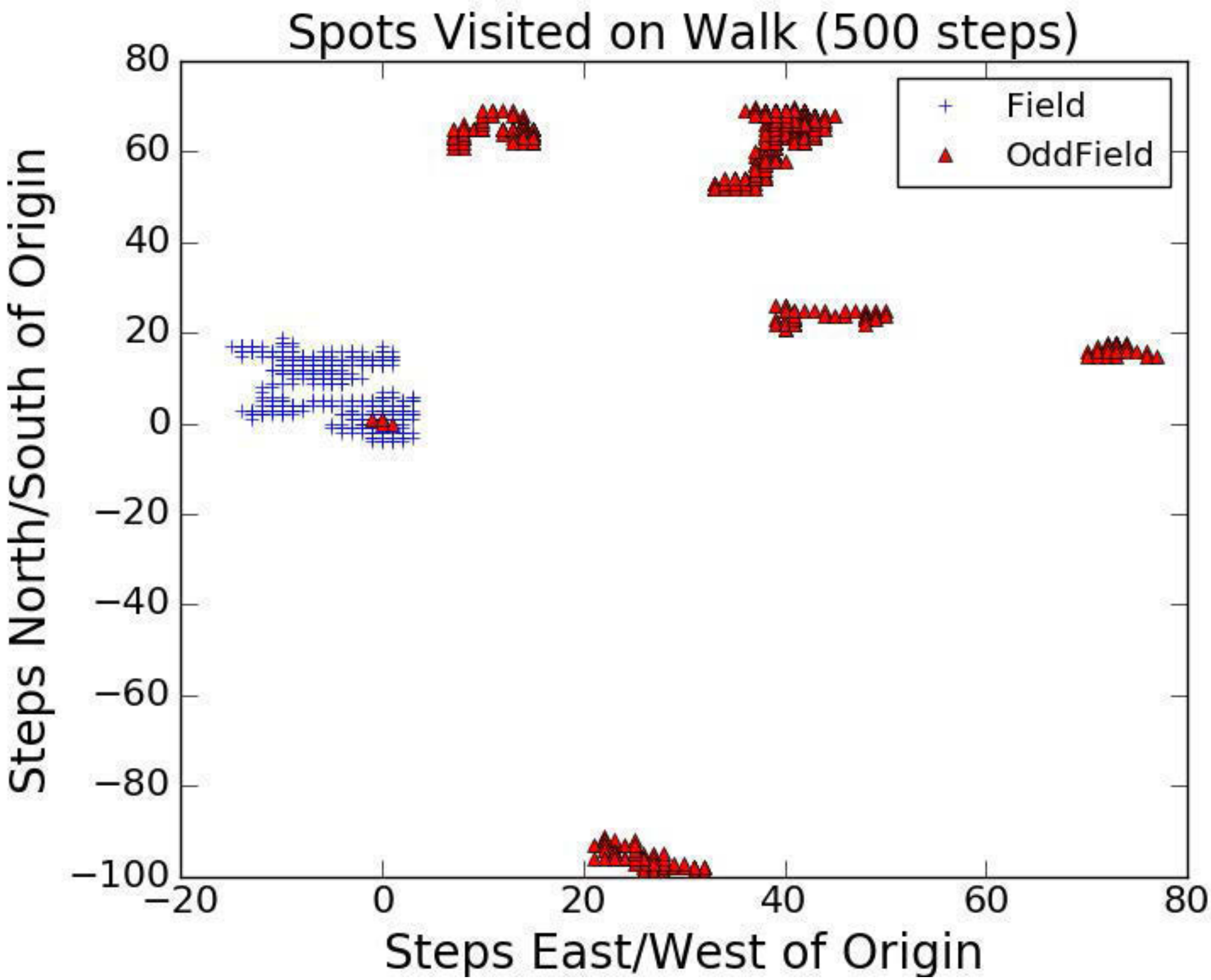
A Subclass of Field, part 1

```
class OddField(Field):
    def __init__(self, numHoles = 1000,
                 xRange = 100, yRange = 100):
        Field.__init__(self)
        self.wormholes = {}
        for w in range(numHoles):
            x = random.randint(-xRange, xRange)
            y = random.randint(-yRange, yRange)
            newX = random.randint(-xRange, xRange)
            newY = random.randint(-yRange, yRange)
            newLoc = Location(newX, newY)
            self.wormholes[(x, y)] = newLoc
```

A Subclass of Field, part 2

```
def moveDrunk(self, drunk):
    Field.moveDrunk(self, drunk)
    x = self.drunks[drunk].getX()
    y = self.drunks[drunk].getY()
    if (x, y) in self.wormholes:
        self.drunks[drunk] = self.wormholes[(x, y)]
```

Spots Reached During One Walk



Summary

- Point is not the simulations themselves, but how we built them
- Started by defining classes
- Built functions corresponding to
  - One trial, multiple trials, result reporting
- Made series of incremental changes to simulation so that we could investigate different questions
  - Get simple version working first
  - Did a sanity check!
  - Elaborate a step at a time
- Showed how to use plots to get insights