Massachusetts Institute of Technology
Department of Electrical Engineering and Computer Science

6.002 – Circuits & Electronics
Spring 2007

Homework #10
Handout S07-048

Issued 04/19/2007 – Due 05/02/2007

Helpful readings for this homework: Chapter 12.1-5, 12.7, Chapter 13.1-3

Exercise 10.1: Exercise 12.3 from Chapter 12 of A&L (page 695).

Exercise 10.2: Exercise 14.4 from Chapter 14 of A&L (page 824). Hint: Use the impedance method.

Problem 10.1: In the network shown below, the inductor and the capacitor have zero current and voltage, respectively, prior to $t = 0$. At $t = 0$, a step in voltage from 0 to V_o is applied by the voltage source indicated.

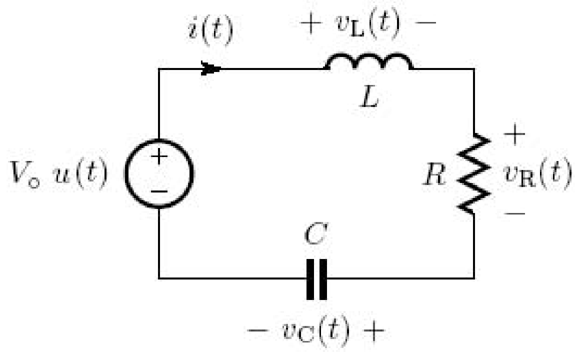

Figure 1: A step-driven series RLC circuit.

(a). Find v_C, v_L, v_R, i, di/dt just after the step at $t = 0$.

(b). Argue that $i = 0$ at $t = \infty$ so that $i(t)$ has no constant component.

(c). Find a second-order differential equation which describes the behavior of $i(t)$ for $t > 0$.

(d). Following parts (a) and (b), the current $i(t)$ takes the form $i(t) = I\sin(\omega t + \phi)e^{-\alpha t}$. Find I, ω, ϕ and α in terms of V_o, R, L and C.

(e). Suppose that the input is a voltage impulse with area Λ_o, where $\Lambda_o = \tau V_o$, V_o is the amplitude of the voltage step shown in Figure 1 and τ a given time constant. Repeat parts (a), (b), (c) and (d) for the network shown in Figure 1. Hint: Assume that the current $i(t)$ takes the form $i(t) = [A\cos(\omega t) + B\sin(\omega t)]e^{-\alpha t}$.

(f). Using the expression for $i(t)$ found in part (d), verify your answer to part (e) by considering the relation between step and impulse responses.

Save copies of your work for the pre-lab of Lab 3.

Problem 10.2: The network shown in Figure 2 is driven in steady-state by the sinusoidal current $i_{IN}(t) = I_{in}\cos(\omega t)$. The output of the network is the voltage $v_{OUT}(t)$, which takes the form $v_{OUT}(t) = V_{out}\cos(\omega t + \phi)$. Find V_{out} and ϕ as functions of ω as follows.

(a). Find a differential equation that can be solved for $v_{OUT}(t)$ given $i_{IN}(t)$. Hint: consider how $v_{OUT}(t)$ is related to the inductor current.

(b). Let $i_{IN}(t) = Re\{I_{in}e^{j\omega t}\}$. Also let $v_{OUT}(t) = Re\{\hat{V}_{out}e^{j\omega t}\}$, where $\hat{V}_{out}$ is a complex function of the circuit parameters, ω and I_{in}. With these definitions, find $\hat{V}_{out}$.

(c). An alternative way to write $v_{OUT}(t)$ is as $v_{OUT}(t) = Re\{|\hat{V}_{out}|e^{j(\omega t + \angle \hat{V}_{out})}\}$. Determine $|\hat{V}_{out}|$ and $\angle\hat{V}_{out}$ as functions of the circuit parameters, ω and I_{in}. Then, find V_{out} and ϕ for the original cosine input, again both as functions of the circuit parameters ω and I_{in}.

(d). Sketch and clearly label V_{out}/I_{in} and ϕ as functions of ω. Identify the low-frequency and high-frequency asymptotes on the sketch.

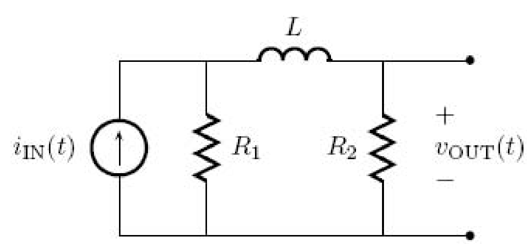

Figure 2: A first-order network driven in steady-state

Problem 10.3: Parts (a), (b) and (c) of Problem 14.16 from Chapter 14 of A&L (page 834).