Photon Momentum and Uncertainty Principle

Outline

- Photons Have Momentum (Compton Scattering)
- Wavepackets - Review
- Resolution of an Optical Microscope
- Heisenberg's Uncertainty Principle

TRUE / FALSE

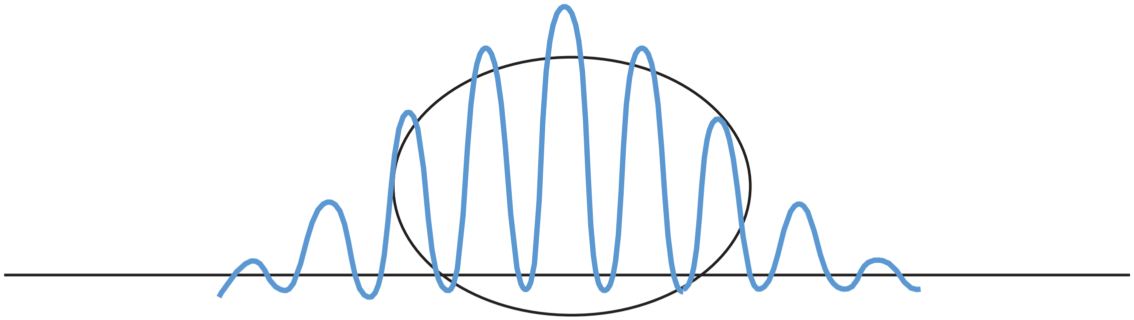
1. The photoelectric effect was used to show that light was composed of packets of energy proportional to its frequency. _____

2. The number of photons present in a beam of light is simply the intensity $\boldsymbol{I}$ divided by the photon energy $\boldsymbol{h\nu}$. _____

3. Infrared light at a wavelength of 1.24 microns has photon energy of 1.5 eV. _____

So is Light a Wave or a Particle ?

Light is always both Wave and Particle !



On macroscopic scales, large number of photons look like they exhibit only wave phenomena.

A single photon is still a wave, but your act of trying to measure it makes it look like a localized particle.

Do Photons Have Momentum ?

What is momentum ?

$$E = \frac{1}{2}mv^2 = \frac{1}{2}(mv)\cdot v = \frac{1}{2}p\cdot v$$

Just like Energy,

TOTAL MOMENTUM IS ALWAYS CONSERVED

Photons have energy and a finite velocity so there must be some momentum associated with photons !

$$p = \frac{E}{c} = \frac{h\nu}{c}$$

Photon Momentum

IN FREE SPACE:

$$E = cp \Rightarrow p = \frac{E}{c} = \frac{\hbar\omega}{c} = \hbar k$$

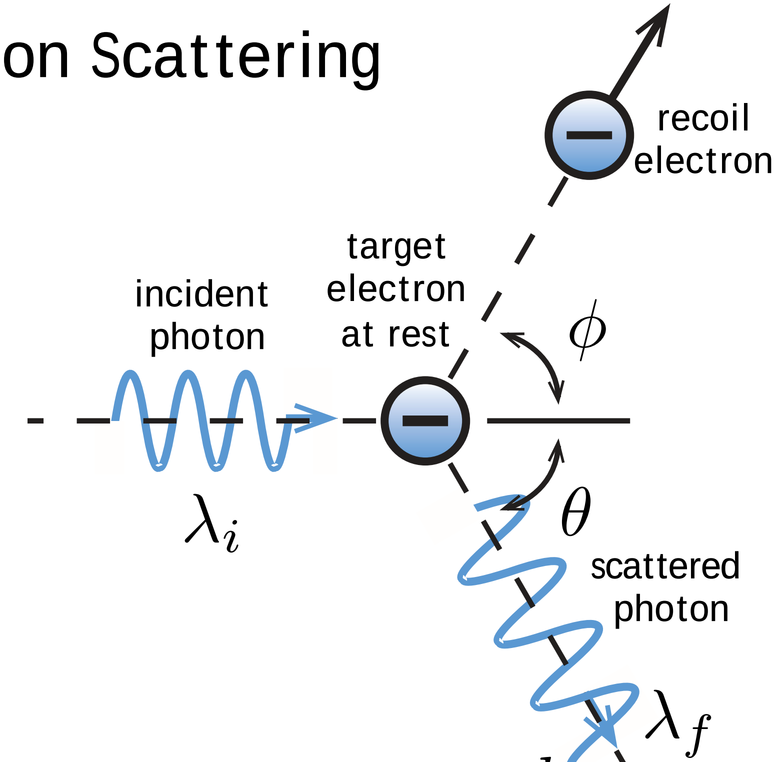
IN OPTICAL MATERIALS:

$$E = v_p p \Rightarrow p = \frac{E}{v_p} = \frac{\hbar\omega}{v_p} = \hbar k_{vac} n$$

Image by GFHund http://commons.wikimedia.org/wiki/File:Compton,Arthur_1929_Chicago.jpg Wikimedia Commons.

In 1924, A. H. Compton performed an experiment
where X-rays impinged on matter,
and he measured the scattered radiation.

Compton Scattering



It was found that the scattered X-ray did not have the same wavelength !

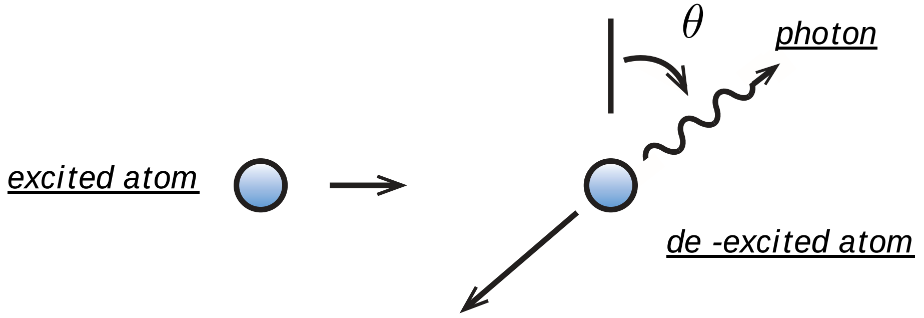
$$\lambda_f - \lambda_i = \Delta\lambda = \frac{h}{m_o c}(1 - \cos\theta)$$

Compton found that if you treat the photons as if they were particles of zero mass, with energy $E = hc/\lambda$ *and momentum* $p = h/\lambda$.

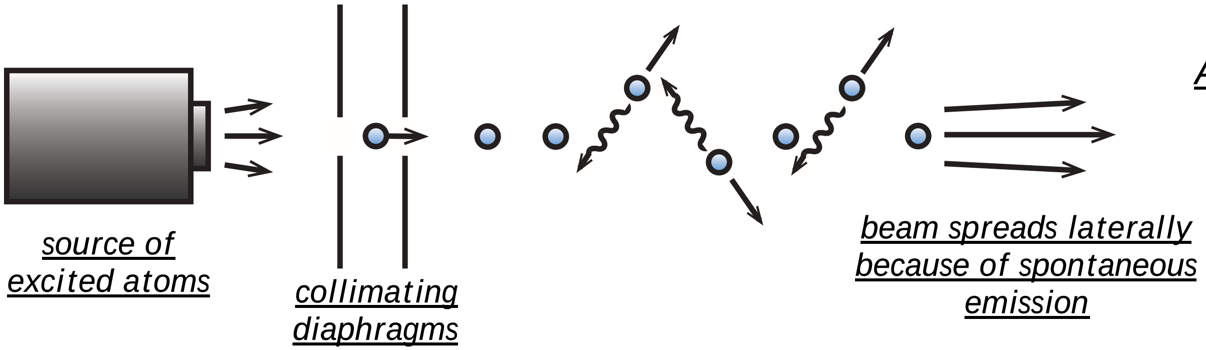
the collision behaves just as if it were two billiard balls colliding !
(with total momentum always conserved)

Manifestation of the Photon Momentum

SOURCE EMITTING A PHOTON

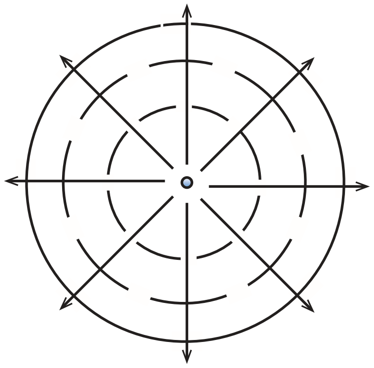


Conservation of linear momentum implies that an atom recoils when it undergoes spontaneous emission. The direction of photon emission (and atomic recoil) is not predictable.



A well-collimated atomic beam of excited atoms will spread laterally because of the recoil associated with spontaneous emission.

SOURCE EMITTING AN EM WAVE



A source emitting a spherical wave cannot recoil, because the spherical symmetry of the wave prevents it from carrying any linear momentum from the source.

Photon Momentum - Moves Solar Sails

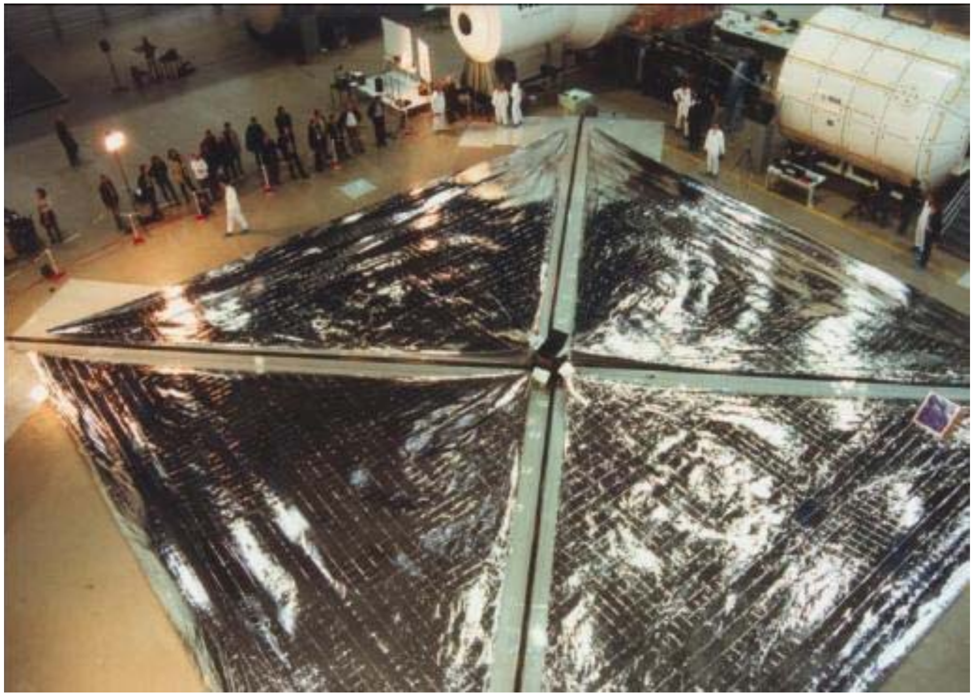


Image by D. Kassing http://en.wikipedia.org/wiki/File:SolarSail-DLR-ESA.jpg on Wikipedia

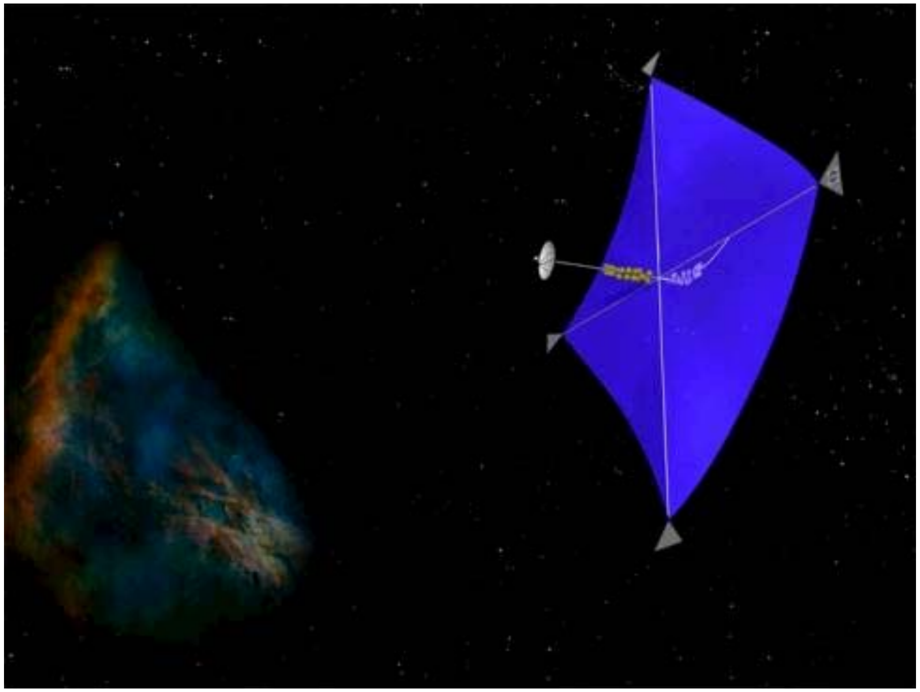
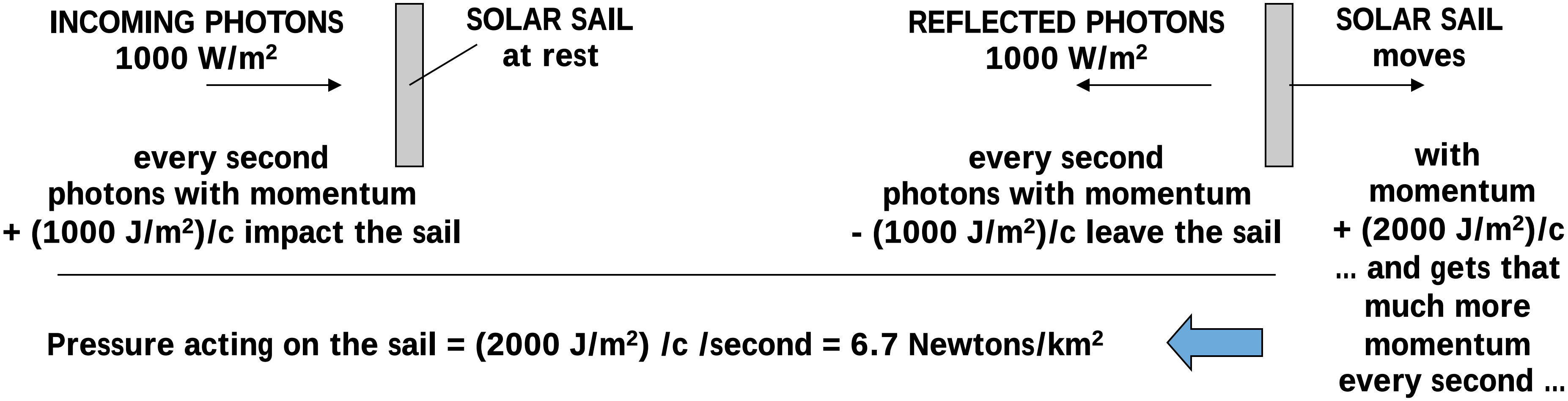


Image in the Public Domain



INCOMING PHOTONS
1000 W/m² →

SOLAR SAIL
at rest

every second
photons with momentum
+ (1000 J/m²)/c impact the sail

REFLECTED PHOTONS
1000 W/m² ←

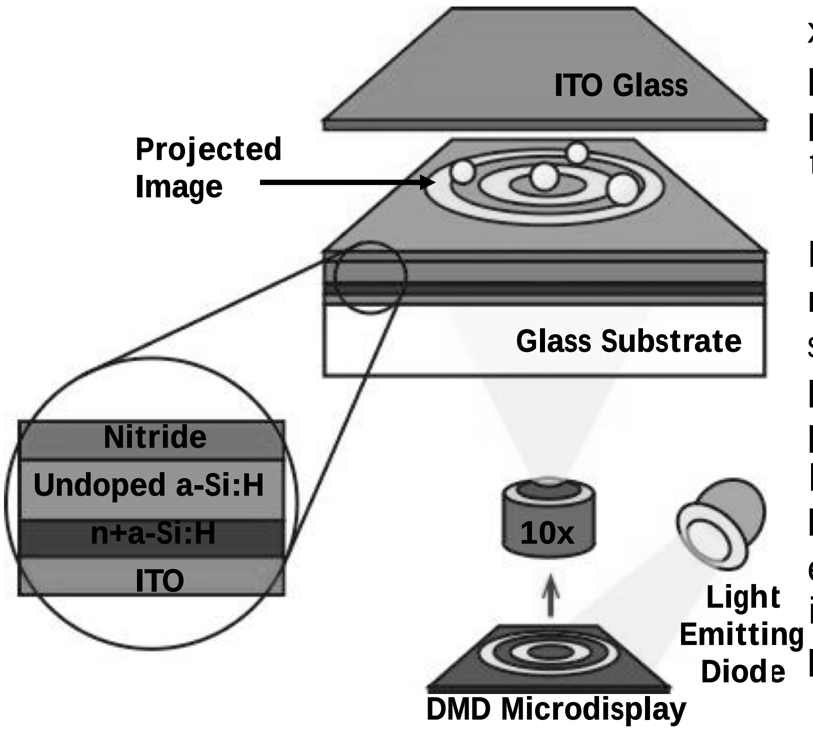
SOLAR SAIL
moves →

every second
photons with momentum
- (1000 J/m²)/c leave the sail

with
momentum
+ (2000 J/m²)/c
… and gets that
much more
momentum
every second …

Pressure acting on the sail = (2000 J/m²) /c /second = 6.7 Newtons/km²

Optoelectric Tweezers



Transforms optical energy to electrical energy through the use of a photoconductive surface. The idea is similar to that used in the ubiquitous office copier machine. In xerography, a document is scanned and transferred onto a photosensitive drum, which attracts dyes of carbon particles that are rolled onto a piece of paper to reproduce the image.

In this case, the researchers use a photosensitive surface made of amorphous silicon, a common material used in solar cells and flat-panel displays. Microscopic polystyrene particles suspended in a liquid were sandwiched between a piece of glass and the photoconductive material. Wherever light would hit the photosensitive material, it would behave like a conducting electrode, while areas not exposed to light would behave like a non-conducting insulator. Once a light source is removed, the photosensitive material returns to normal.

Depending upon the properties of the particles or cells being studied, they will either be attracted to or repelled by the electric field generated by the optoelectronic tweezer. Either way, the researchers can use that behavior to scoot particles where they want them to go.

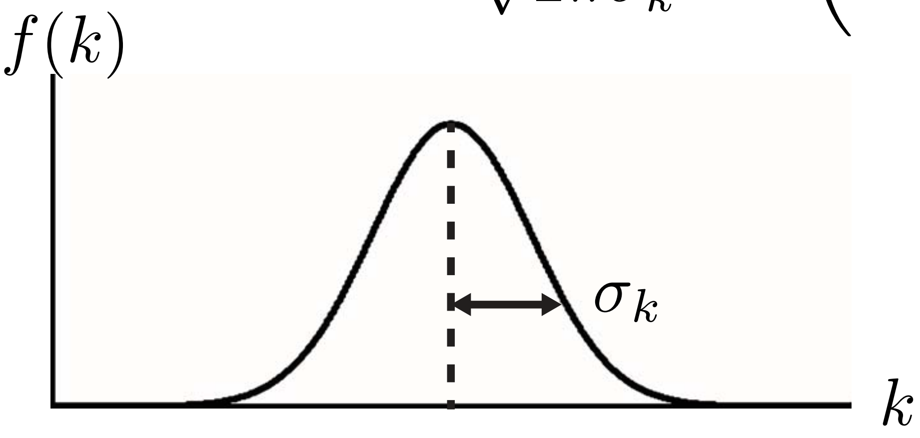
Review of Wavepackets

WHAT WOULD WE GET IF WE SUPERIMPOSED WAVES OF MANY DIFFERENT FREQUENCIES ?

$$\vec{E} = \vec{E}_o \int_{-\infty}^{+\infty} f\left(k\right) e^{+j(\omega t - kz)} dk$$

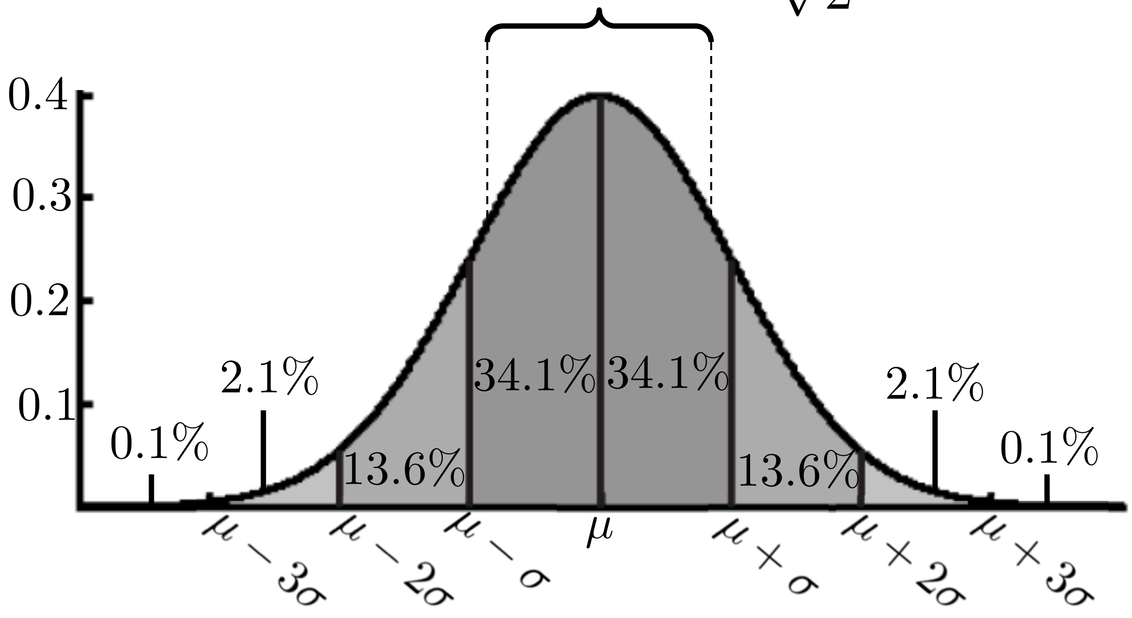
LET'S SET THE FREQUENCY DISTRIBUTION as GAUSSIAN

$$f\left(k\right) = \frac{1}{\sqrt{2\pi \sigma_k}} exp\left(-\frac{\left(k - k_o\right)^2}{2\sigma_k^2} \right)$$



Reminder: Gaussian Distribution

50% of data within $\pm \frac{\sigma}{\sqrt{2}}$



$$f\left(x\right) = \frac{1}{\sqrt{2\pi\sigma}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

μ specifies the position of the bell curve's central peak
σ specifies the half-distance between inflection points

FOURIER TRANSFORM OF A GAUSSIAN IS A GAUSSIAN

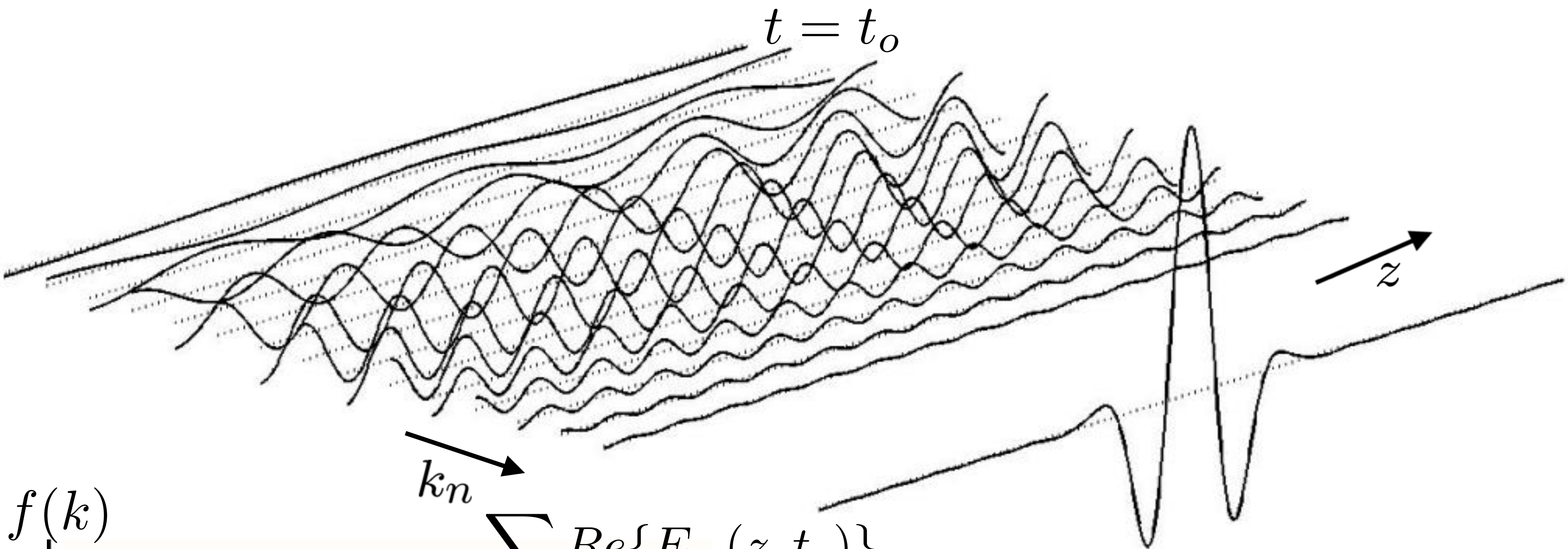
$$\mathcal{F}_x\left[e^{-ax^2}\right](k) = \sqrt{\frac{\pi}{a}} e^{-\frac{\pi^2 k^2}{a}}$$

REMEMBER:

$$k = \frac{n}{c}\omega$$

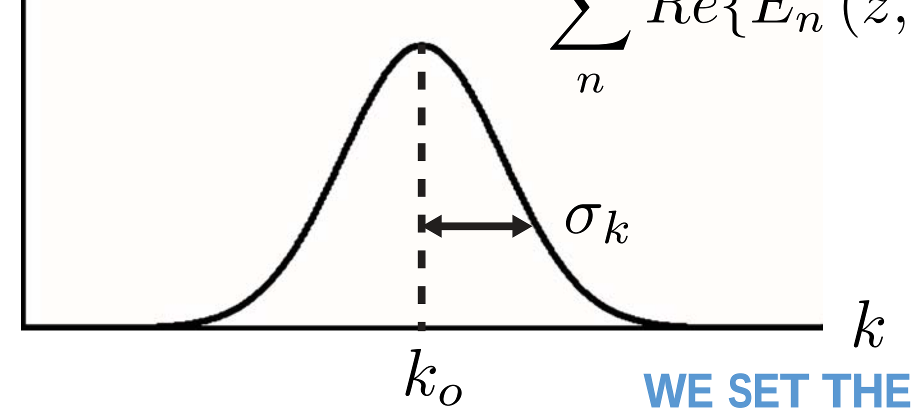
Gaussian Wavepacket in Space

$$Re\{E_n\,(z, t_o)\}$$



$$\sum_n Re\{E_n\,(z, t_o)\}$$

SUM OF SINUSOIDS = WAVEPACKET

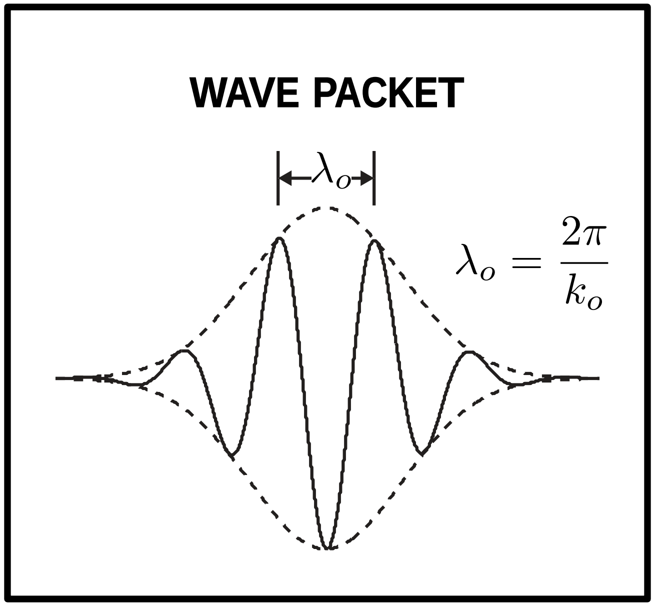


$$f\,(k) = \frac{1}{\sqrt{2\pi\sigma_k}} exp\left(-\frac{(k-k_o)^2}{2\sigma_k^2}\right)$$

WE SET THE FREQUENCY DISTRIBUTION as GAUSSIAN

Gaussian Wavepacket in Space

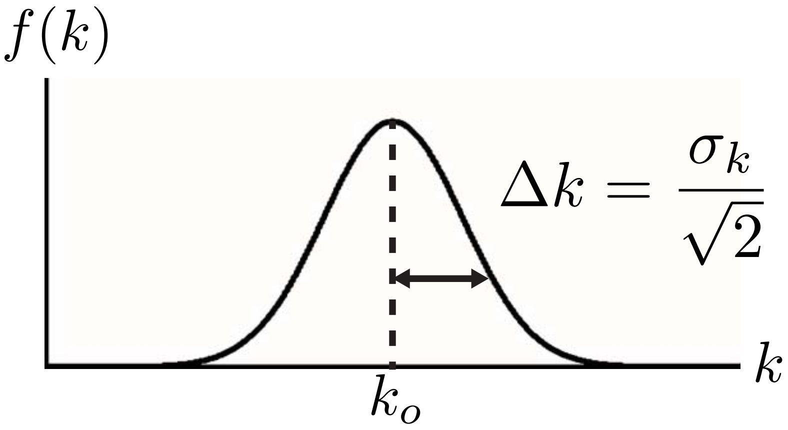
$$E(z,t) = E_o \underbrace{exp\left(-\frac{\sigma_k^2}{2}(ct-z)^2\right)}_{\text{GAUSSIAN ENVELOPE}} \cos\left(\omega_o t - k_o z\right)$$



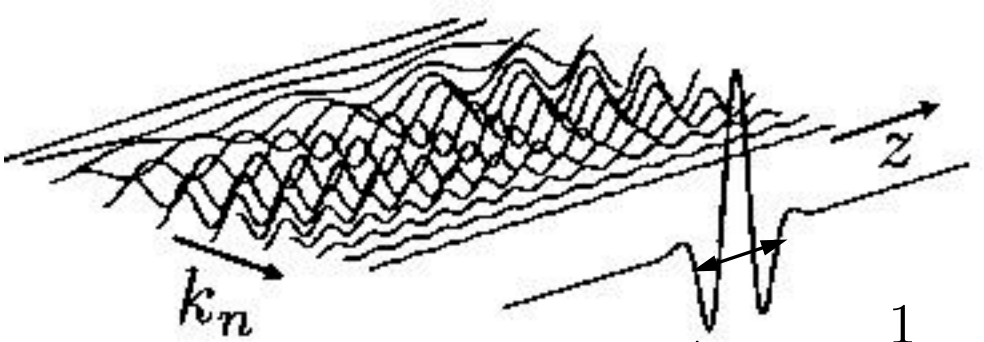
In free space ...

$$k = \frac{\omega}{c} = \frac{2\pi E}{hc}$$

... this plot then shows the PROBABILITY OF WHICH k (or ENERGY) EM WAVES are MOST LIKELY TO BE IN THE WAVEPACKET



$$\Delta k = \frac{\sigma_k}{\sqrt{2}}$$

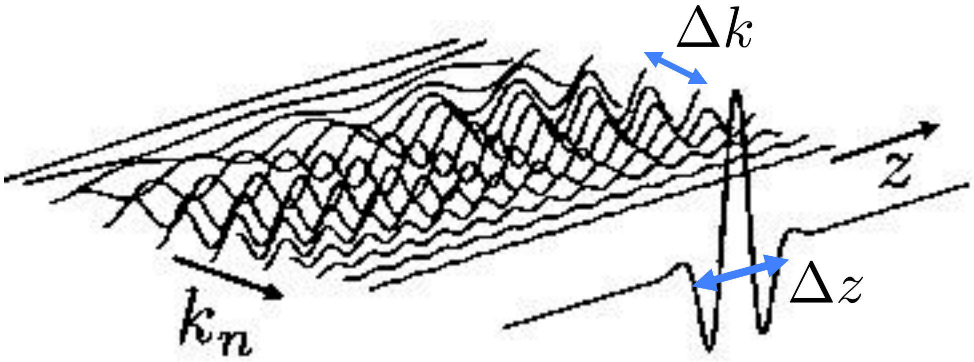


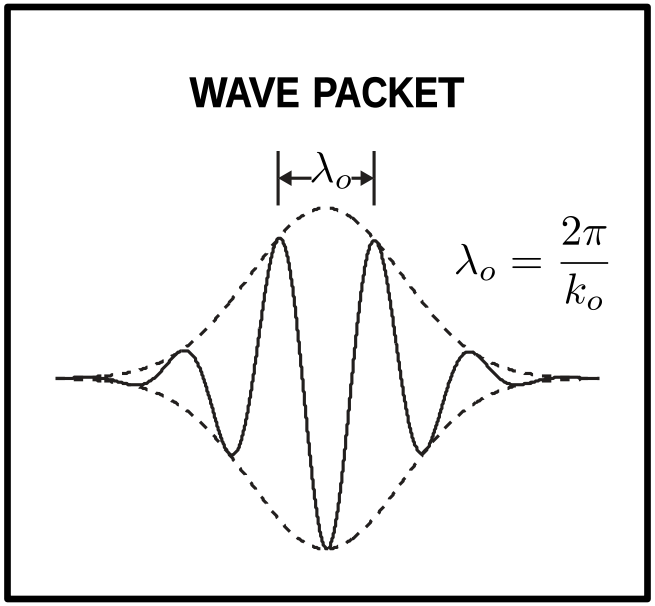
$$\Delta z = \frac{1}{\sqrt{2}\sigma_k}$$

$$\Rightarrow \boxed{\Delta k \Delta z = 1/2}$$

Gaussian Wavepacket in Time

$$E(z,t) = E_o exp\left(-\frac{\sigma_k^2}{2}(ct-z)^2\right)\cos\left(\omega_o t - k_o z\right)$$





UNCERTAINTY RELATIONS

$$\Delta z = \frac{c}{n}\Delta t$$

$$\Delta k = \frac{n}{c}\Delta\omega$$

$$\Delta k \Delta z = 1/2$$

$$\Delta\omega\Delta t = 1/2$$

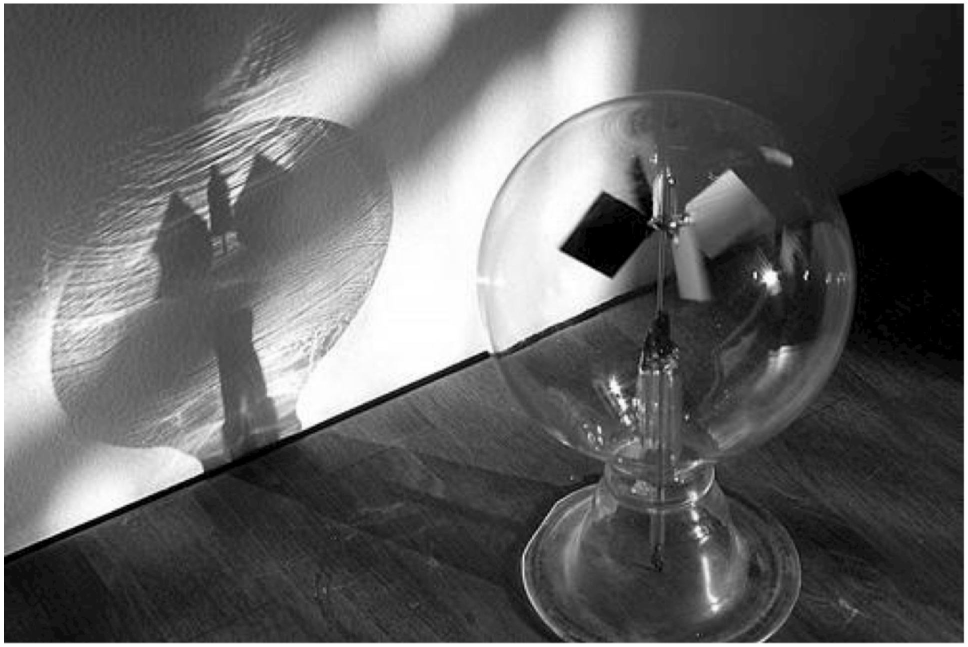
$$\boxed{\Delta p \Delta z = \hbar/2}$$

$$\boxed{\Delta E \Delta t = \hbar/2}$$

Today's Culture Moment

Crookes Radiometer

The crookes radiometer spins because of thermal processes, but he initially guessed it was due to photon momentum...



invented in 1873
by the chemist
Sir William Crookes

Image by Rob Ireton
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Heisenberg realized that …

- In the world of very small particles, one cannot measure any property of a particle without interacting with it in some way
- This introduces an unavoidable uncertainty into the result
- One can never measure all the properties exactly

Werner Heisenberg (1901-1976)

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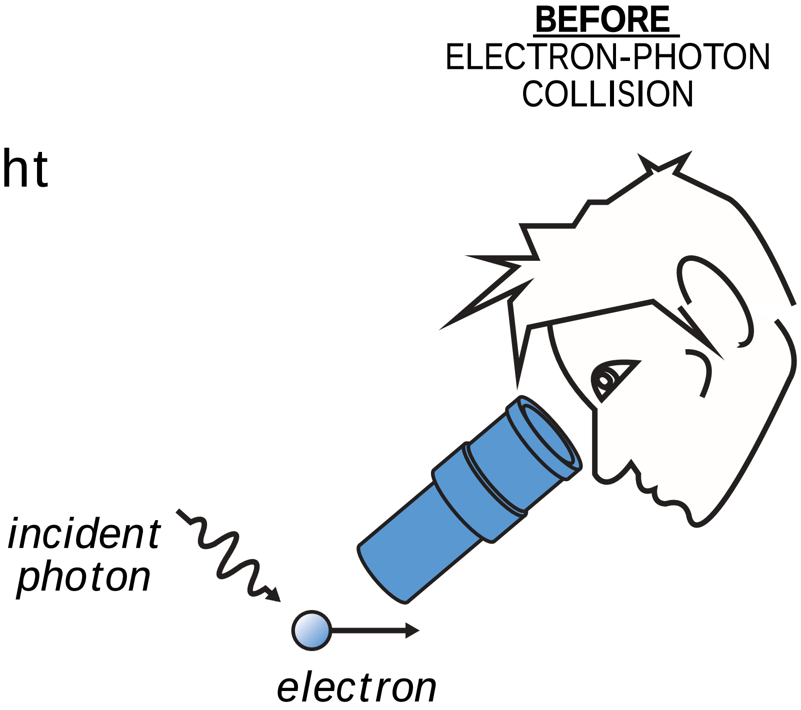
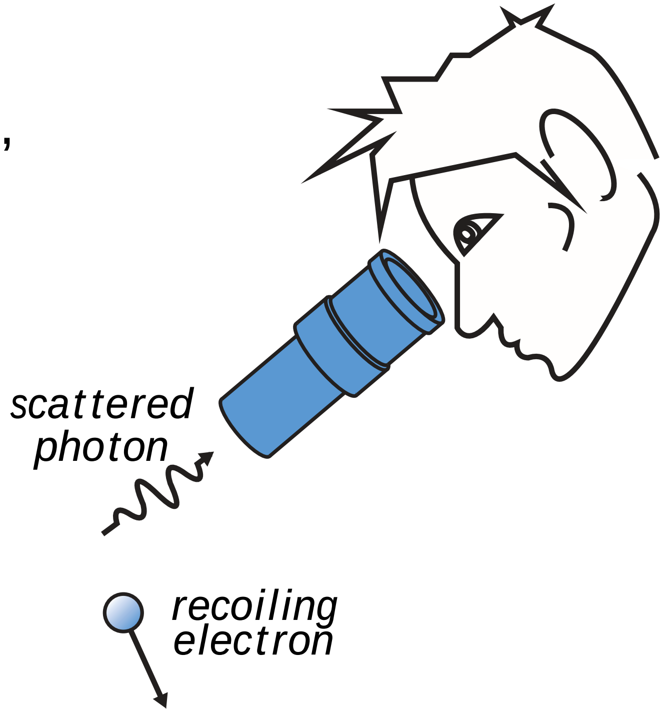
Measuring Position and Momentum of an Electron

- Shine light on electron and detect reflected light using a microscope
- Minimum uncertainty in position is given by the wavelength of the light
- So to determine the position accurately, it is necessary to use light with a short wavelength



Measuring Position and Momentum of an Electron

- By Planck's law $E = hc/\lambda$, a photon with a short wavelength has a large energy
- Thus, it would impart a large 'kick' to the electron
- But to determine its momentum accurately, electron must only be given a small kick
- This means using light of long wavelength !



AFTER
ELECTRON-PHOTON
COLLISION

Implications

- It is impossible to know *both* the position and momentum exactly, i.e., $\Delta x=0$ and $\Delta p=0$
- These uncertainties are inherent in the physical world and have nothing to do with the skill of the observer
- Because h is so small, these uncertainties are not observable in normal everyday situations

$$\hbar = 1.054 \times 10^{-34}\,[\mathrm{J} \cdot \mathrm{s}]$$

Example of Baseball

- A pitcher throws a 0.1-kg baseball at 40 m/s
- So momentum is 0.1 x 40 = 4 kg m/s
- Suppose the momentum is measured to an accuracy of 1 percent , i.e.,

$$\Delta p = 0.01\,p = 4 \times 10^{-2} \text{ kg m/s}$$

Example of Baseball (cont'd)

- The uncertainty in position is then

$$\Delta x \geq \frac{h}{4\pi\Delta p} = 1.3 \times 10^{-33}\mathrm{m}$$

- No wonder one does not observe the effects of the uncertainty principle in everyday life!

Example of Electron

- Same situation, but baseball replaced by an electron which has mass 9.11 x 10^{-31} kg traveling at 40 m/s

- So momentum = 3.6 x 10^{-29} kg m/s
  and its uncertainty = 3.6 x 10^{-31} kg m/s

- The uncertainty in position is then

$$\Delta x \geq \frac{h}{4\pi \Delta p} = 1.4 \times 10^{-4}\mathrm{m}$$

Classical World

- The observer is objective and passive
- Physical events happen independently of whether there is an observer or not
- This is known as objective reality

Role of an Observer in Quantum Mechanics

- The observer is *not* objective and passive
- The act of observation changes the physical system irrevocably
- This is known as subjective reality

One might ask:

"If light can behave like a particle, might particles act like waves"?

YES !

Particles, like photons, also have a wavelength given by:

$$\boxed{\lambda = h/p = h/mv}$$

The wavelength of a particle depends on its momentum, just like a photon!

The main difference is that matter particles have mass, and photons don't!

Matter Waves

Compute the wavelength of a 10 [g] bullet moving at 1000 [m/s].

$\lambda = h/mv = 6.6\text{x}10^{-34}$ [J s] / (0.01 [kg])(1000 [m/s])

= **$6.6\text{x}10^{-35}$ [m]**

This is **immeasureably small**

For ordinary "everyday objects,"
we don't experience that

MATTER CAN BEHAVE AS A WAVE

But, what about small particles ?

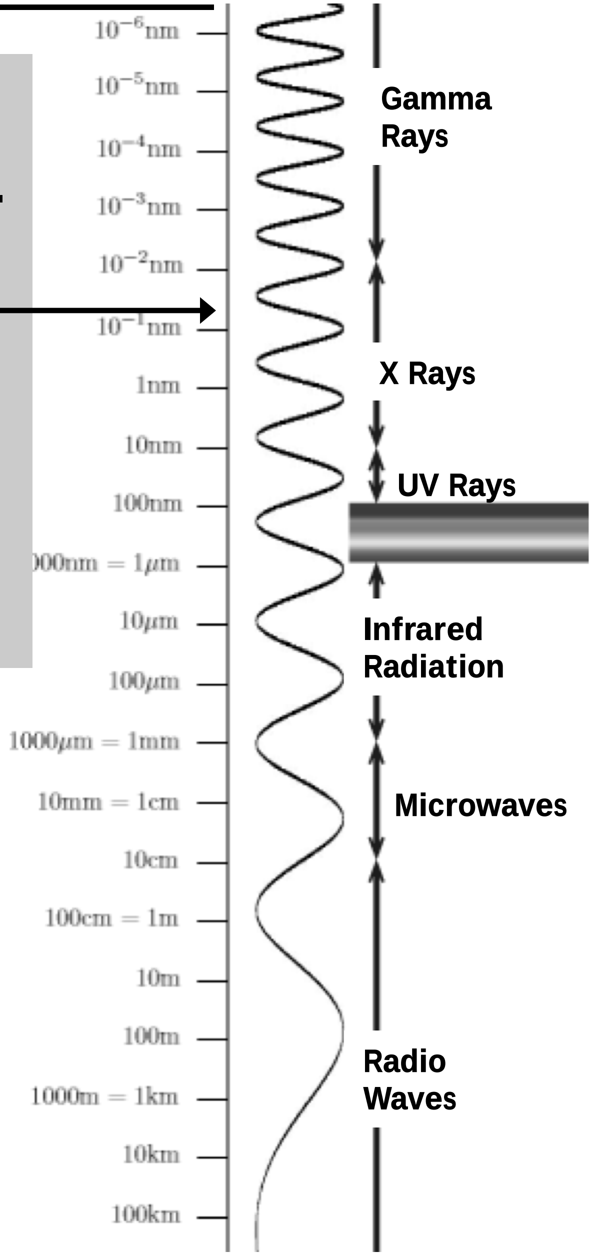
Compute the wavelength of an electron ($m = 9.1 \times 10^{-31}$ [kg]) moving at 1×10^{7} [m/s].

$$\lambda = h/mv$$

$$= 6.6 \times 10^{-34} \text{ [J s]}/(9.1 \times 10^{-31} \text{ [kg]})(1 \times 10^{7} \text{ [m/s]})$$

$$= \mathbf{7.3 \times 10^{-11} \text{ [m]}}.$$

$$= \mathbf{0.073 \text{ [nm]}}$$

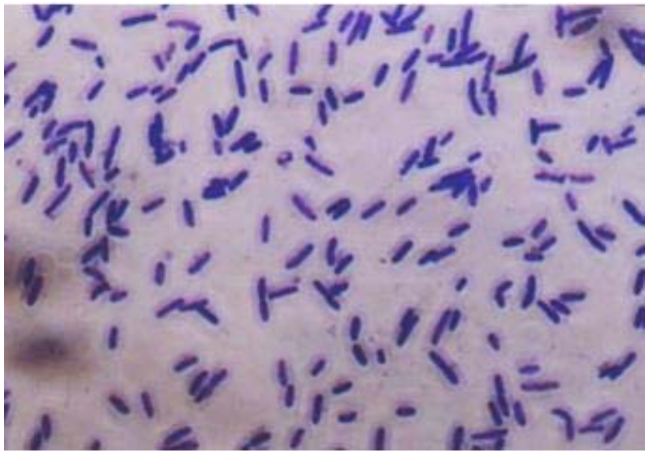


These electrons have a wavelength in the region of X-rays

Wavelength versus Size

With a visible light microscope, we are limited to being able to resolve objects which are at least about
<u>$0.5*10^{-6}$ m</u> = <u>0.5 μm</u> = <u>500 nm</u> in size.

This is because visible light, with a wavelength of ~500 nm cannot resolve objects whose size is smaller than it's wavelength.



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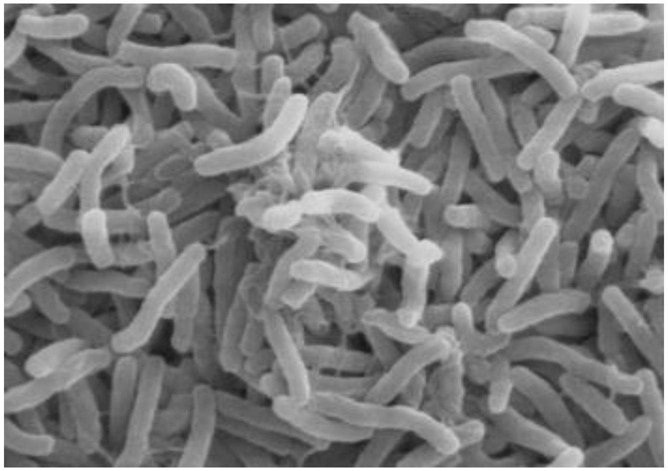
Bacteria, as viewed using visible light



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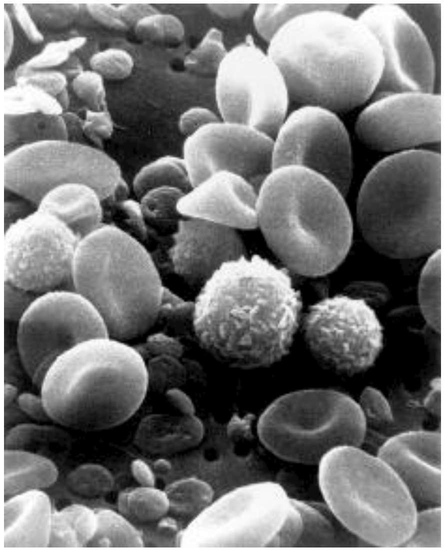
Bacteria, as viewed using electrons!

Electron Microscope

The electron microscope is a device which uses the wave behavior of electrons to make images which are otherwise too small for visible light!

This image was taken with a Scanning Electron Microscope (SEM).

SEM can resolve features as small as 5 nm. This is about 100 times better than can be done with visible light microscopes!



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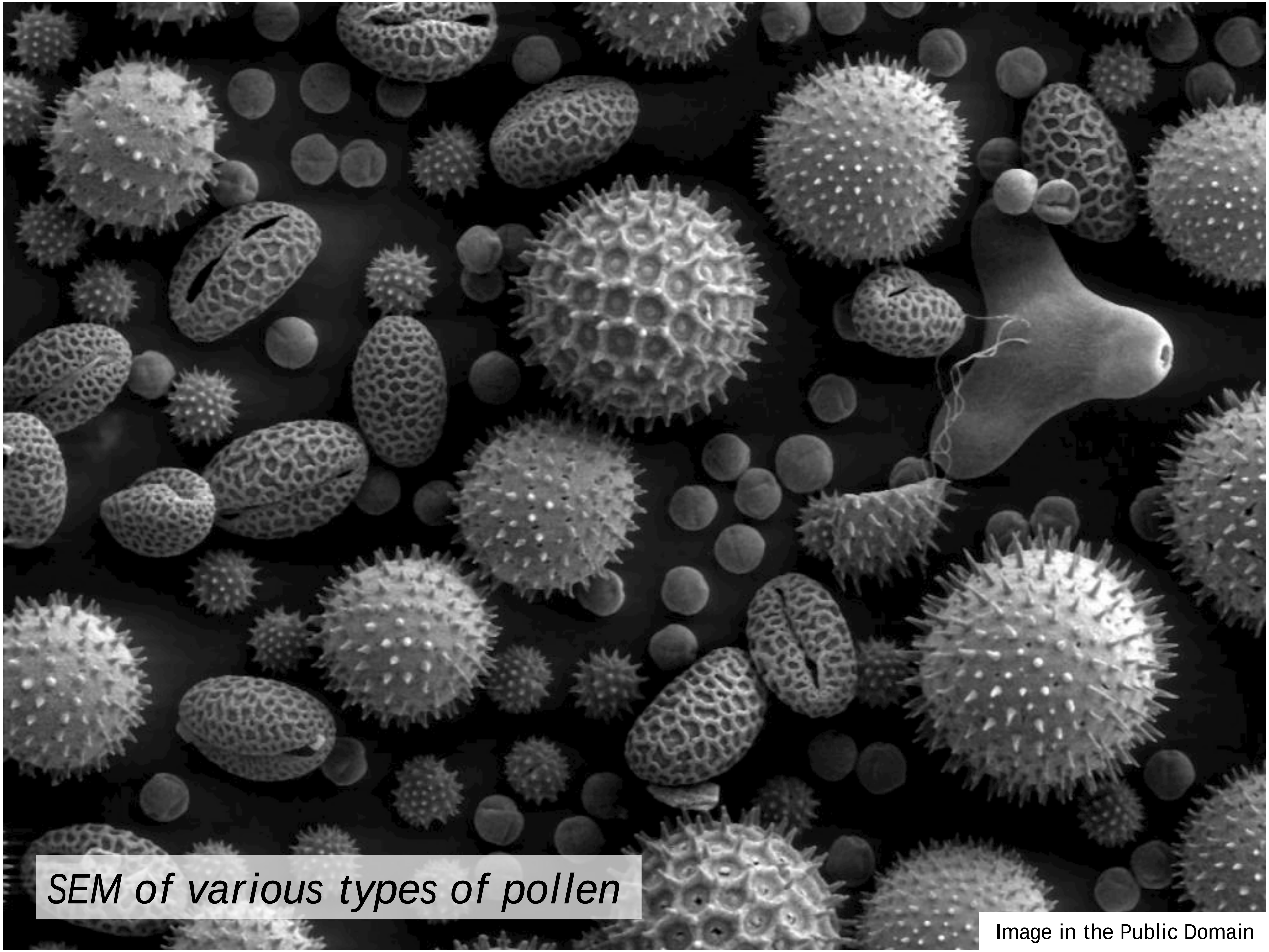
IMPORTANT POINT:
High energy particles can be used to reveal the structure of matter !



SEM of various types of pollen

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SEM of an ant head

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Summary

- Light is made up of photons, but in macroscopic situations it is often fine to treat it as a wave.

- Photons carry both energy & momentum.

$$E = hc/\lambda \qquad p = E/c = h/\lambda$$

- Matter also exhibits wave properties. For an object of mass *m*, and velocity, v, the object has a wavelength, $\lambda = h / m$v

- One can probe 'see' the fine details of matter by using high energy particles (they have a small wavelength !)

6.007 Electromagnetic Energy: From Motors to Lasers
Spring 2011