**6.012 Microelectronic Devices and Circuits**
Formula Sheet for Exam One, Fall 2009

## Parameter Values:

$q = 1.6x10^{-19}\ Coul$
$\varepsilon_o = 8.854x10^{-14}\ F/cm$
$\varepsilon_{r,Si} = 11.7, \quad \varepsilon_{Si} \approx 10^{-12}\ F/cm$
$n_i[Si@R.T] \approx 10^{10}\ cm^{-3}$
$kT/q \approx 0.025\ V; \quad (kT/q)\ln 10 \approx 0.06\ V$
$1\ \mu m = 1x10^{-4}\ cm$

## Periodic Table:

| III | IV | V |
|---|---|---|
| B | C | N |
| Al | Si | P |
| Ga | Ge | As |
| In | Sn | Sb |

## Drift/Diffusion:

Drift velocity: $\bar{s}_x = \pm \mu_m E_x$

Conductivity: $\sigma = q(\mu_e n + \mu_h p)$

Diffusion flux: $F_m = -D_m \dfrac{\partial C_m}{\partial x}$

Einstein relation: $\dfrac{D_m}{\mu_m} = \dfrac{kT}{q}$

## Electrostatics:

$$\varepsilon \frac{dE(x)}{dx} = \rho(x) \qquad E(x) = \frac{1}{\varepsilon}\int \rho(x)dx$$

$$-\frac{d\phi(x)}{dx} = E(x) \qquad \phi(x) = -\int E(x)dx$$

$$-\varepsilon \frac{d^2\phi(x)}{dx^2} = \rho(x) \qquad \phi(x) = -\frac{1}{\varepsilon}\iint \rho(x)dxdx$$

## The Five Basic Equations:

Electron continuity: $$\frac{\partial n(x,t)}{\partial t} - \frac{1}{q}\frac{\partial J_e(x,t)}{\partial x} = g_L(x,t) - \left[n(x,t)\cdot p(x,t) - n_i^2\right]r(T)$$

Hole continuity: $$\frac{\partial p(x,t)}{\partial t} + \frac{1}{q}\frac{\partial J_h(x,t)}{\partial x} = g_L(x,t) - \left[n(x,t)\cdot p(x,t) - n_i^2\right]r(T)$$

Electron current density: $$J_e(x,t) = q\mu_e n(x,t)E(x,t) + qD_e \frac{\partial n(x,t)}{\partial x}$$

Hole current density: $$J_h(x,t) = q\mu_h p(x,t)E(x,t) - qD_h \frac{\partial p(x,t)}{\partial x}$$

Poisson's equation: $$\frac{\partial E(x,t)}{\partial x} = \frac{q}{\varepsilon}\left[p(x,t) - n(x,t) + N_d^+(x) - N_a^-(x)\right]$$

## Uniform doping, full ionization, TE

n-type, $N_d >> N_a$

$$n_o \approx N_d - N_a \equiv N_D, \quad p_o = n_i^2/n_o, \quad \phi_n = \frac{kT}{q}\ln\frac{N_D}{n_i}$$

p-type, $N_a >> N_d$

$$p_o \approx N_a - N_d \equiv N_A, \quad n_o = n_i^2/p_o, \quad \phi_p = -\frac{kT}{q}\ln\frac{N_A}{n_i}$$

## Uniform optical excitation, uniform doping

$$n = n_o + n' \qquad p = p_o + p' \qquad n' = p' \qquad \frac{dn'}{dt} = g_l(t) - (p_o + n_o + n')n'r$$

Low level injection, $n', p' << p_o + n_o$: $$\frac{dn'}{dt} + \frac{n'}{\tau_{min}} = g_l(t) \quad \text{with} \quad \tau_{min} \approx (p_o r)^{-1}$$

Flow problems (uniformly doped quasineutral regions with quasi-static excitation and low level injection; p-type example):

Minority carrier excess: $$\frac{d^2 n'(x)}{dx^2} - \frac{n'(x)}{L_e^2} = -\frac{1}{D_e} g_L(x) \qquad L_e \equiv \sqrt{D_e \tau_e}$$

Minority carrier current density: $$J_e(x) \approx qD_e \frac{dn'(t)}{dx}$$

Majority carrier current density: $$J_h(x) = J_{Tot} - J_e(x)$$

Electric field: $$E_x(x) \approx \frac{1}{q\mu_h p_o}\left[J_h(x) + \frac{D_h}{D_e} J_e(x)\right]$$

Majority carrier excess: $$p'(x) \approx n'(x) + \frac{\varepsilon}{q}\frac{dE_x(x)}{dx}$$

Short base, infinite lifetime limit:

Minority carrier excess: $$\frac{d^2 n'(x)}{dx^2} \approx -\frac{1}{D_e} g_L(x) \quad \Rightarrow \quad n'(x) \approx -\frac{1}{D_e}\iint g_L(x)\,dx\,dx$$

Non-uniformly doped semiconductor sample in thermal equilibrium

$$\frac{d^2\phi(x)}{dx^2} = \frac{q}{\varepsilon}\left\{n_i\left[e^{q\phi(x)/kT} - e^{-q\phi(x)/kT}\right] - \left[N_d(x) - N_a(x)\right]\right\}$$
$$n_o(x) = n_i e^{q\phi(x)/kT}, \quad p_o(x) = n_i e^{-q\phi(x)/kT}, \quad p_o(x) n_o(x) = n_i^2$$

Depletion approximation for abrupt p-n junction:

$$\rho(x) = \begin{cases} 0 & \text{for} \quad x < -x_p \\ -qN_{Ap} & \text{for} \quad -x_p < x < 0 \\ qN_{Dn} & \text{for} \quad 0 < x < x_n \\ 0 & \text{for} \quad x_n < x \end{cases}$$

$$N_{Ap} x_p = N_{Dn} x_n$$

$$\phi_b \equiv \phi_n - \phi_p = \frac{kT}{q}\ln\frac{N_{Dn}N_{Ap}}{n_i^2}$$

$$w(v_{AB}) = \sqrt{\frac{2\varepsilon_{Si}(\phi_b - v_{AB})}{q}\frac{(N_{Ap}+N_{Dn})}{N_{Ap}N_{Dn}}}$$

$$\left|E_{pk}\right| = \sqrt{\frac{2q(\phi_b - v_{AB})}{\varepsilon_{Si}}\frac{N_{Ap}N_{Dn}}{(N_{Ap}+N_{Dn})}}$$

$$q_{DP}(v_{AB}) = -AqN_{Ap}x_p(v_{AB}) = -A\sqrt{2q\varepsilon_{Si}(\phi_b - v_{AB})\frac{N_{Ap}N_{Dn}}{(N_{Ap}+N_{Dn})}}$$

Ideal p-n junction diode i-v relation:

$$n(-x_p) = \frac{n_i^2}{N_{Ap}} e^{qv_{AB}/kT}, \quad n'(-x_p) = \frac{n_i^2}{N_{Ap}}\left(e^{qv_{AB}/kT} - 1\right); \qquad p(x_n) = \frac{n_i^2}{N_{Dn}} e^{qv_{AB}/kT}, \quad p'(x_n) = \frac{n_i^2}{N_{Dn}}\left(e^{qv_{AB}/kT} - 1\right)$$

$$i_D = Aqn_i^2\left[\frac{D_h}{N_{Dn}w_{n,eff}} + \frac{D_e}{N_{Ap}w_{p,eff}}\right]\left[e^{qv_{AB}/kT} - 1\right] \qquad w_{m,eff} = \begin{cases} w_m - x_m & \text{if } L_m >> w_m \\ L_m \tanh\left[(w_m - x_m)/L_m\right] & \text{if } L_m \sim w_m \\ L_m & \text{if } L_m << w_m \end{cases}$$

$$q_{QNR,p\text{-}side} = Aq\int_{-w_p}^{-x_p} n'(x)\,dx, \qquad q_{QNR,n\text{-}side} = Aq\int_{x_n}^{w_n} p'(x)\,dx, \qquad \text{Note: } p'(x) \approx n'(x) \text{ in QNRs}$$

Small Signal Linear Equivalent Circuit for a p-n Diode

($n^+$-p doping assumed for $C_d$)

$$g_d \equiv \left.\frac{\partial i_D}{\partial v_{AB}}\right|_Q = \frac{q}{kT} I_S\, e^{qV_{AB}/kT} \approx \frac{qI_D}{kT}, \qquad C_d = C_{dp} + C_{df},$$

$$\text{where} \quad C_{dp}(V_{AB}) = A\sqrt{\frac{q\varepsilon_{Si}N_{Ap}}{2(\phi_b - V_{AB})}}, \quad \text{and} \quad C_{df}(V_{AB}) = \frac{qI_D}{kT}\frac{\left[w_p - x_p\right]^2}{2D_e} = g_d\tau_d \quad \text{with} \quad \tau_d \equiv \frac{\left[w_p - x_p\right]^2}{2D_e}$$

Large signal BJT Model in Forward Active Region (FAR):

(npn with base width modulation)

$$i_B(v_{BE}, v_{CE}) = I_{BS}\left(e^{qv_{BE}/kT} - 1\right)$$

$$i_C(v_{BE}, v_{BC}) = \beta_F\, i_B(v_{BE}, v_{CE})\left[1 + \lambda v_{CE}\right] = \beta_F I_{BS}\left(e^{qv_{BE}/kT} - 1\right)\left[1 + \lambda v_{CE}\right]$$

$$\text{with:} \quad I_{BS} \equiv \frac{I_{ES}}{(\beta_F + 1)} = \frac{Aqn_i^2}{(\beta_F + 1)}\left(\frac{D_h}{N_{DE}w_{E,eff}} + \frac{D_e}{N_{AB}w_{B,eff}}\right), \quad \beta_F \equiv \frac{\alpha_F}{(1 - \alpha_F)}, \text{ and} \quad \lambda \equiv \frac{1}{V_A}$$

$$\text{Also,} \quad \alpha_F = \frac{(1 - \delta_B)}{(1 + \delta_E)} \quad \text{and} \quad \beta_F \approx \frac{(1 - \delta_B)}{(\delta_E + \delta_B)} \quad \text{with} \quad \delta_E = \frac{D_h}{D_e}\cdot\frac{N_{AB}}{N_{DE}}\cdot\frac{w_{B,eff}}{w_{E,eff}} \quad \text{and} \quad \delta_B = \frac{w_{B,eff}^2}{2L_{eB}^2}$$

$$\text{When} \quad \delta_B \approx 0 \quad \text{then} \quad \alpha_F \approx \frac{1}{(1 + \delta_E)} \quad \text{and} \quad \beta_F \approx \frac{1}{\delta_E}$$