MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Department of Electrical Engineering and Computer Science

6.012 MICROELECTRONIC DEVICES AND CIRCUITS

Answers to Exam 2 - Spring 2008

Problem 1: Graded by Prof. Fonstad

a) i) n-channel MOSFET: Enhancement mode because $V_T \geq 0$, which for an n-channel MOSFET means that with $v_{GS} = 0$ there is no channel.

p-channel MOSFET: Depletion mode because $V_T \geq 0$, which for a p-channel MOSFET means that with $v_{GS} = 0$ there is already a channel.

ii) Negative because there is a hole inversion layer in the p-channel device, i.e. electrons depleted from the area near the interface and holes attracted to the interface, even with $v_{GS} = 0$. Concurrently, it takes more gate voltage to invert the channel of the n-channel device, meaning more positive charge must be put on the gate electrode which is balances by negative ions at the interface.

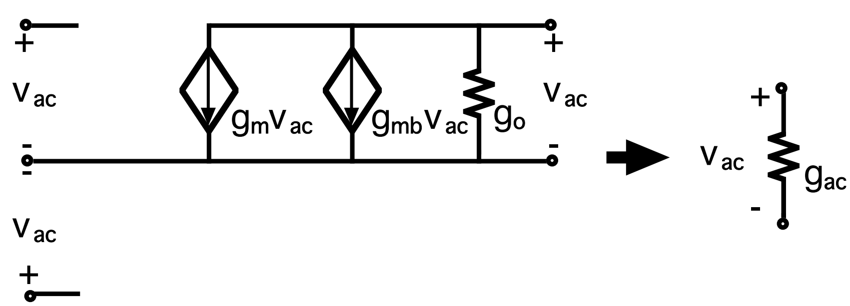
iii) Larger, because the charging current is the current in saturation through the p-channel device, and since the current in saturation for both of the devices is proportional to $(V_{DD} - |V_T|)^2$, this current is larger for the p-channel device since for it $|V_T|$ is smaller.

iv) Larger, because for both devices $|V_T| = |V_{FB}| + |2\phi_{Si}| + (t_{ox}/\varepsilon_{ox})(\ldots.)^{1/2}$. Thus making t_{ox} thicker, makes $|V_T|$ larger.

b) i) V_{GS}: 2V V_{BS}: -1V V_{DS}: 2V

v_{gs}: v_{AC} v_{ds}: v_{AC} v_{bs}: v_{AC}

The LEC is a single resistor: $r_{ac} = 1/g_{ac} = 1/[g_m+g_{mb}+g_o]= 1/[g_m(1+\eta)+g_o]$



LEC:

ii) n-channel, because for a transistor in strong inversion and biased in saturation, $g_m = (2KI_D)^{1/2}$ and we can assume that $K_n > K_p$ because in general $\mu_e > \mu_h$.

iii) Similar, because for a transistor operating sub-threshold $g_m = qI_D/nkT$, and the I_D's are equal and the n's will be similar.

iv) In the linear region $i_D = K(v_{GS}-V_T-v_{DS}/2)v_{DS}$

Transconductance: $g_m = \partial i_D/\partial v_{GS}|_Q = KV_{DS}$

Output conductance: $g_o = \partial i_D/\partial v_{DS}|_Q = [K(v_{GS}-V_T-v_{DS}/2)-Kv_{DS}/2]|_Q=K(V_{GS}-V_T-V_{DS})$

Problem 2: Graded by Prof. Palacios

a) The i_D-v_{DS} plot shows us that for the gate voltage applied the MOSFET saturates at a v_{DS} of 5 V. We can find out what the gate voltage is using: $V_{DS,sat} = V_{GS} - V_T$:

$$V_{GS} = V_{DS,sat} + V_T = 5\text{ V} + 1\text{ V} = 6\text{ V}$$

b) Near the origin, $i_D = K(v_{GS}-V_T-v_{DS}/2)v_{DS}$ and the slope is $K(v_{GS}-V_T-v_{DS})$. Evaluating this at the origin, we find that

$$\partial i_D/\partial v_{DS}\ (@\ V_{DS} = 0) = K(V_{GS} - V_T) = KV_{DS,sat}$$

where $(V_{GS} - V_T) = 5$ V. To find K and evaluate this we can use the saturation current, 10 mA, and saturation voltage, 5 V, in $i_{D,sat} = K(v_{GS} - V_T)^2/2 = K(V_{DS,sat})^2/2$ to find:

$$K = 2i_{D,sat}/(V_{DS,sat})^2 = 2\ (0.01/25) = 0.0008\ \text{A/V}^2$$

With this the slope, $KV_{DS,sat}$, is: $8 \times 10^{-4} \times 5 = 4 \times 10^{-3}$ S

c) At any point in the channel, $q_N^*(y) = -(\varepsilon_{ox}/t_{ox})(V_{GC}-V_T) = -(\varepsilon_{ox}/t_{ox})(V_{GS}-V_{CS}-V_T)$

i) At the source end of the channel, $V_{CS} = 0$, and we find:

$$q_N^*(0) = -(\varepsilon_{ox}/t_{ox})(V_{GS}-V_T) = -(3.5 \times 10^{-13}/10^{-6})5 = -1.75 \times 10^{-6}\ \text{C/cm}^2$$

ii) At the drain end of the channel, $V_{CS} = V_{DS} = 2.5$ V, and we find:

$$q_N^*(L) = -(\varepsilon_{ox}/t_{ox})(V_{GS}-V_{DS}-V_T) = -(3.5 \times 10^{-13}/10^{-6})\ 2.5 = -8.75 \times 10^{-7}\ \text{C/cm}^2$$

d) The drift velocity can be found by remembering that i_D is the channel current and that the current at any point y along the channel is the sheet charge density in the channel, $q_N^*(y)$, times the drift velocity of the carriers composing this charge, times the width of the channel: $i_D = -\ W\ q_N^*(y)\ s_{e\text{-}Drift}(y)$. Thus: $s_{e\text{-}Drift}(y) = -\ i_D/W\ q_N^*(y)$

i) At the source end of the channel we find:

$$s_{e\text{-}Drift}(0) = i_D/W\ q_N^*(y) = 7.5 \times 10^{-3}/5 \times 10^{-3} \times 1.75 \times 10^{-6} = 8.57 \times 10^5\ \text{cm/s}$$

ii) At the drain end of the channel we find:

$$s_{e\text{-}Drift}(L) = i_D/W\ q_N^*(y) = 7.5 \times 10^{-3}/5 \times 10^{-3} \times 8.75 \times 10^{-7} = 1.71 \times 10^6\ \text{cm/s}$$

e) Now the relationship between the drain current and the charge density at y = L must be $i_D = -\ W\ q_N^*(L)\ s_{sat}(y)$. We want the charge density, so we solve for that:

$$q_N^*(L) = -\ i_D/W\ s_{sat}(y) = -10^{-2}/5 \times 10^{-3} \times 10^7 = -2 \times 10^{-7}\ \text{C/cm}^2$$

f) This is a difficult question and almost nobody got it correct. This was anticipated before the exam, but we decided to leave the question in with a precautionary note added, just to see if anyone could do it. That said, the idea is to go back to the expression for i_D along the channel, and to integrate it from y = 0 to y = L/2, the point along the channel at which we want to know v_{CS}. Since we know i_D already, we can find $v_{CS}(L/2)$. The sequence leading to the expression sought is:

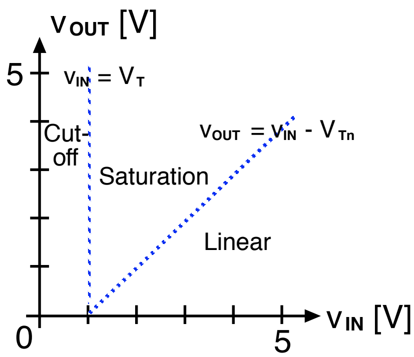
$$i_{D,sat} = -W\ q_N^*(y)\ \mu_e\ dv_{CS}/dy = W\ (\varepsilon_{ox}/t_{ox})[V_{GS}-v_{CS}(y)-V_T]\ \mu_e\ dv_{CS}/dy$$

$$i_{D,sat}dy = W\ (\varepsilon_{ox}/t_{ox})[V_{GS}-v_{CS}(y)-V_T]\ \mu_e\ dv_{CS}$$

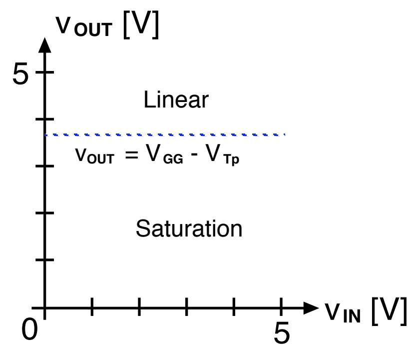
$$\int_0^y i_{D,sat}dy = i_{D,sat}\ y = \int_0^{V_{CS}(y)} W\ (\varepsilon_{ox}/t_{ox})[V_{GS}-v_{CS}-V_T]\ \mu_e\ dv_{CS}$$

Problem 3: Graded by Prof. Palacios

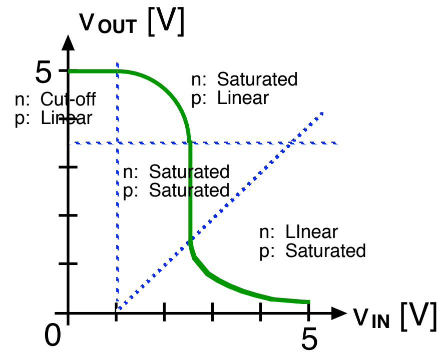
a) i)



ii)



b)



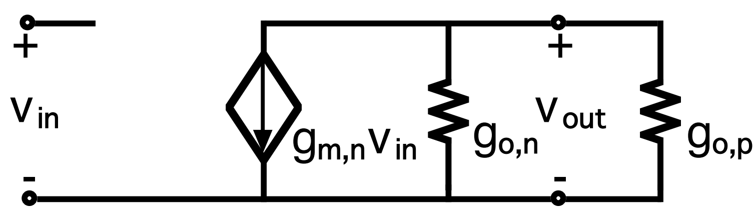
c) The threshold point occurs where both transistors are in saturation. The equation for the curve there is:

$$I_{D\text{-sat n}} = (K_n/2)(V_{GSn}-V_{Tn})^2 = I_{D\text{-sat p}} = (K_p/2)(V_{SGp}-|V_{Tp}|)^2$$

We know $K_n = K_p$, $V_{Tn} = |V_{Tp}|$, $V_{SGp} = V_{DD} - V_{GG}$, and $V_{GSn} = V_{IN} = V_{DD}/2$, so we find

$$V_{DD}/2 - V_{Tn} = V_{DD} - V_{GG} - |V_{Tp}| \;\Rightarrow\; V_{GG} = V_{DD}/2 = 2.5\text{ V}$$

d) When both devices are in saturation at this bias point, the LEC is:



We find $A_v = -g_{mn}/(g_{on}+g_{op})$. There are several ways to write g_m in terms of the bias current and voltages, but the most useful here is $g_{mn} = 2I_{Dn}/(V_{GSn}-V_{Tn})$. This along with $g_{on} = \lambda_n I_{Dn}$ and $g_{op} = \lambda_p I_{Dp}$, and using $I_{Dn} = I_{Dp} = I_D$, $\lambda_n = \lambda_p = 0.1\text{ V}^{-1}$, $V_{Tn} = 1$ V, and $V_{GSn} = 2.5$ V, lead us to:

$$A_v = -g_{mn}/(g_{on}+g_{op}) = 2/[(V_{GSn}-V_{Tn})(\lambda_n+\lambda_p)] = 2/(1.5 \times 0.2) = -6.6$$

e) When $V_{IN} = 5$ V the n-channel device is on, and the p-channel device is saturated with $I_D = K_p(V_{DD}-V_{GG}-|V_{Tp}|)^2/2$. The corresponding static power dissipation is $I_D V_{DD}$. Working through this we find

$$P_{Static}(V_{IN} = 5\text{V}) = I_D V_{DD} = [(W_P/L_P)\mu_h C^*_{ox}(V_{DD}-V_{GG}-|V_{Tp}|)^2/2] \times V_{DD}$$

$$= (25 \times 200 \times 6 \times 10^{-7} \times 1.5^2) \times 5/2 = 16.8 \times 10^{-3}\text{ W} = 16.8\text{ mW}$$

You did not have to include channel length modulation in this sub-section, but if you did you had an additional term $[1 + \lambda(V_{DS}-V_{DS,sat})] = 1 + 0.1(5 - 1.5) = 1.35$ multiplying the current, and found:

$$P_{Static}(V_{IN} = 5\text{V}) = 16.8 \times 1.35 = 22.7\text{ mW}$$

f) When $V_{IN} = 5$ V the n-channel device is off, and the only static power dissipation is due to the sub-threshold current, $I_{D,s\text{-}t}$, and is $I_{D,s\text{-}t}V_{DD}$. This is probably negligible, but just to be sure, we start by calculating the sub-threshold current when $v_{GS} = 0$:

$$i_{D,s\text{-}t} = [(W_N/L_N)\mu_e C^*_{ox}]\,(kT/q)^2\,(n-1)\exp(-qV_T/nkT)$$
$$= (10 \times 5 \times 10^2 \times 6 \times 10^{-7})\,(2.5 \times 10^{-2})^2\,(1.5-1)\exp[-1/(1.5 \times 0.025)]$$
$$\approx 9.3 \times 10^{-7}\,e^{-27}\text{ A} \approx 9.3 \times 10^{-7} \times 1.6 \times 10^{-12} \approx 1.5 \times 10^{-18}$$

From this we have $P_{Static}\,(V_{IN} = 0V) = 1.5 \times 10^{-18} \times 5 \approx 10^{-17}$ W ≈ 0.

g) The low to high transition time is due to charging through the p-channel device with its saturation current, $I_D = K_p(V_{DD}\text{-}V_{GG}\text{-}|V_{Tp}|)^2/2$ [= 3.36 mA from Part (e)], so we have:

$$\tau_{Lo\text{-}Hi} = C_L\,V_{DD}/[K_p(V_{DD}\text{-}V_{GG}\text{-}|V_{Tp}|)^2/2] = 10^{-13} \times 5/(3.36 \times 10^{-3}) = 1.48 \times 10^{-10}\text{ s}$$

Exam Statistics

Average/Standard deviation:

Problem 1	26.6	6.9
Problem 2	21.7	6.6
Problem 3	22.0	6.9
Total	70.3	16.6

Class median: 73

Distribution to nearest 5:

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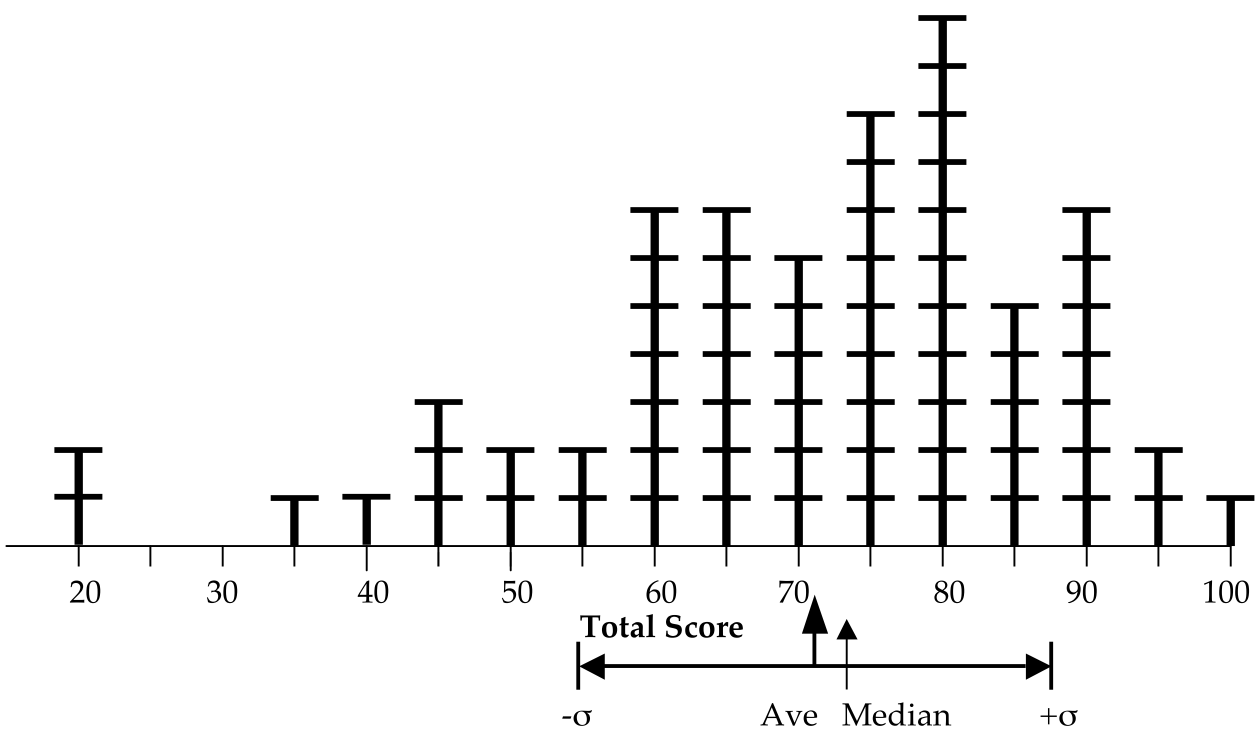

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