6.035

Spring 2010

Lecture 10: Introduction to Dataflow Analysis

Value Numbering Summary

- Forward symbolic execution of basic block
- Maps
  - Var2Val – symbolic value for each variable
  - Exp2Val – value of each evaluated expression
  - Exp2Tmp – tmp that holds value of each evaluated expression
- Algorithm
  - For each statement
    - If variables in RHS not in the Var2Val add it with a new value
    - If RHS expression in Exp2Tmp use that Temp
    - If not add RHS expression to Exp2Val with new value
    - Copy the value into a new tmp and add to EXp2Tmp

Copy Propagation Summary

- Forward Propagation within basic block
- Maps
  - tmp2var: tells which variable to use instead of a given temporary variable
  - var2set: inverse of tmp to var. tells which temps are mapped to a given variable by tmp to var
- Algorithm
  - For each statement
    - If any tmp variable in the RHS is in tmp2var replace it with var
    - If LHS var in var2set remove the variables in the set in tmp2var

Dead Code Elimination Summary

- Backward Propagation within basic block
- Map
  - A set of variables that are needed later in computation
- Algorithm
  - Every statement encountered
    - If LHS is not in the set, remove the statement
    - Else put all the variables in the RHS into the set

Summary So far… what's next

- Till now: How to analyze and transform within a basic block

- Next: How to do it for the entire procedure

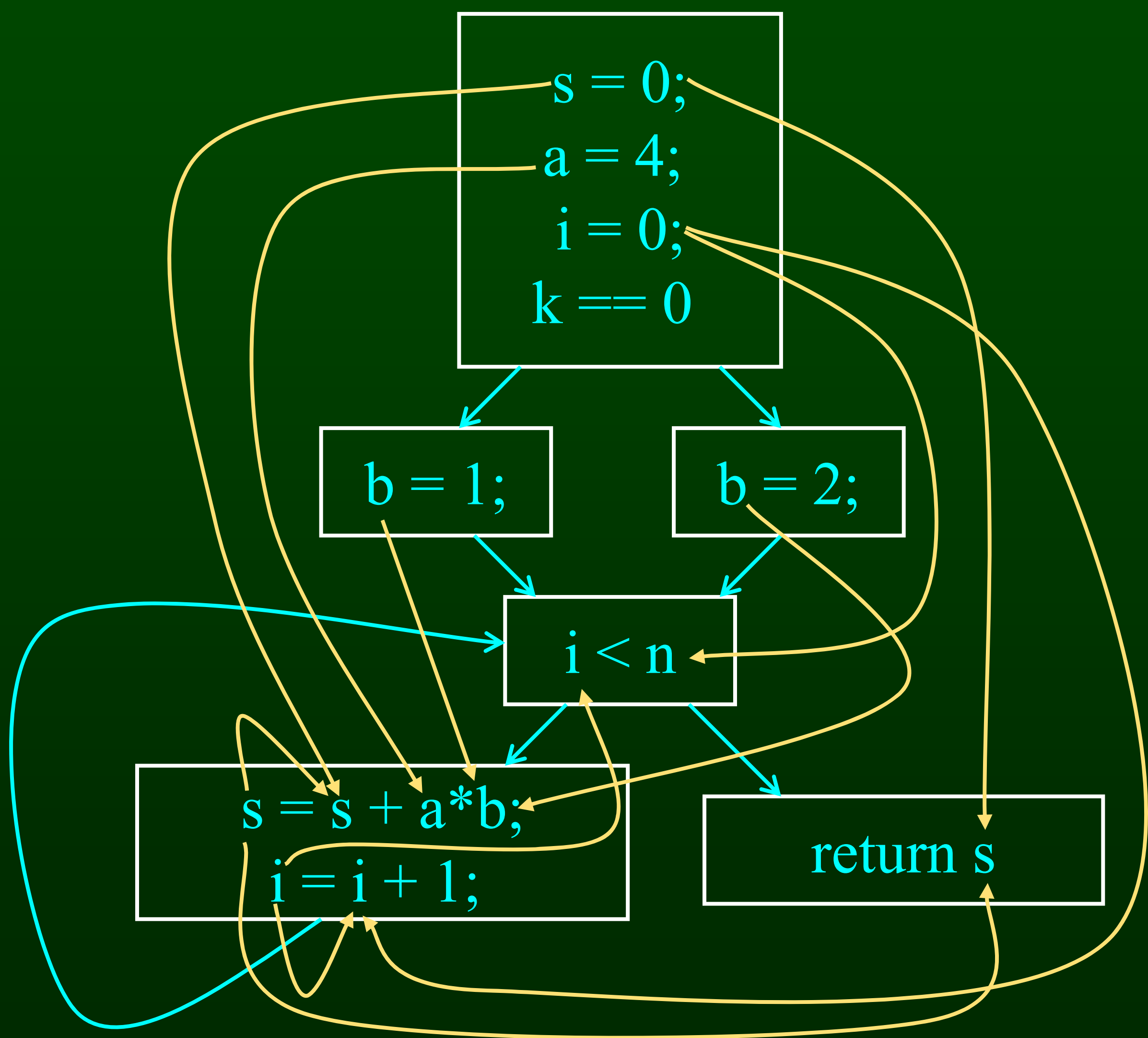
Outline

- Reaching Definitions
- Available Expressions
- Liveness

Reaching Definitions

- Concept of definition and use
  - a = x+y
  - is a definition of a
  - is a use of x and y
- A definition reaches a use if
  - value written by definition
  - may be read by use

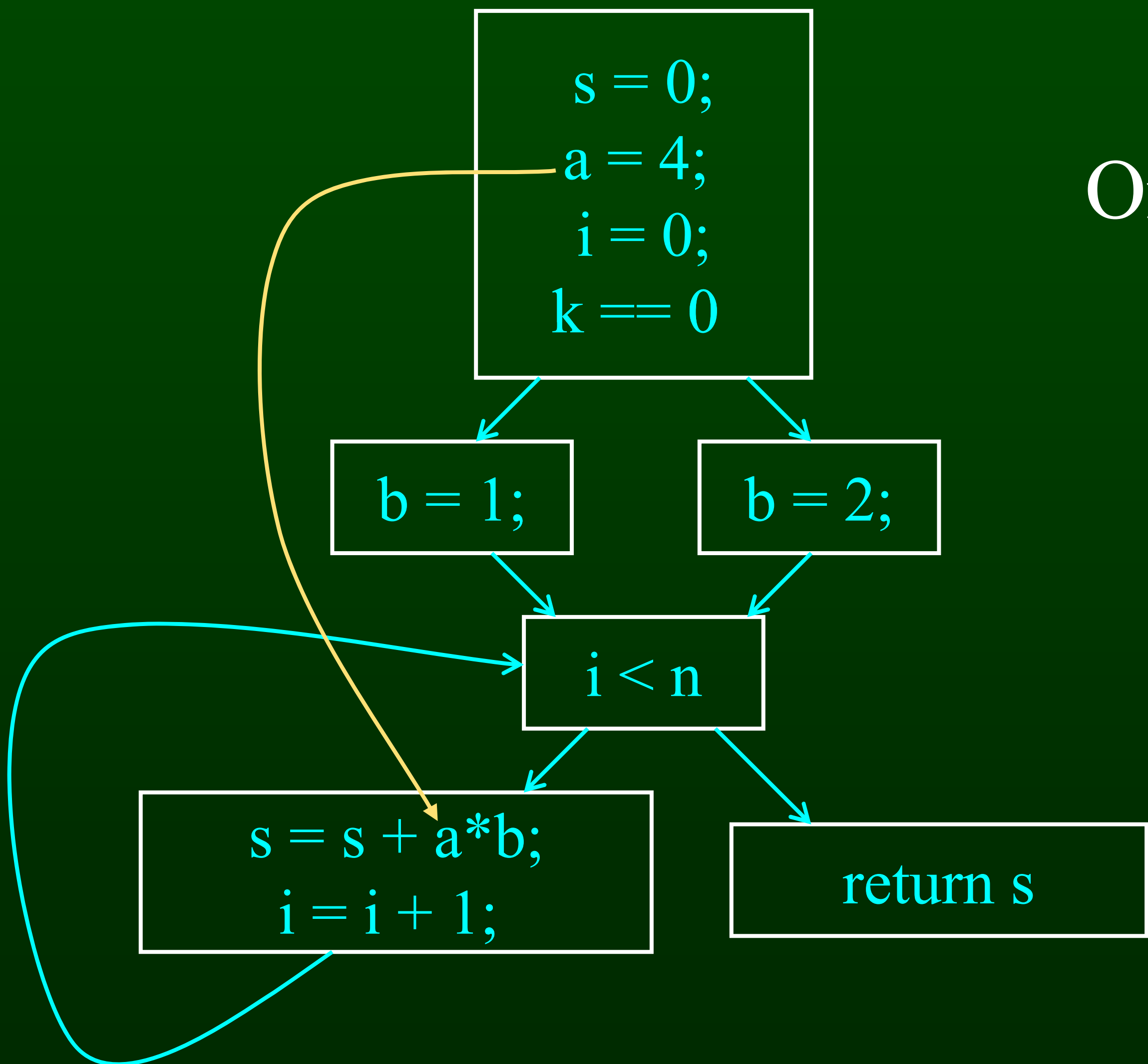
Reaching Definitions



Reaching Definitions and Constant Propagation

- Is a use of a variable a constant?
  - Check all reaching definitions
  - If all assign variable to same constant
  - Then use is in fact a constant
- Can replace variable with constant

Is a Constant in s = s+a*b?

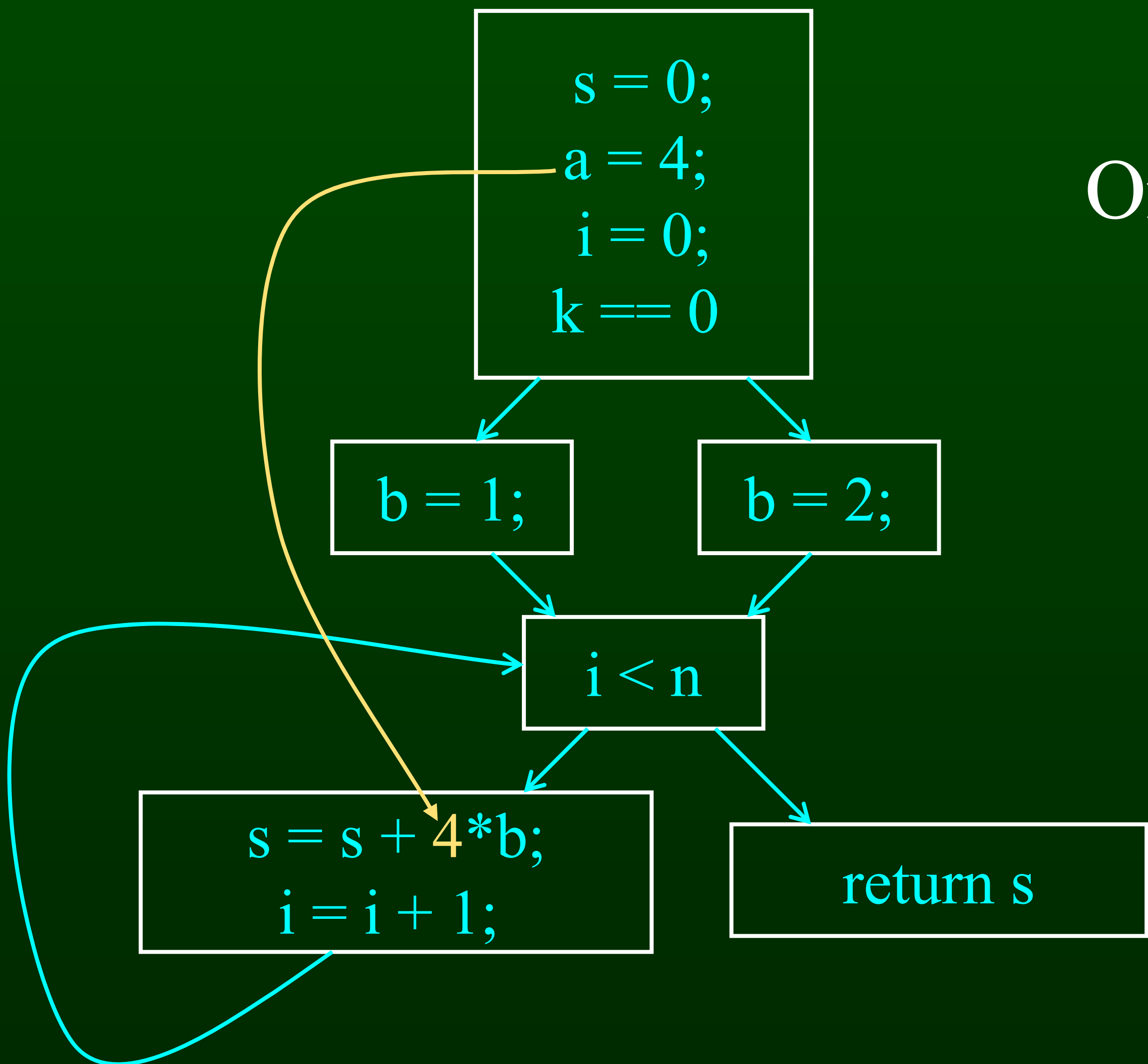


Yes!

On all reaching definitions

a = 4

Constant Propagation Transform

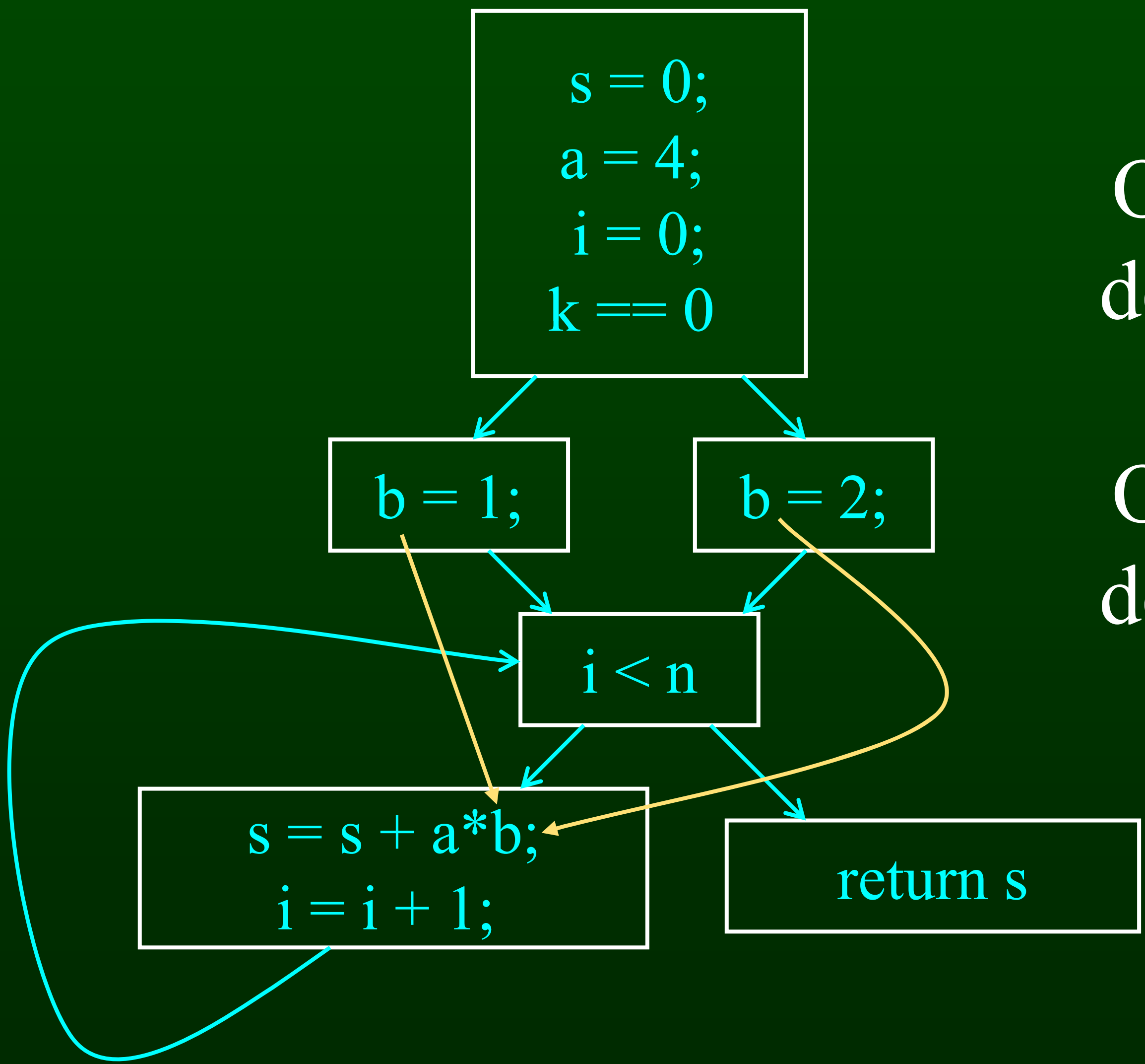


Yes!

On all reaching definitions

a = 4

Is b Constant in s = s+a*b?



No!

One reaching definition with

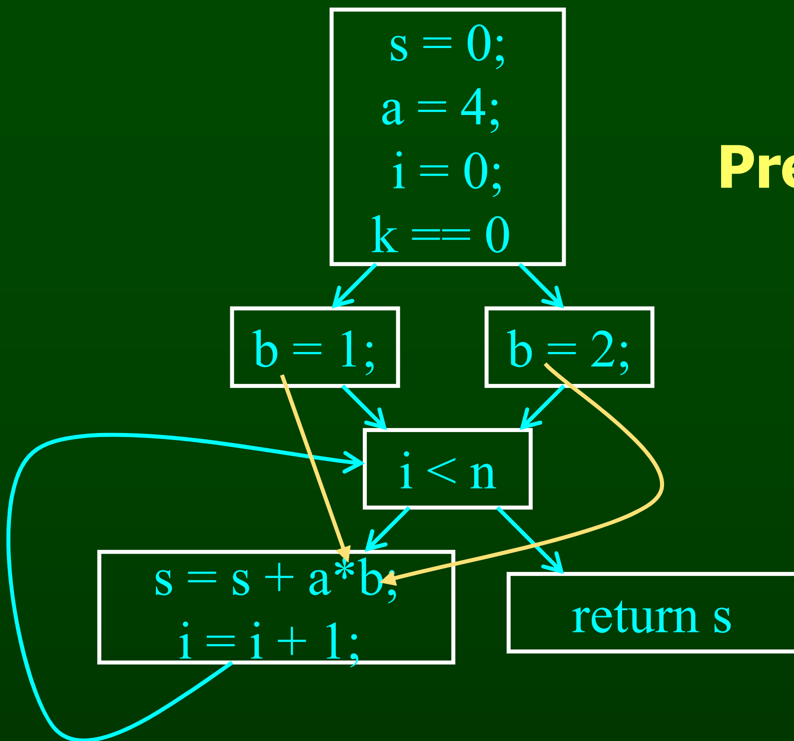
b = 1

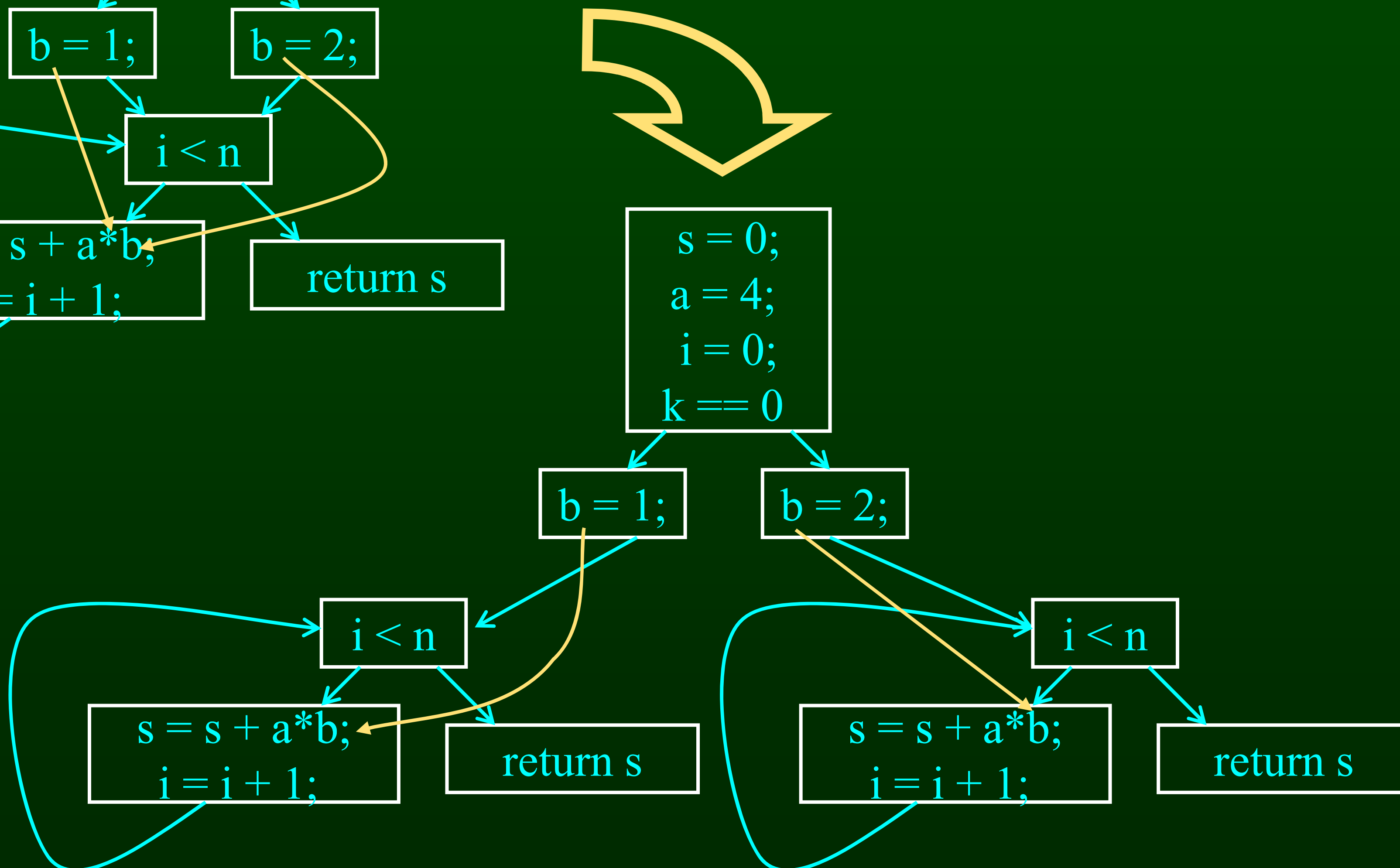
One reaching definition with

b = 2

Splitting

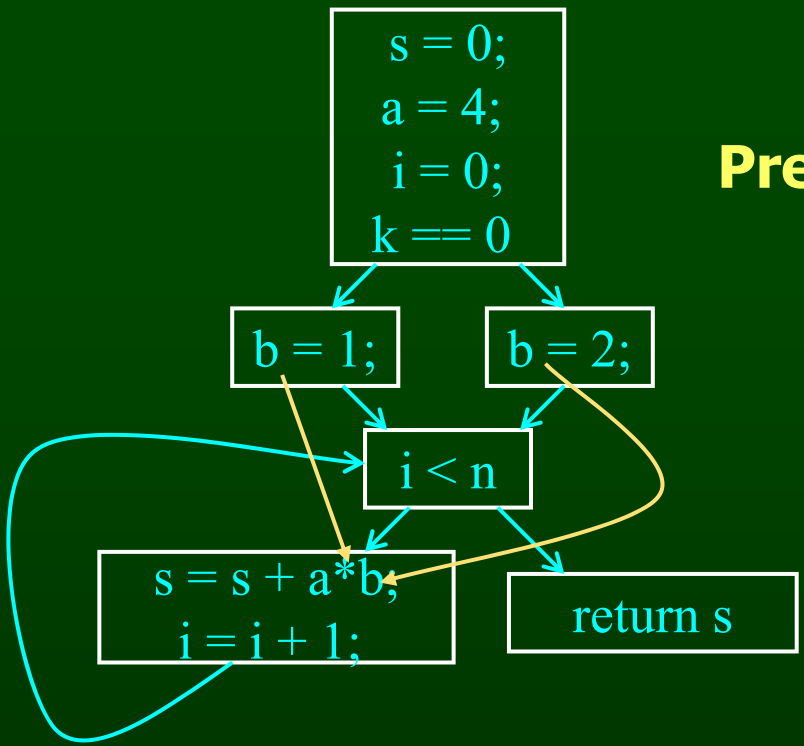
Preserves Information Lost At Merges

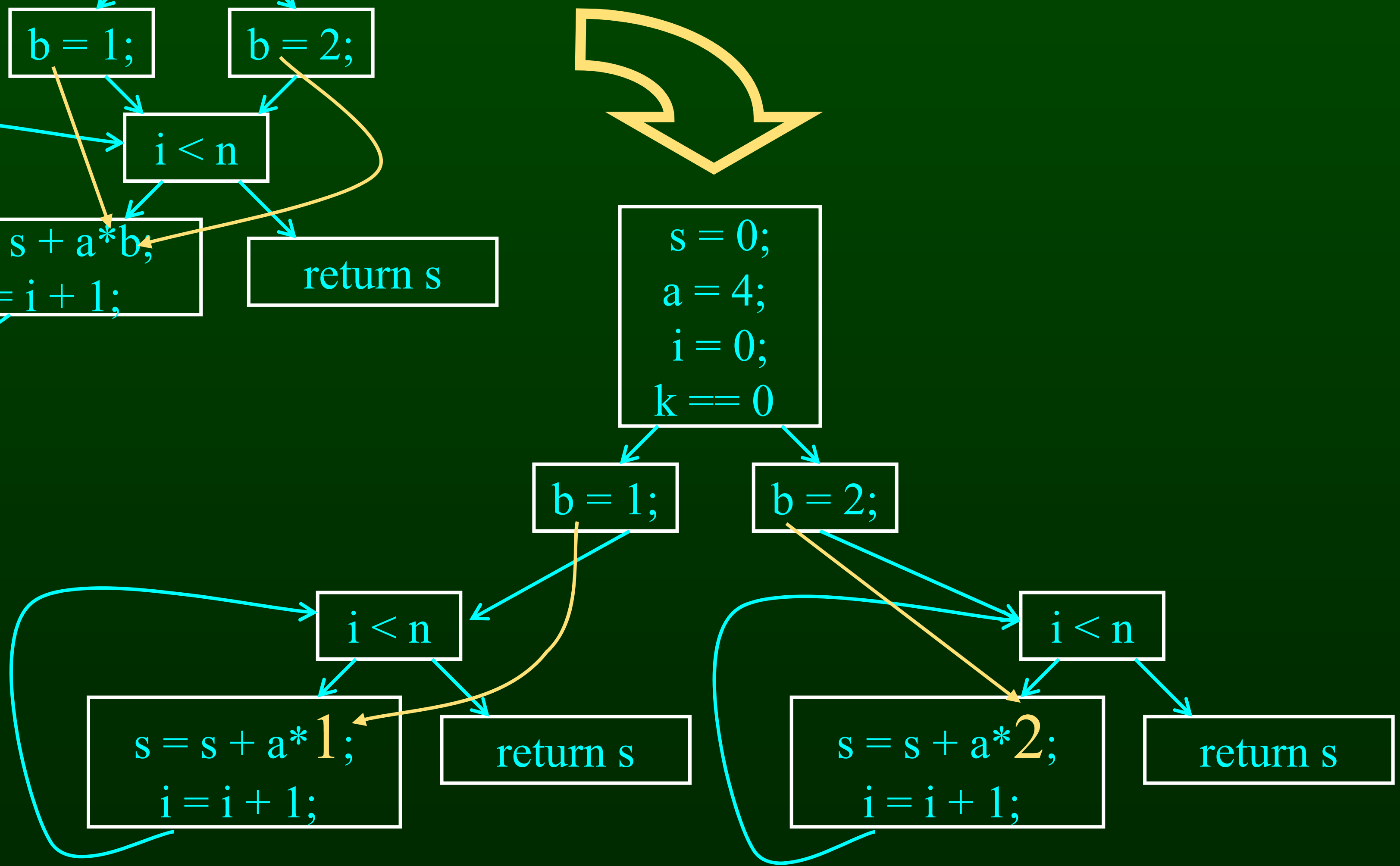




Splitting

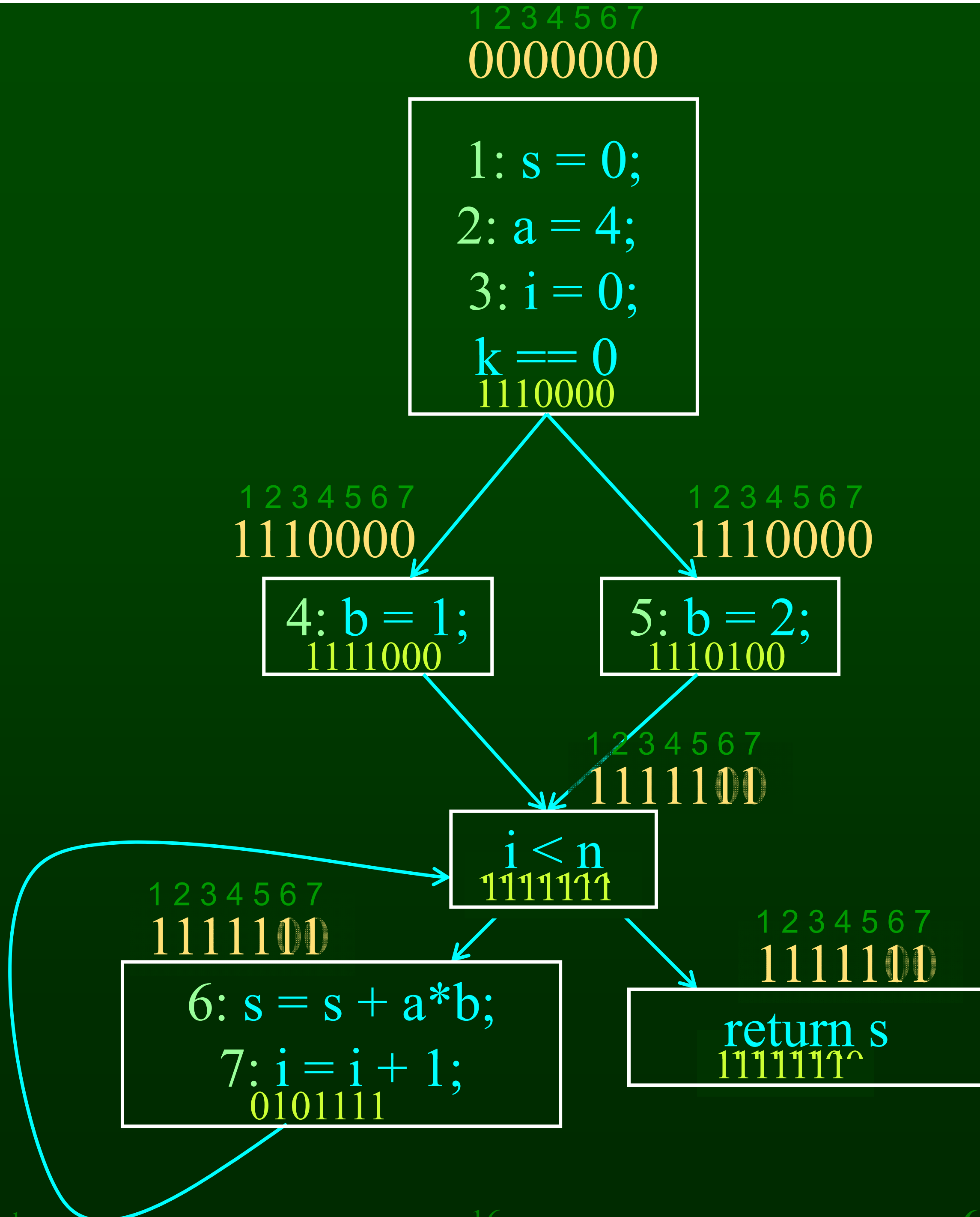
Preserves Information Lost At Merges





Computing Reaching Definitions

- Compute with sets of definitions
  - represent sets using bit vectors
  - each definition has a position in bit vector
- At each basic block, compute
  - definitions that reach start of block
  - definitions that reach end of block
- Do computation by simulating execution of program until reach fixed point


1 2 3 4 5 6 7
0000000
1: s = 0;
2: a = 4;
3: i = 0;
k == 0
1110000
1 2 3 4 5 6 7
1110000
4: b = 1;
1111000
1 2 3 4 5 6 7
1110000
5: b = 2;
1110100
1 2 3 4 5 6 7
1111111
i < n
1111111
1 2 3 4 5 6 7
1111111
6: s = s + a*b;
7: i = i + 1;
0101111
1 2 3 4 5 6 7
1111111
return s
1111111

Formalizing Analysis

- Each basic block has
  - IN - set of definitions that reach beginning of block
  - OUT - set of definitions that reach end of block
  - GEN - set of definitions generated in block
  - KILL - set of definitions killed in block
- GEN[s = s + a*b; i = i + 1;] = 0000011
- KILL[s = s + a*b; i = i + 1;] = 1010000
- Compiler scans each basic block to derive GEN and KILL sets

Dataflow Equations

- IN[b] = OUT[b1] U ... U OUT[bn]
  - where b1, ..., bn are predecessors of b in CFG
- OUT[b] = (IN[b] - KILL[b]) U GEN[b]
- IN[entry] = 0000000
- Result: system of equations

Solving Equations

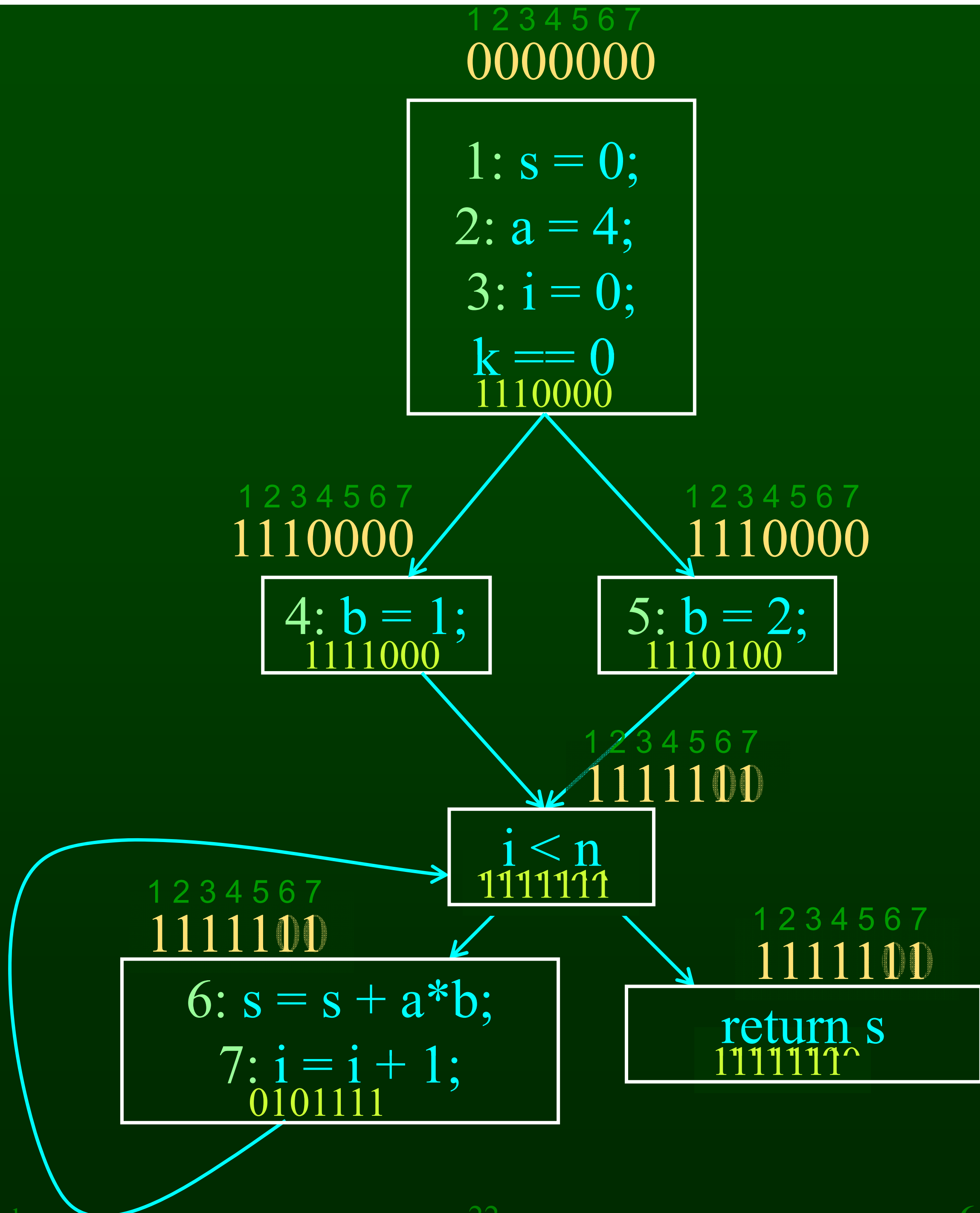
- Use fixed point algorithm
- Initialize with solution of OUT[b] − 0000000
- Repeatedly apply equations
  – IN[b] − OUT[b1] U ... U OUT[bn]
  – OUT[b] = (IN[b] - KILL[b]) U GEN[b]
- Until reach fixed point
- Until equation application has no further effect
- Use a worklist to track which equation applications may have a further effect

Reaching Definitions Algorithm

```
for all nodes n in N
    OUT[n] = emptyset; // OUT[n] = GEN[n];
IN[Entry] = emptyset;
OUT[Entry] = GEN[Entry];
Changed = N - { Entry }; // N = all nodes in graph

while (Changed != emptyset)
    choose a node n in Changed;
    Changed = Changed - { n };

    IN[n] = emptyset;
    for all nodes p in predecessors(n)
        IN[n] = IN[n] U OUT[p];

    OUT[n] = GEN[n] U (IN[n] - KILL[n]);

    if (OUT[n] changed)
        for all nodes s in successors(n)
            Changed = Changed U { s };
```

Questions

- Does the algorithm halt?
  - yes, because transfer function is monotonic
  - if increase IN, increase OUT
  - in limit, all bits are 1
- If bit is 0, does the corresponding definition ever reach basic block?
- If bit is 1, is does the corresponding definition always reach the basic block?



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5: b = 2;
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1 2 3 4 5 6 7
1111100
i < n
1111111
1 2 3 4 5 6 7
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6: s = s + a*b;
7: i = i + 1;
0101111
1 2 3 4 5 6 7
1111100
return s
1111111

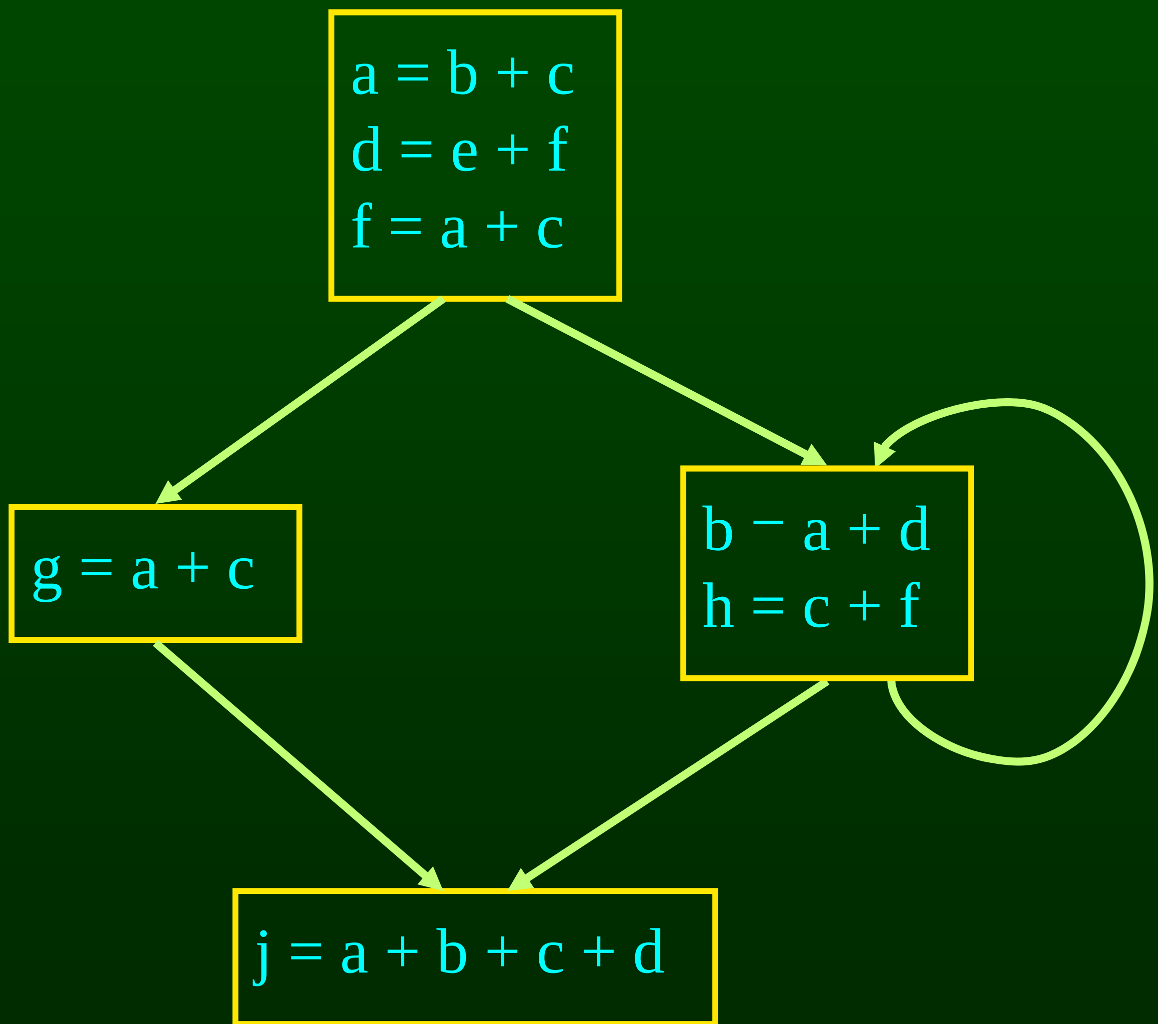
Outline

- Reaching Definitions
- **Available Expressions**
- Liveness

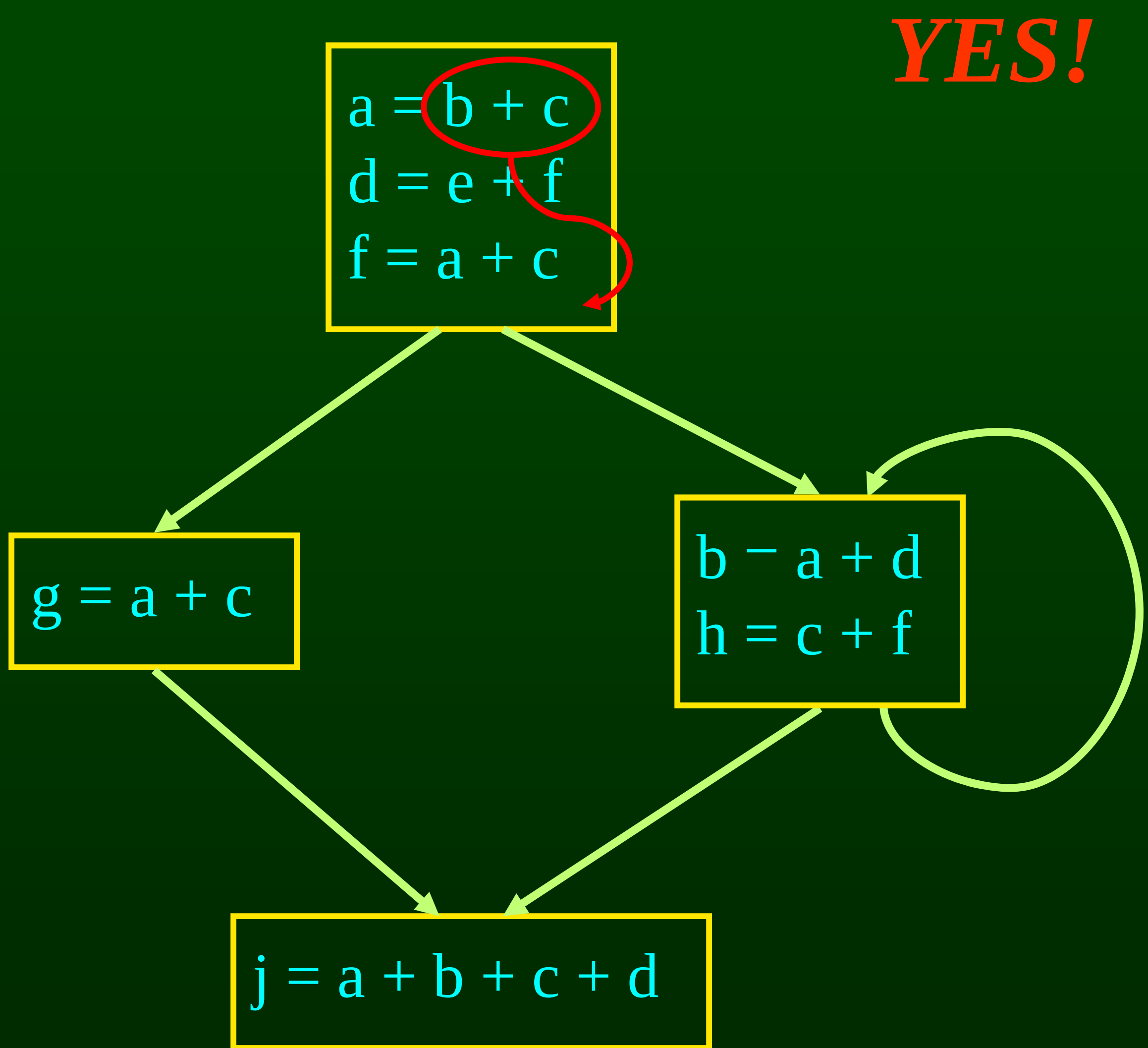
Available Expressions

- An expression x+y is available at a point p if
  - every path from the initial node to p must evaluate x+y before reaching p,
  - and there are no assignments to x or y after the evaluation but before p.
- Available Expression information can be used to do global (across basic blocks) CSE
- If expression is available at use, no need to reevaluate it

Example: Available Expression

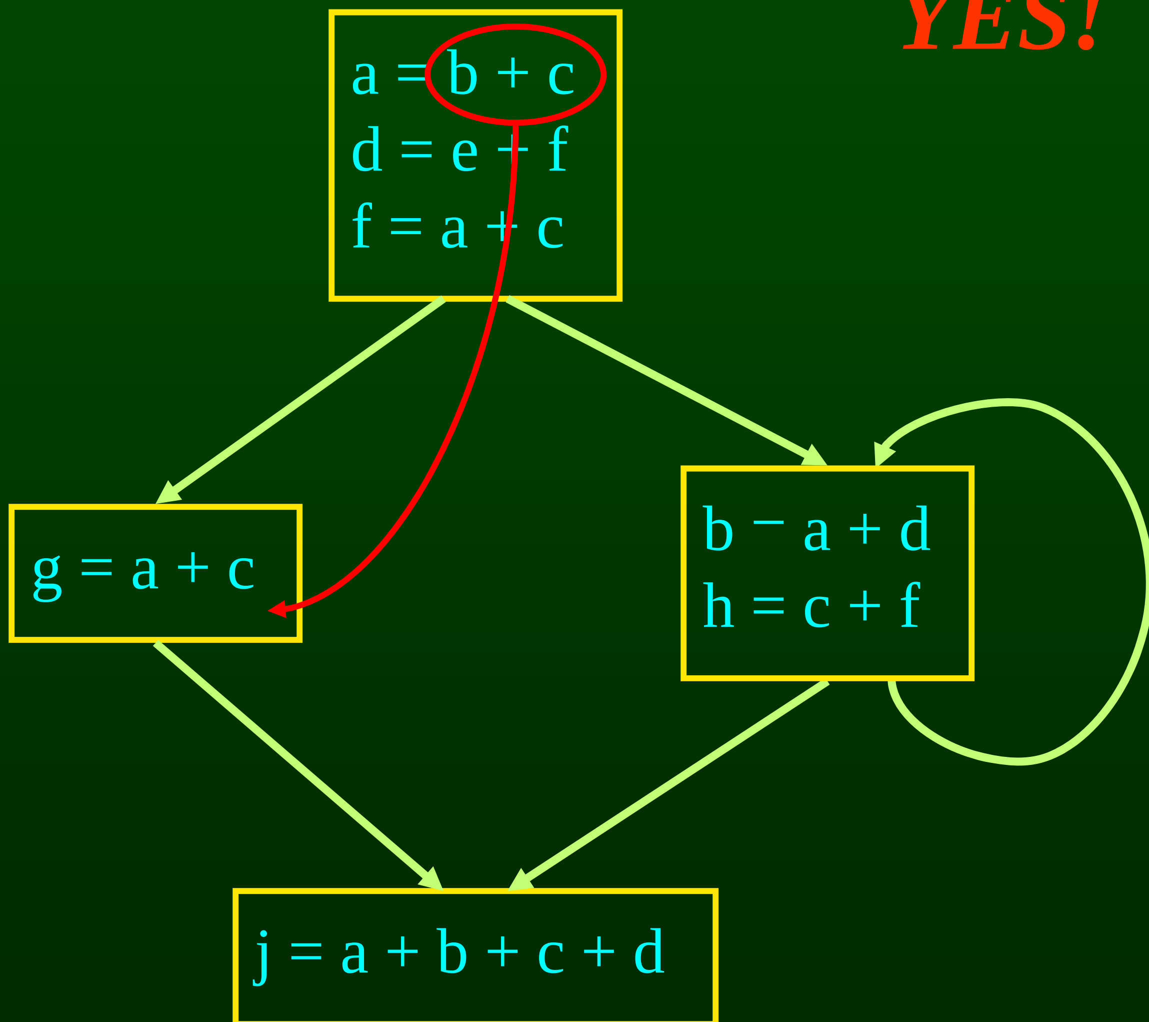


Is the Expression Available?

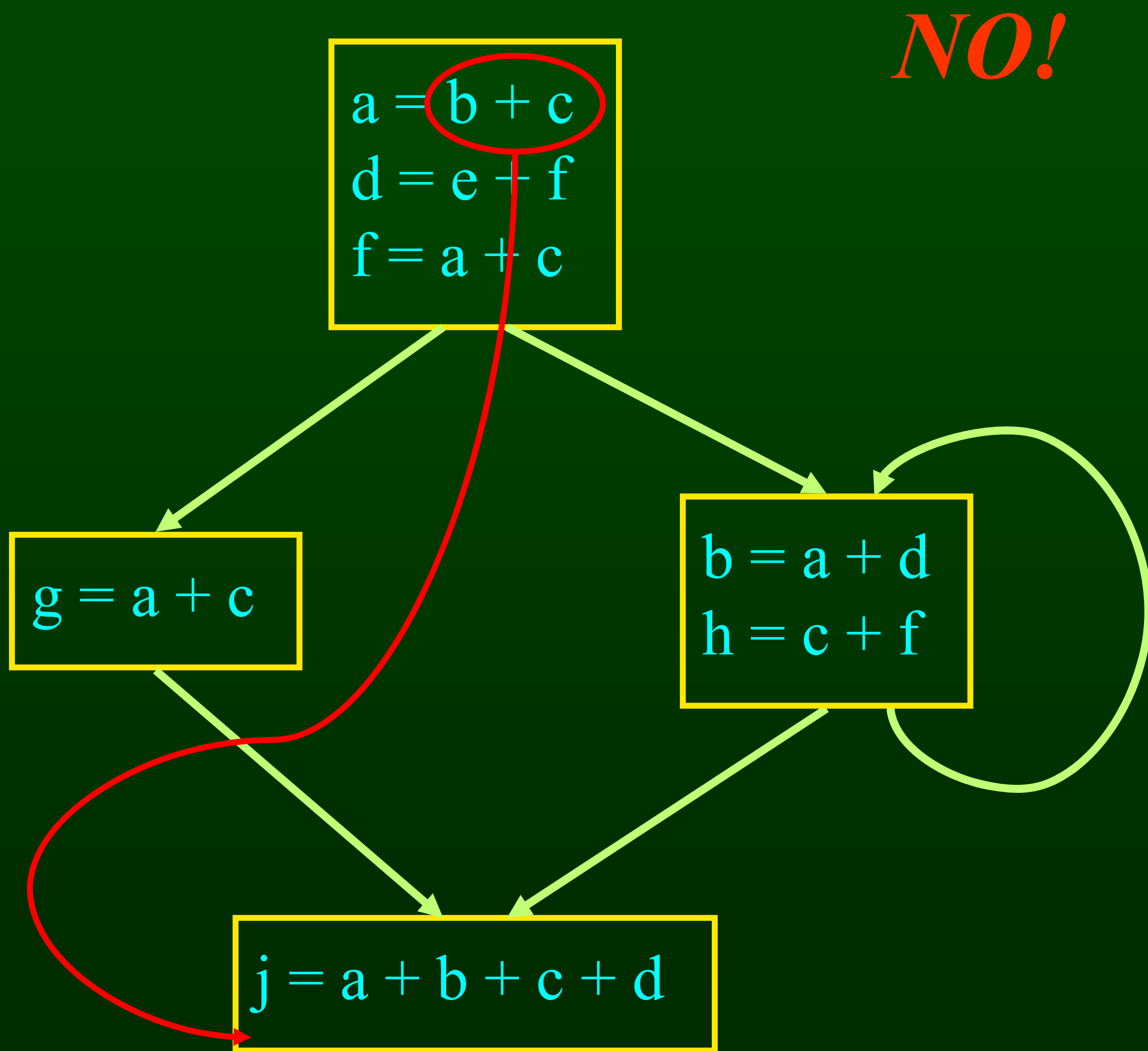


Is the Expression Available?

YES!

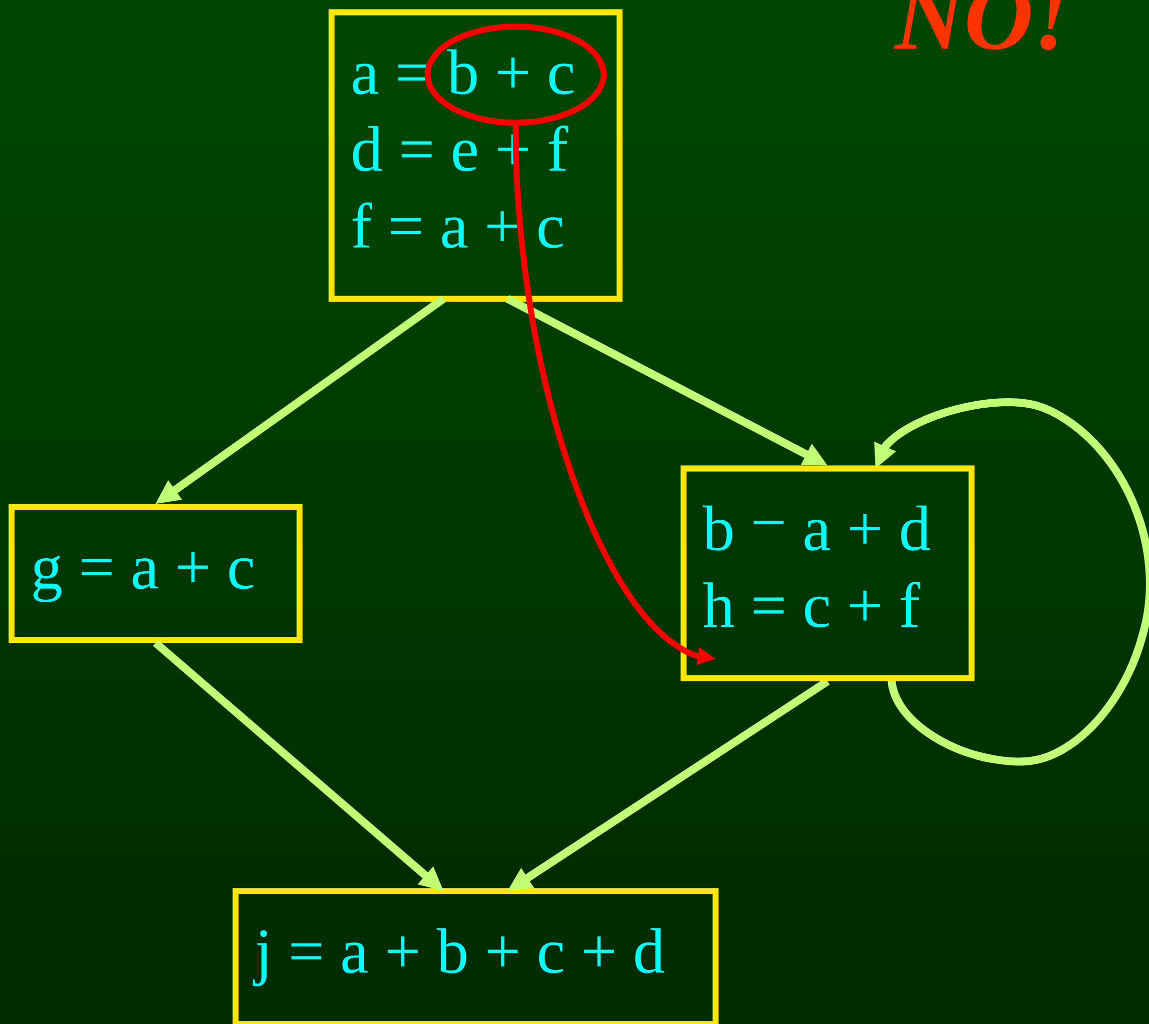


Is the Expression Available?



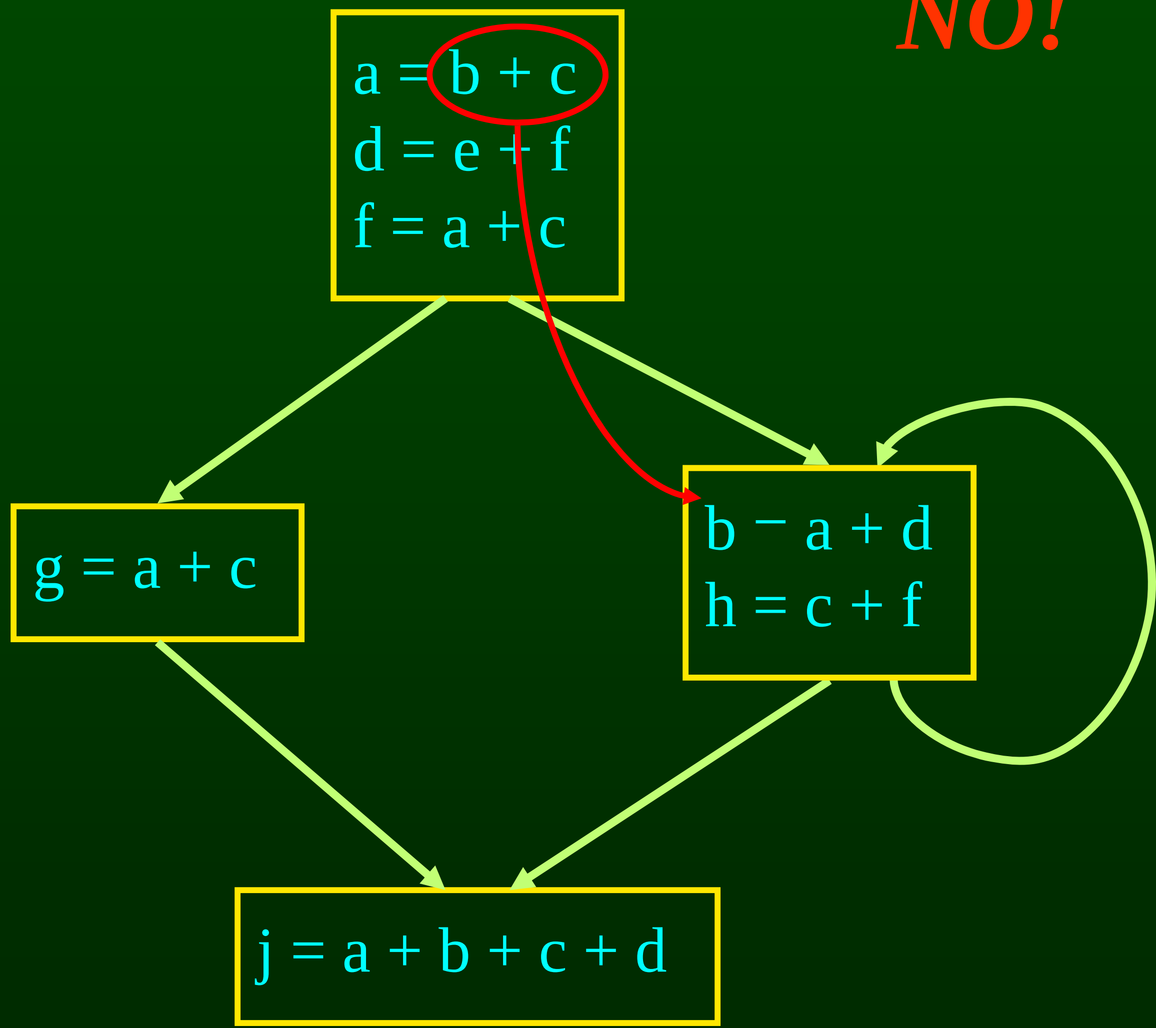
Is the Expression Available?

NO!



Is the Expression Available?

NO!



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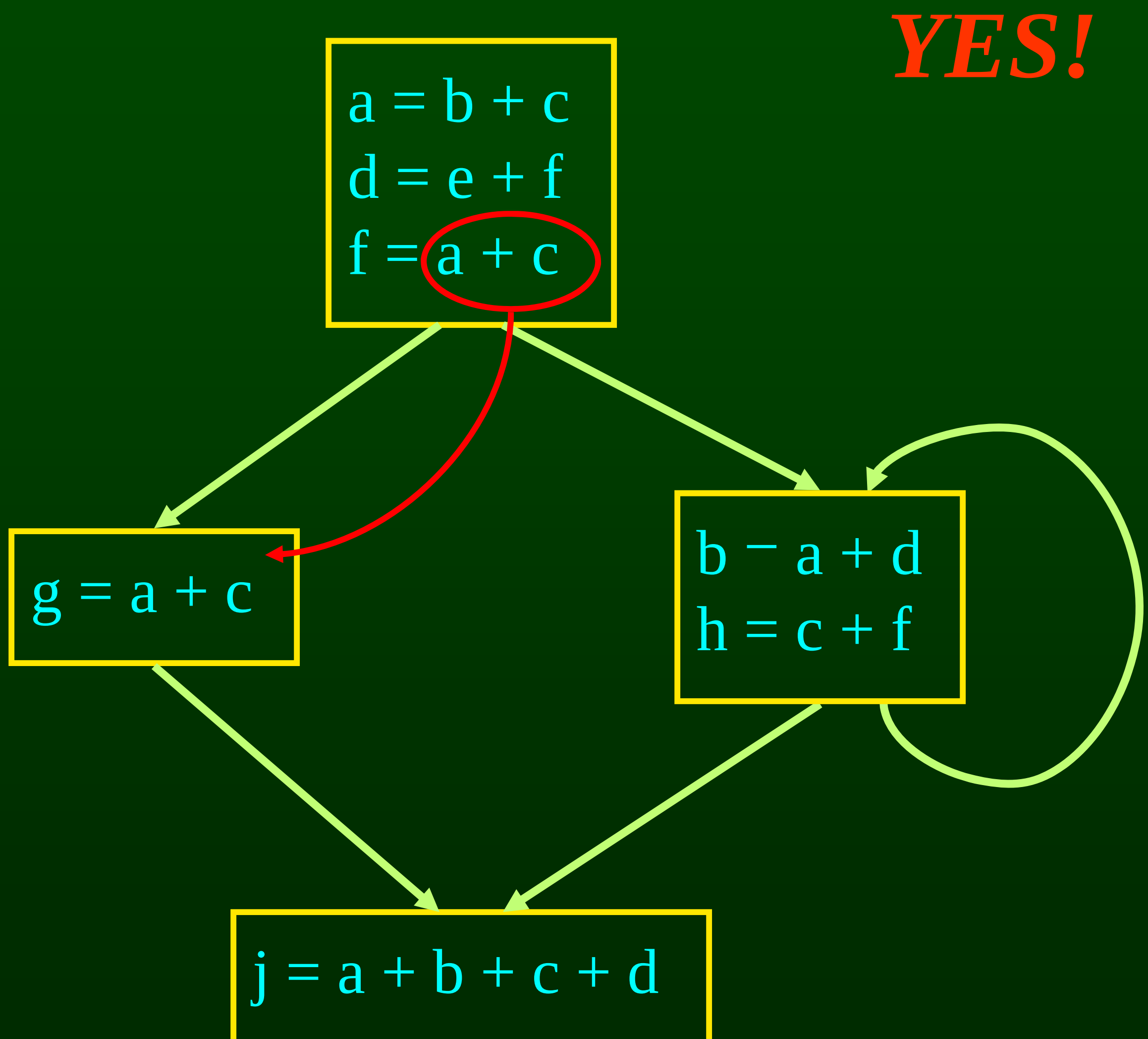
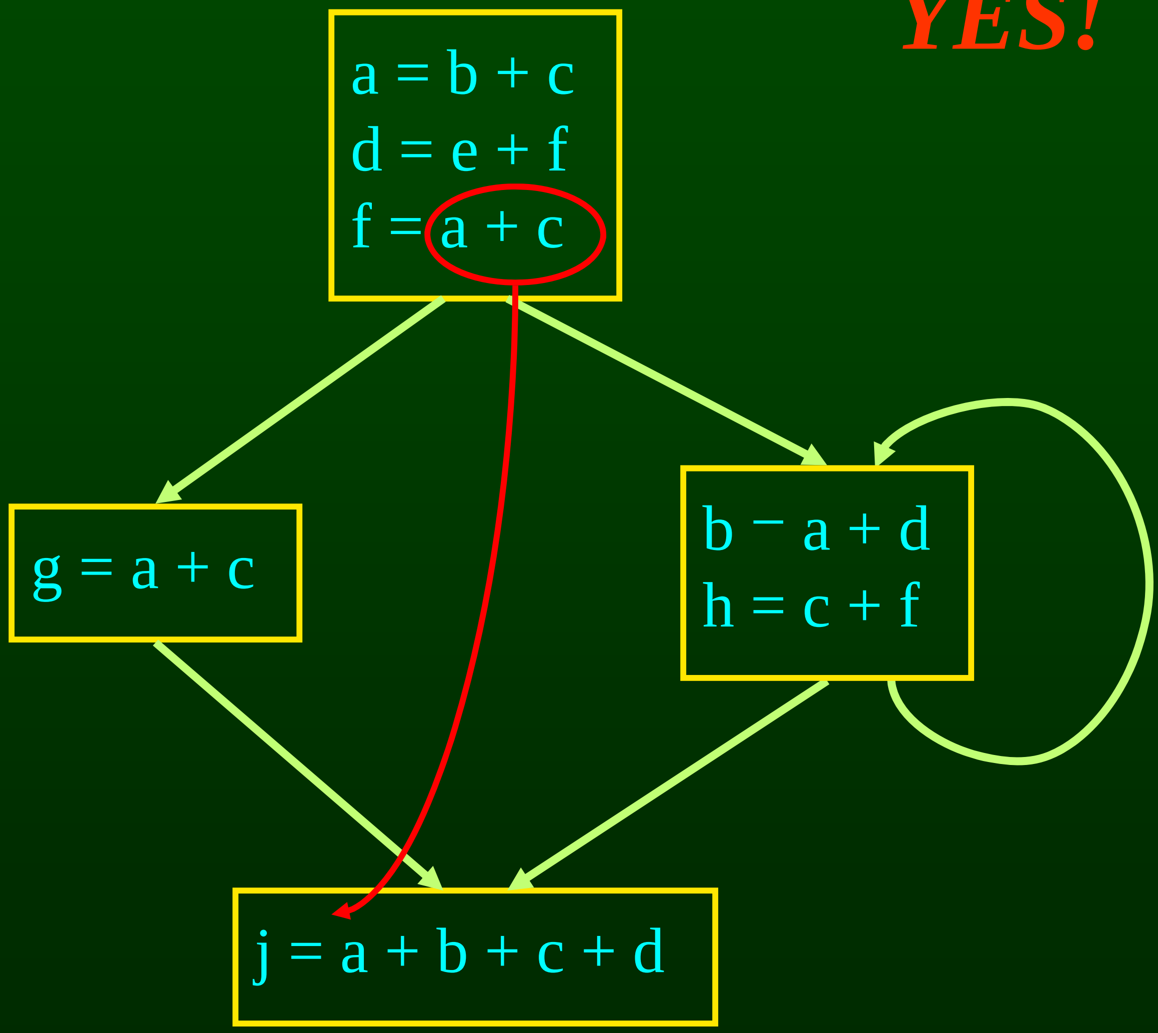
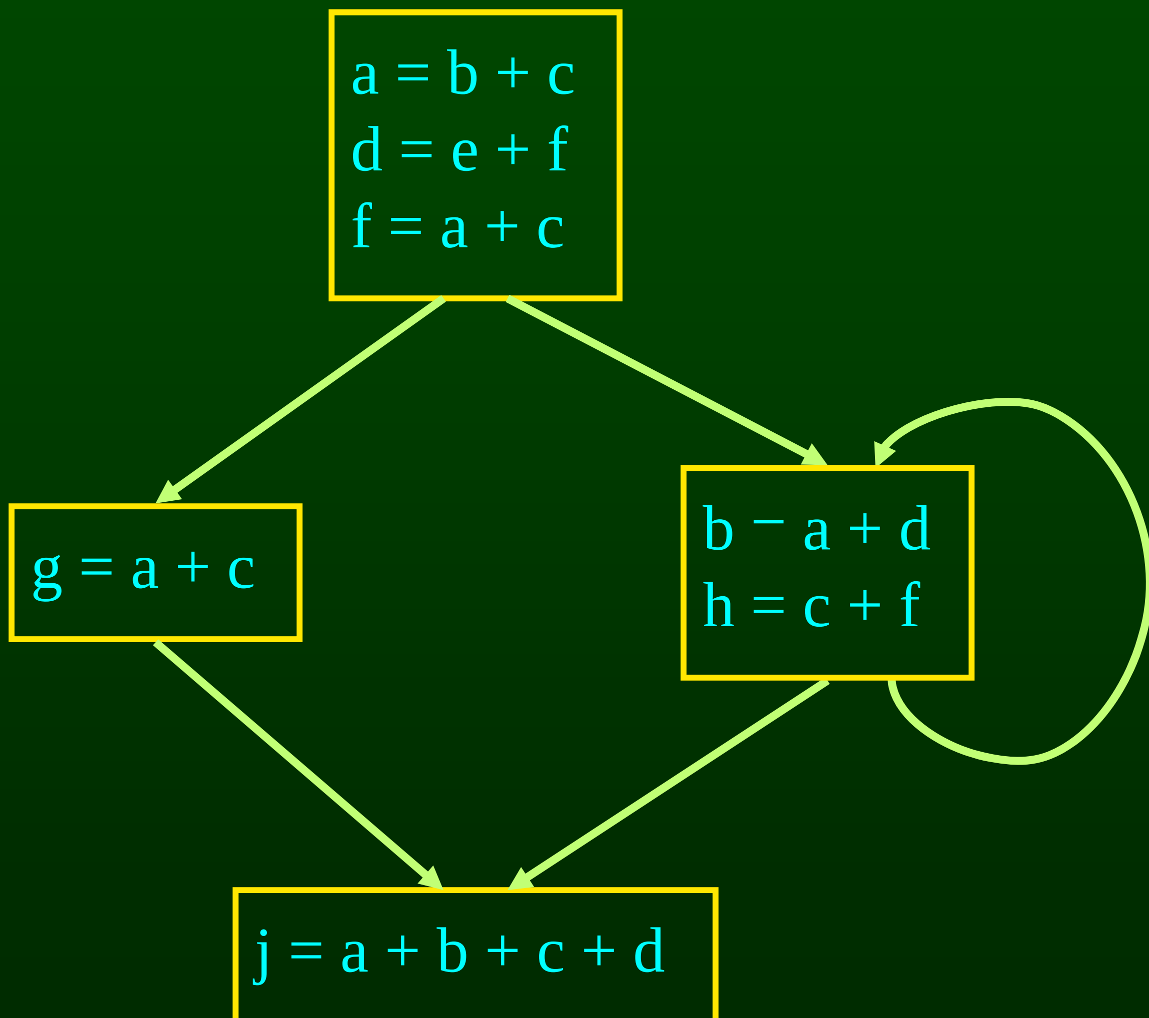


Is the Expression Available?

YES!



Use of Available Expressions



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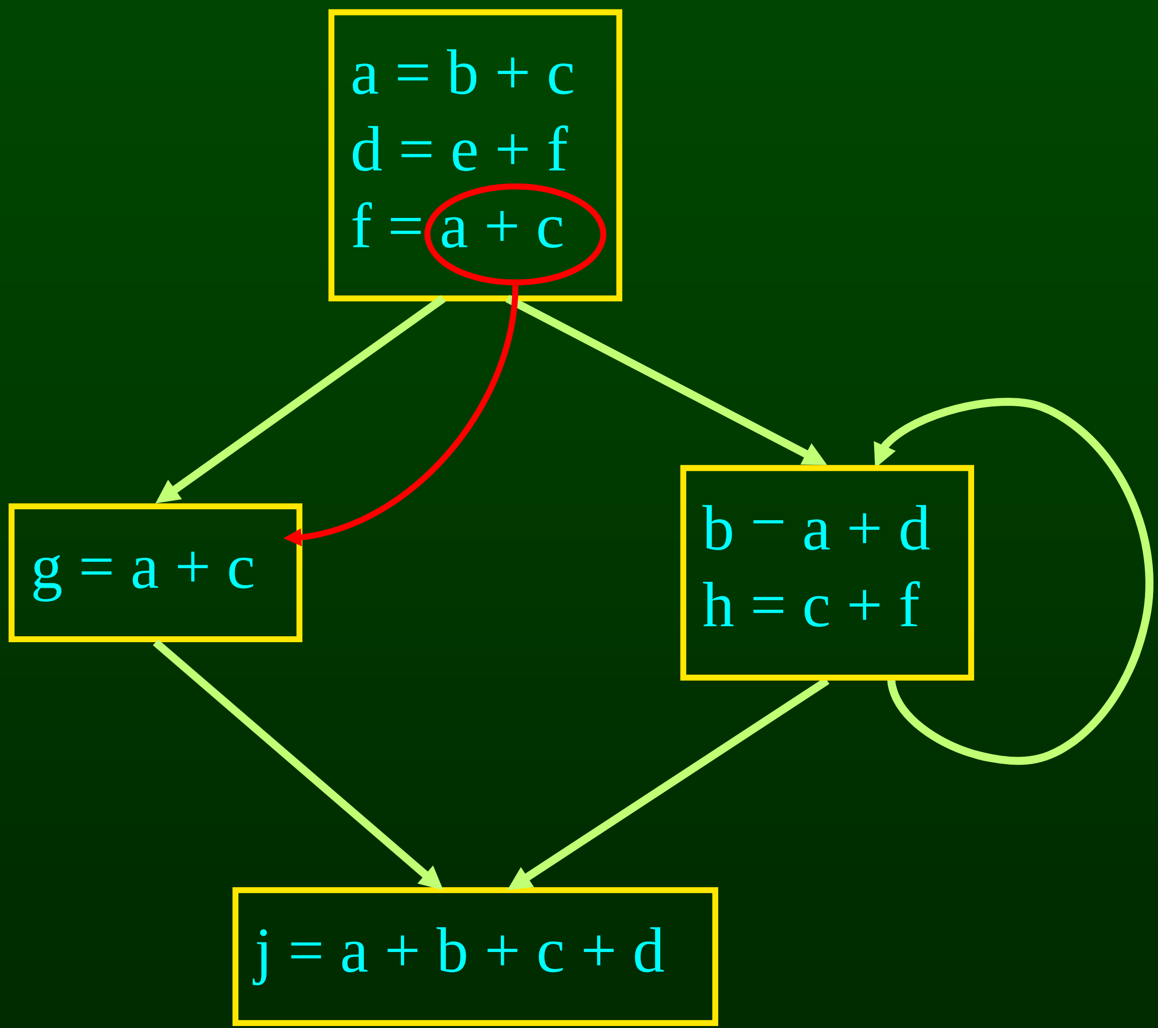

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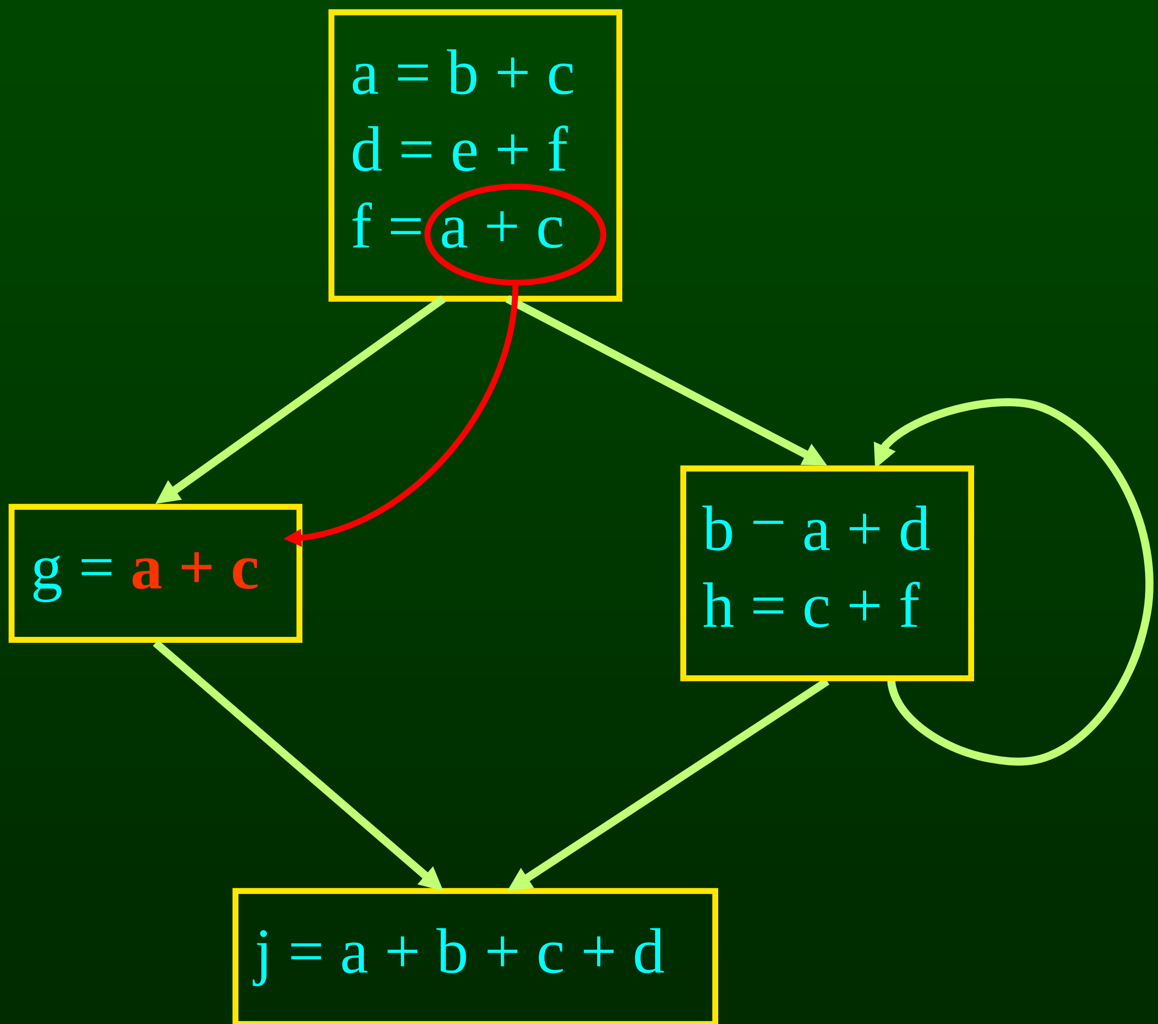

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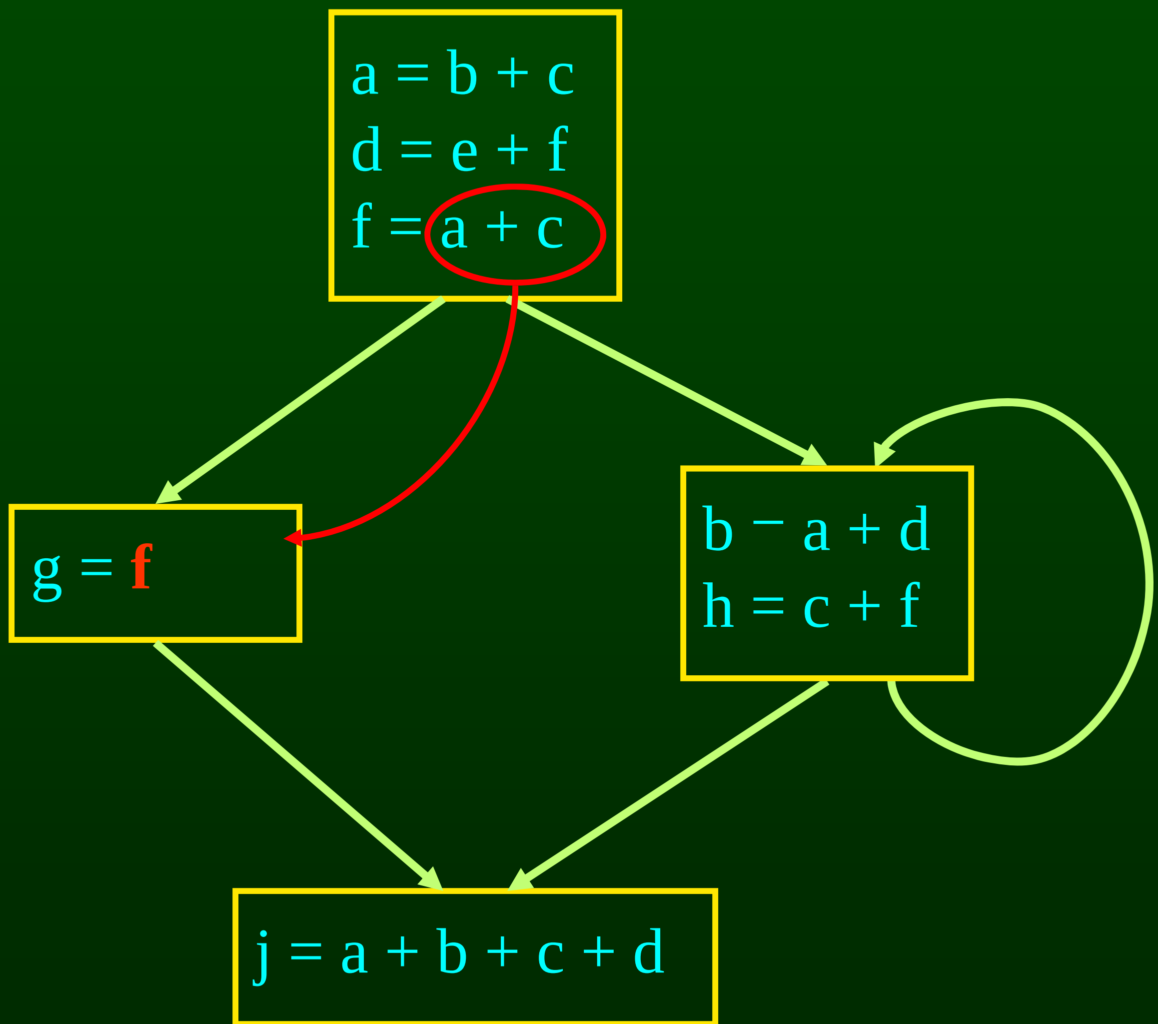
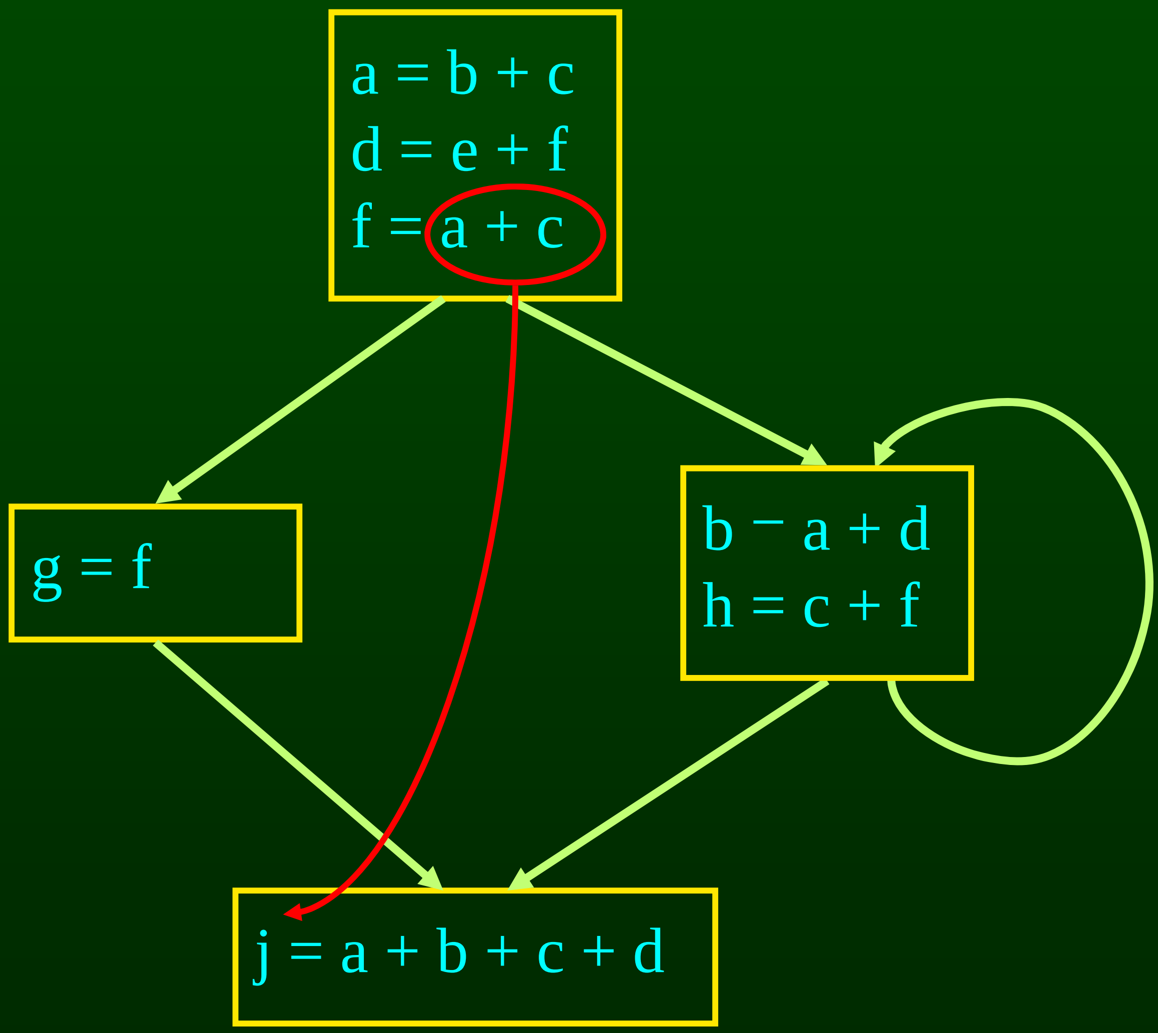
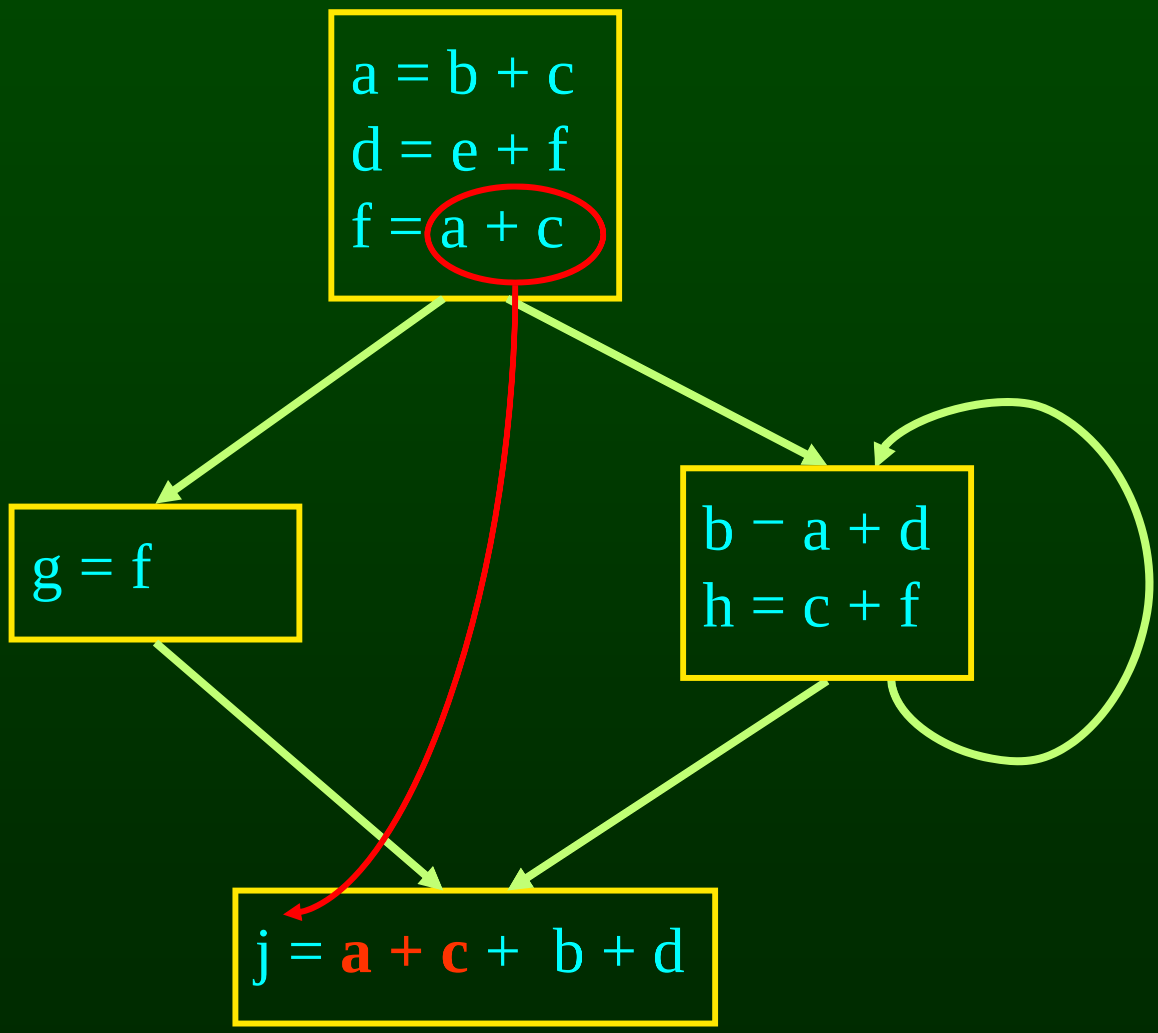


Use of Available Expressions



Use of Available Expressions



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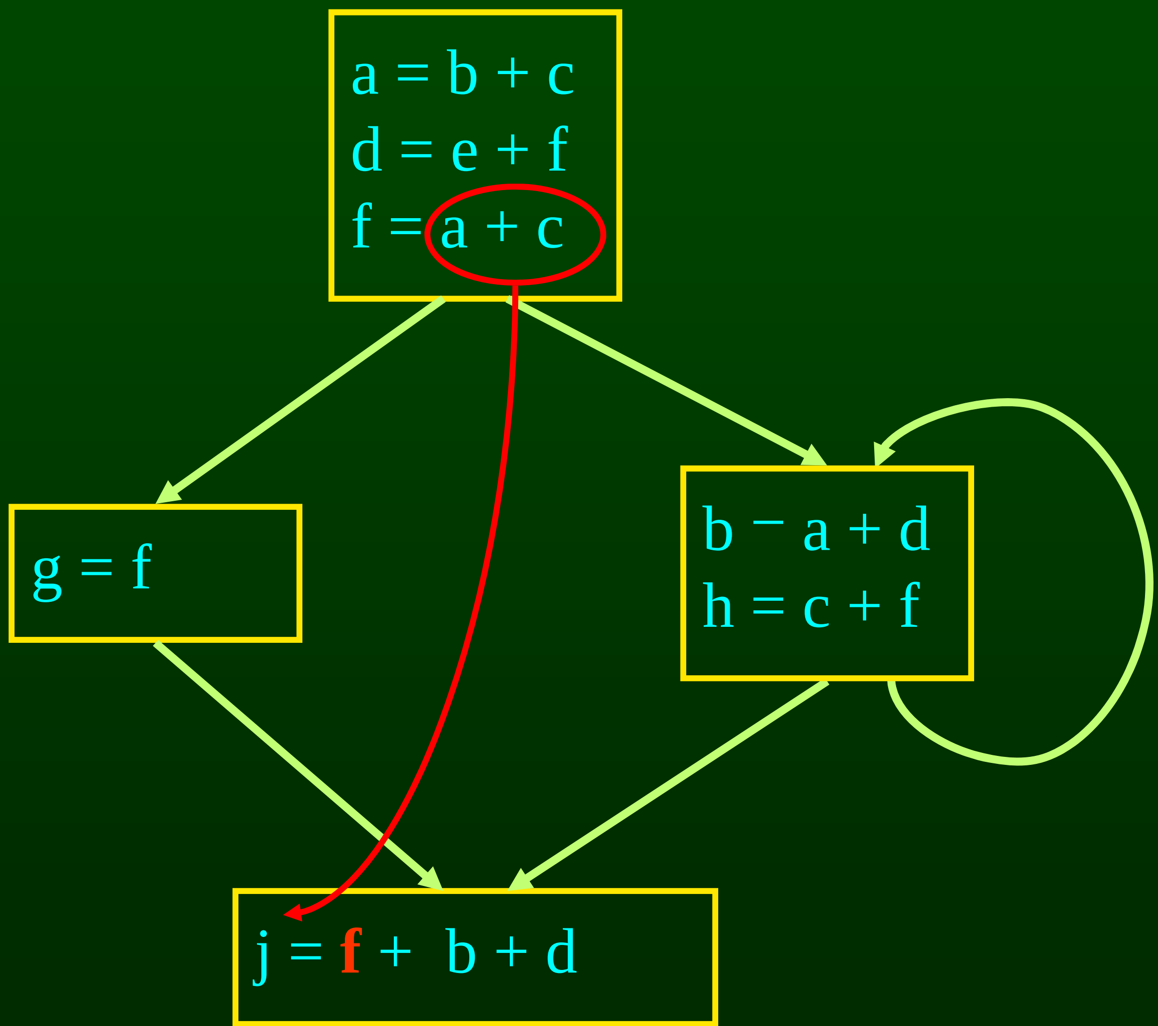

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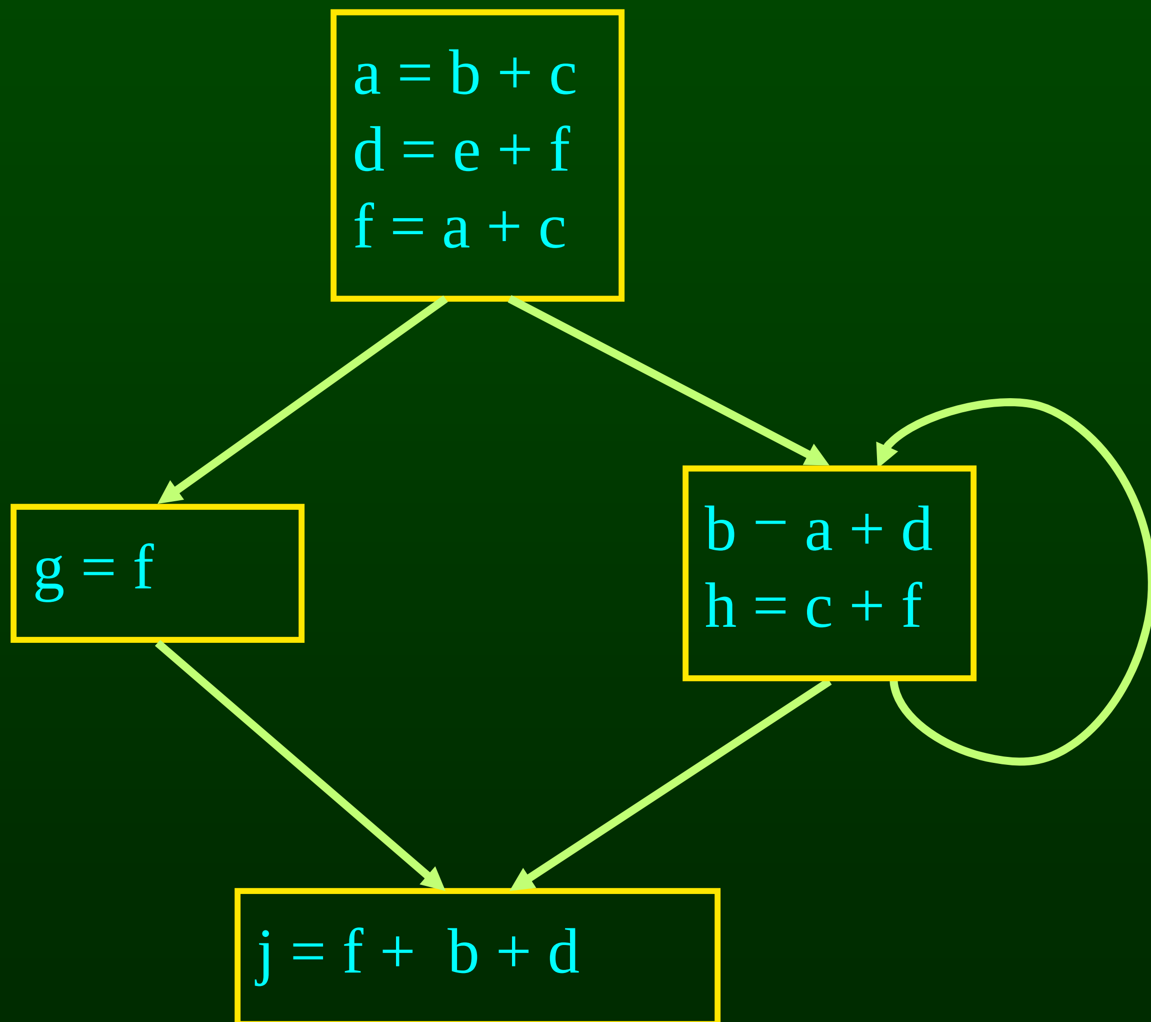


Computing Available Expressions

- Represent sets of expressions using bit vectors
- Each expression corresponds to a bit
- Run dataflow algorithm similar to reaching definitions
- Big difference
  - definition reaches a basic block if it comes from ANY predecessor in CFG
  - expression is available at a basic block only if it is available from ALL predecessors in CFG

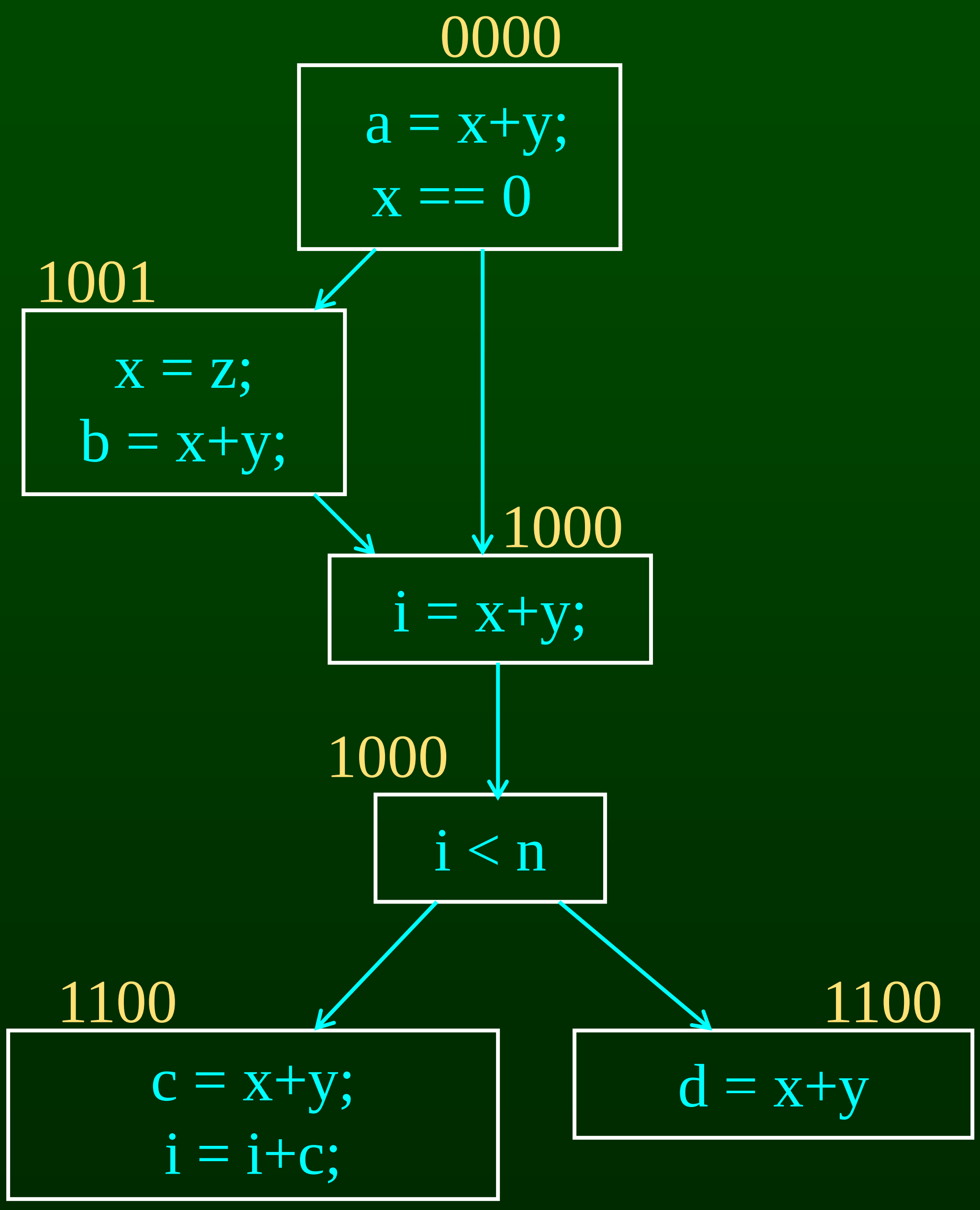
Expressions

1: x+y

2: i<n

3: i+c

4: x==0



Global CSE Transform

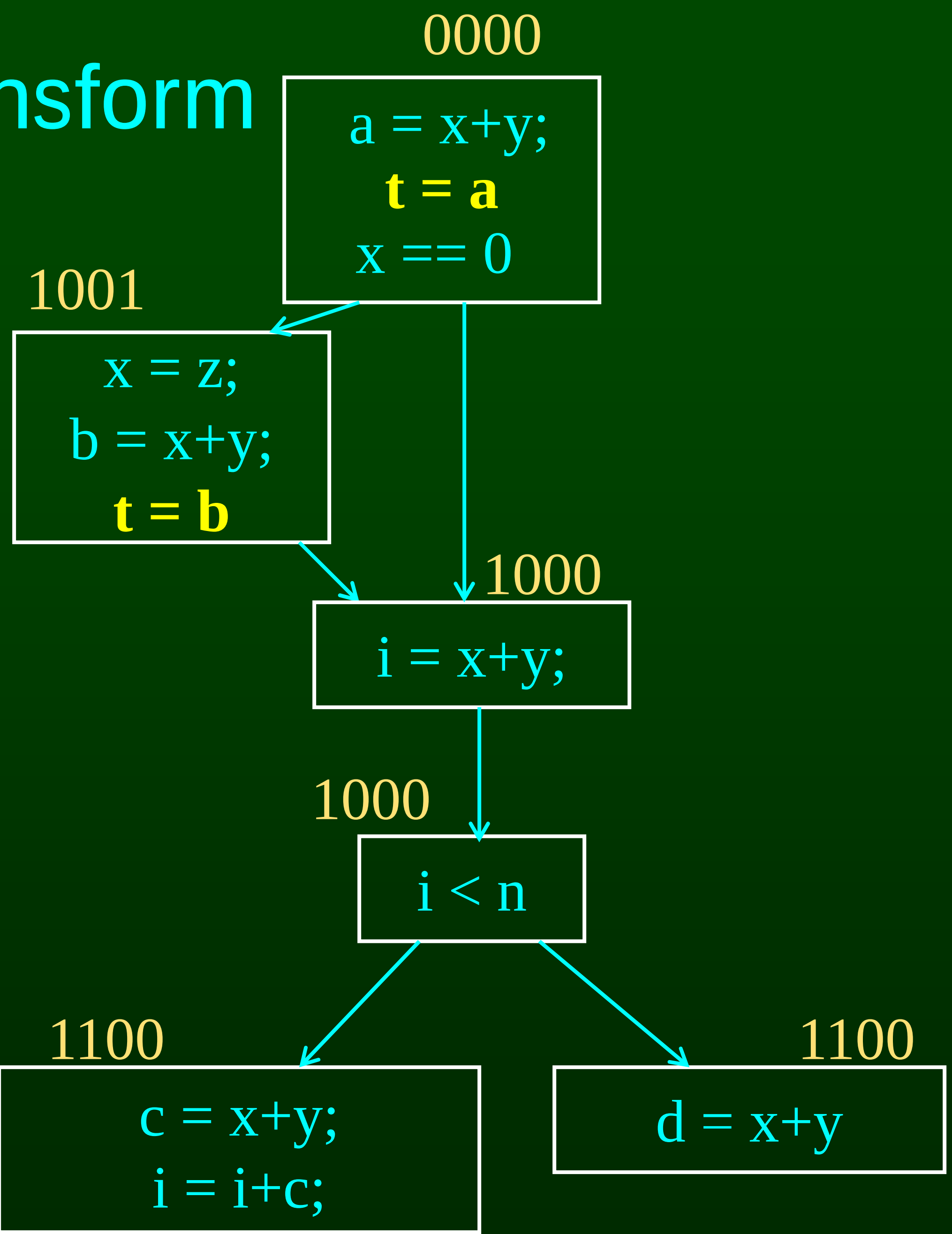
Expressions

1: x+y

2: i<n

3: i+c

4: x==0



must use same temp
for CSE in all blocks

Global CSE Transform

Expressions
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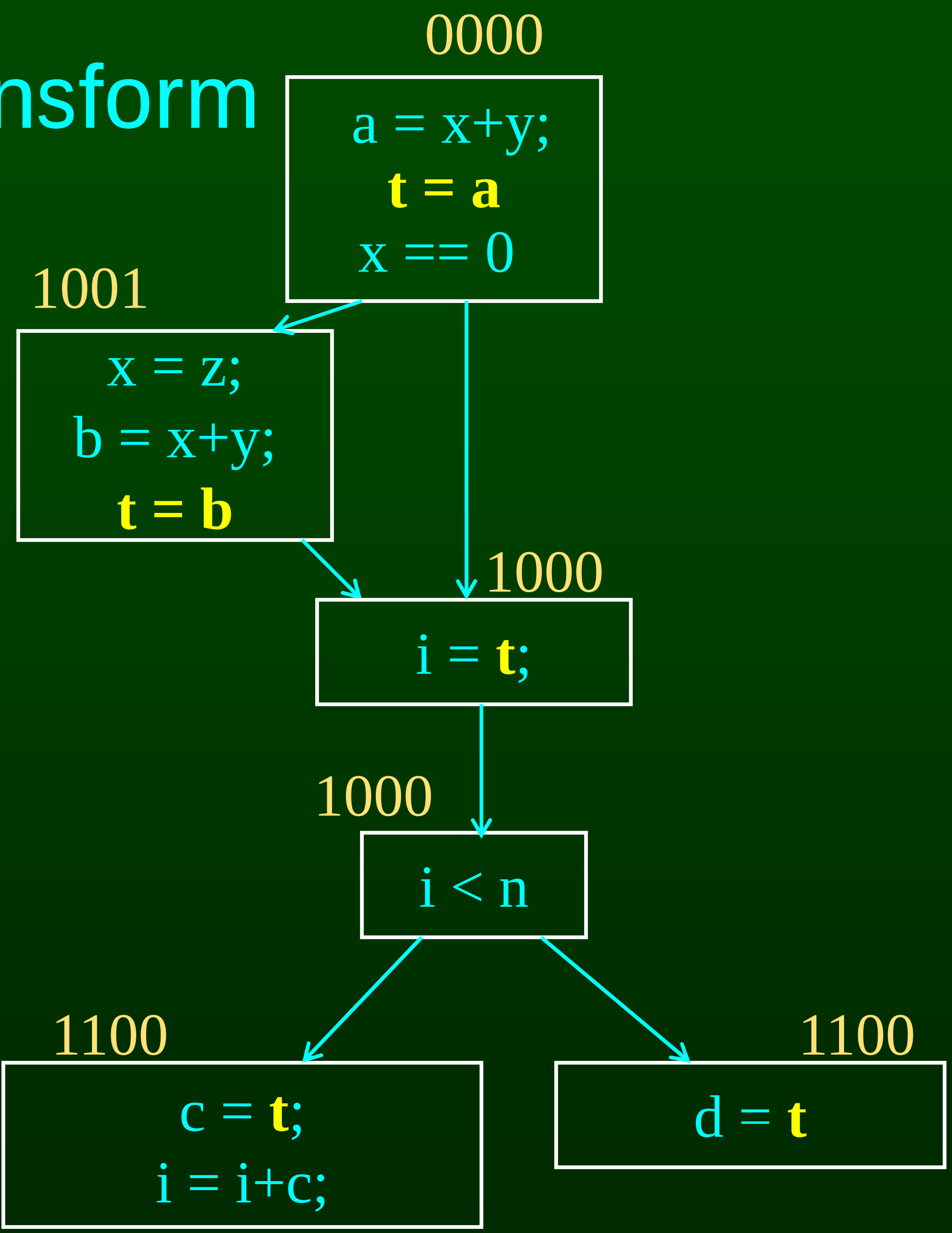


must use same temp
for CSE in all blocks

Formalizing Analysis

- Each basic block has
  - IN set of expressions available at start of block
  - OUT - set of expressions available at end of block
  - GEN - set of expressions computed in block
  - KILL - set of expressions killed in in block
- GEN[x = z; b = x+y] = 1000
- KILL[x = z; b = x+y] = 1001
- Compiler scans each basic block to derive GEN and KILL sets

Dataflow Equations

- $IN[b] = OUT[b1] \cap \ldots \cap OUT[bn]$
  – where b1, ..., bn are predecessors of b in CFG
- $OUT[b] = (IN[b] - KILL[b]) \cup GEN[b]$
- $IN[entry] = 0000$
- Result: system of equations

Solving Equations

- Use fixed point algorithm
- IN[entry] = 0000
- Initialize OUT[b] = 1111
- Repeatedly apply equations
  - IN[b] = OUT[b1] $\cap$ ... $\cap$ OUT[bn]
  - OUT[b] = (IN[b] - KILL[b]) U GEN[b]
- Use a worklist algorithm to reach fixed point

Available Expressions Algorithm

```
for all nodes n in N
    OUT[n] = E;  // OUT[n] = E - KILL[n];
IN[Entry] = emptyset;
OUT[Entry] = GEN[Entry];
Changed = N - { Entry }; // N = all nodes in graph

while (Changed != emptyset)
    choose a node n in Changed;
    Changed = Changed - { n };

    IN[n] = E; // E is set of all expressions
    for all nodes p in predecessors(n)
        IN[n] = IN[n] ∩ OUT[p];

    OUT[n] = GEN[n] U (IN[n] - KILL[n]);

    if (OUT[n] changed)
        for all nodes s in successors(n)
            Changed = Changed U { s };
```

Questions

- Does algorithm always halt?
- If expression is available in some execution, is it always marked as available in analysis?
- If expression is not available in some execution, can it be marked as available in analysis?

General Correctness

- Concept in actual program execution
  - Reaching definition: definition D, execution E at program point P
  - Available expression: expression X, execution E at program point P
- Analysis reasons about all possible executions
- For all executions E at program point P,
  - if a definition D reaches P in E
  - then D is in the set of reaching definitions at P from analysis
- Other way around
  - if D is not in the set of reaching definitions at P from analysis
  - then D never reaches P in any execution E
- For all executions E at program point P,
  - if an expression X is in set of available expressions at P from analysis
  - then X is available in E at P
- Concept of being conservative

Duality In Two Algorithms

- Reaching definitions
  – Confluence operation is set union
  – OUT[b] initialized to empty set
- Available expressions
  – Confluence operation is set intersection
  – OUT[b] initialized to set of available expressions
- General framework for dataflow algorithms.
- Build parameterized dataflow analyzer once, use for all dataflow problems

Outline

- Reaching Definitions
- Available Expressions
- **Liveness**

Liveness Analysis

- A variable v is live at point p if
  - v is used along some path starting at p, and
  - no definition of v along the path before the use.
- When is a variable v dead at point p?
  - No use of v on any path from p to exit node, or
  - If all paths from p redefine v before using v.

What Use is Liveness Information?

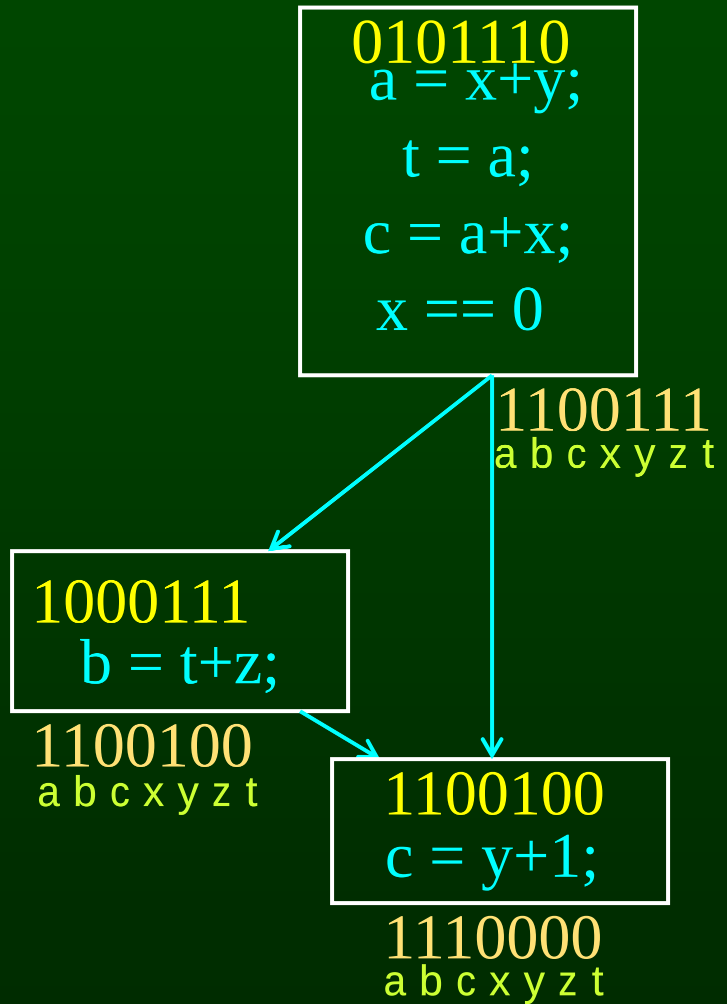
- Register allocation.
  – If a variable is dead, can reassign its register
- Dead code elimination.
  – Eliminate assignments to variables not read later.
  – But must not eliminate last assignment to variable (such as instance variable) visible outside CFG.
  – Can eliminate other dead assignments.
  – Handle by making all externally visible variables live on exit from CFG

Conceptual Idea of Analysis

- Simulate execution
- But start from exit and go backwards in CFG
- Compute liveness information from end to beginning of basic blocks

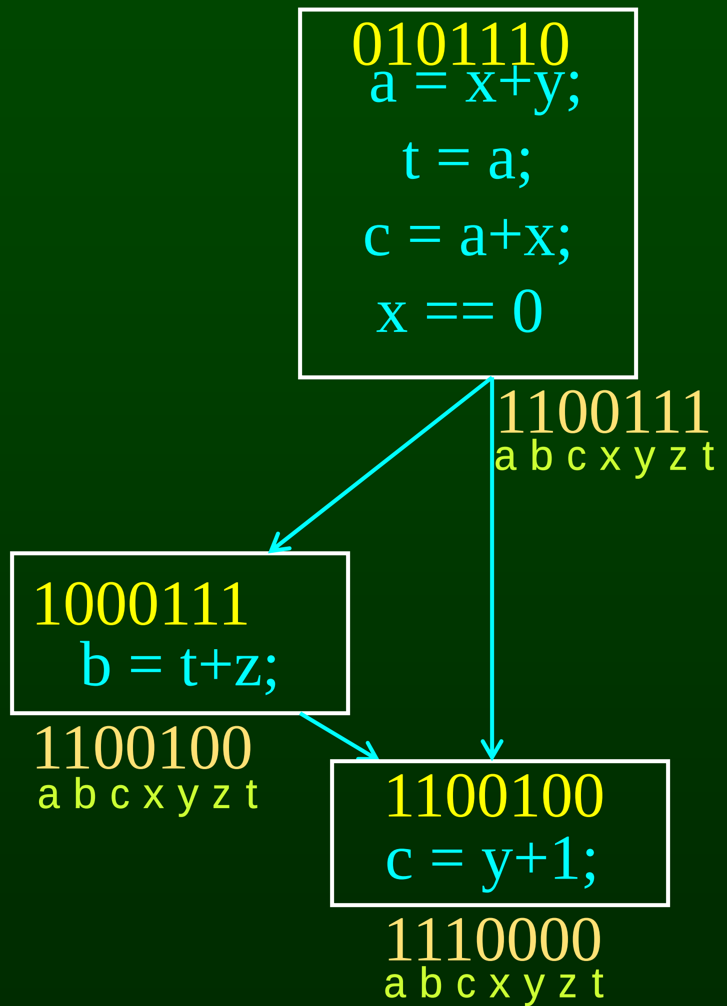
Liveness Example

- Assume a,b,c visible outside method
- So are live on exit
- Assume x,y,z,t not visible
- Represent Liveness Using Bit Vector
  - order is abcxyzt



Dead Code Elimination

- Assume a,b,c visible outside method
- So are live on exit
- Assume x,y,z,t not visible
- Represent Liveness Using Bit Vector
  - order is abcxyzt



Formalizing Analysis

- Each basic block has
    - IN set of variables live at start of block
    - OUT - set of variables live at end of block
    - USE - set of variables with upwards exposed uses in block
    - DEF - set of variables defined in block
- USE[x = z; x = x+1;] = { z } (x not in USE)
- DEF[x = z; x = x+1;y = 1;] = {x, y}
- Compiler scans each basic block to derive USE and DEF sets

Algorithm

```
for all nodes n in N - { Exit }
    IN[n] = emptyset;
OUT[Exit] = emptyset;
IN[Exit] = use[Exit];
Changed = N - { Exit };

while (Changed != emptyset)
    choose a node n in Changed;
    Changed = Changed - { n };

    OUT[n] = emptyset;
    for all nodes s in successors(n)
        OUT[n] = OUT[n] U IN[p];

    IN[n] – use[n] U (out[n]   def[n]);

    if (IN[n] changed)
        for all nodes p in predecessors(n)
            Changed = Changed U { p };
```

Similar to Other Dataflow Algorithms

- Backwards analysis, not forwards
- Still have transfer functions
- Still have confluence operators
- Can generalize framework to work for both forwards and backwards analyses

Comparison

Reaching Definitions

```
for all nodes n in N
    OUT[n] = emptyset;
IN[Entry] = emptyset;
OUT[Entry] = GEN[Entry];
Changed = N - { Entry };

while (Changed != emptyset)
    choose a node n in Changed;
    Changed − Changed   { n };

    IN[n] = emptyset;
    for all nodes p in predecessors(n)
        IN[n] = IN[n] U OUT[p];

    OUT[n] = GEN[n] U (IN[n] - KILL[n]);

    if (OUT[n] changed)
        for all nodes s in successors(n)
            Changed = Changed U { s };
```

Available Expressions

```
for all nodes n in N
    OUT[n] = E;
IN[Entry] = emptyset;
OUT[Entry] = GEN[Entry];
Changed = N - { Entry };

while (Changed != emptyset)
    choose a node n in Changed;
    Changed − Changed   { n };

    IN[n] = E;
    for all nodes p in predecessors(n)
        IN[n] = IN[n] ∩ OUT[p];

    OUT[n] = GEN[n] U (IN[n] - KILL[n]);

    if (OUT[n] changed)
            for all nodes s in successors(n)
            Changed = Changed U { s };
```

Liveness

```
for all nodes n in N - { Exit }
    IN[n] = emptyset;
OUT[Exit] = emptyset;
IN[Exit] = use[Exit];
Changed = N - { Exit };

while (Changed != emptyset)
    choose a node n in Changed;
    Changed − Changed   { n };

    OUT[n] = emptyset;
    for all nodes s in successors(n)
        OUT[n] = OUT[n] U IN[p];

    IN[n] = use[n] U (out[n] - def[n]);

    if (IN[n] changed)
        for all nodes p in predecessors(n)
            Changed = Changed U { p };
```

Comparison

Reaching Definitions

```
for all nodes n in N
    OUT[n] = emptyset;
IN[Entry] = emptyset;
OUT[Entry] = GEN[Entry];
Changed = N - { Entry };

while (Changed != emptyset)
    choose a node n in Changed;
    Changed − Changed   { n };

    IN[n] = emptyset;
    for all nodes p in predecessors(n)
        IN[n] = IN[n] U OUT[p];

    OUT[n] = GEN[n] U (IN[n] - KILL[n]);

    if (OUT[n] changed)
        for all nodes s in successors(n)
            Changed = Changed U { s };
```

Available Expressions

```
for all nodes n in N
    OUT[n] = E;
IN[Entry] = emptyset;
OUT[Entry] = GEN[Entry];
Changed = N - { Entry };

while (Changed != emptyset)
    choose a node n in Changed;
    Changed − Changed   { n };

    IN[n] = E;
    for all nodes p in predecessors(n)
        IN[n] = IN[n] ∩ OUT[p];

    OUT[n] = GEN[n] U (IN[n] - KILL[n]);

    if (OUT[n] changed)
            for all nodes s in successors(n)
            Changed = Changed U { s };
```

Comparison

Reaching Definitions

```
for all nodes n in N
    OUT[n] = emptyset;
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OUT[Entry] = GEN[Entry];
Changed = N - { Entry };

while (Changed != emptyset)
    choose a node n in Changed;
    Changed − Changed   { n };

    IN[n] = emptyset;
    for all nodes p in predecessors(n)
        IN[n] = IN[n] U OUT[p];

    OUT[n] = GEN[n] U (IN[n] - KILL[n]);

    if (OUT[n] changed)
        for all nodes s in successors(n)
            Changed = Changed U { s };
```

Liveness

```
for all nodes n in N
    IN[n] = emptyset;
OUT[Exit] = emptyset;
IN[Exit] = use[Exit];
Changed = N - { Exit };

while (Changed != emptyset)
    choose a node n in Changed;
    Changed − Changed   { n };

    OUT[n] = emptyset;
    for all nodes s in successors(n)
        OUT[n] = OUT[n] U IN[p];

    IN[n] = use[n] U (out[n] - def[n]);

    if (IN[n] changed)
        for all nodes p in predecessors(n)
            Changed = Changed U { p };
```

Analysis Information Inside Basic Blocks

- One detail:
  - Given dataflow information at IN and OUT of node
  - Also need to compute information at each statement of basic block
  - Simple propagation algorithm usually works fine
  - Can be viewed as restricted case of dataflow analysis

Pessimistic vs. Optimistic Analyses

- Available expressions is optimistic
  (for common sub-expression elimination)
  - Assume expressions are available at start of analysis
  - Analysis eliminates all that are not available
  - Cannot stop analysis early and use current result
- Live variables is pessimistic (for dead code elimination)
  - Assume all variables are live at start of analysis
  - Analysis finds variables that are dead
  - Can stop analysis early and use current result
- Dataflow setup same for both analyses
- Optimism/pessimism depends on intended use

Summary

- Basic Blocks and Basic Block Optimizations
  - Copy and constant propagation
  - Common sub-expression elimination
  - Dead code elimination
- Dataflow Analysis
  - Control flow graph
  - IN[b], OUT[b], transfer functions, join points
- Paired analyses and transformations
  - Reaching definitions /constant propagation
  - Available expressions/common sub-expression elimination
  - Liveness analysis/Dead code elimination
- Stacked analysis and transformations work together