# 6.041/6.431 Fall 2010 Quiz 2
# Tuesday, November 2, 7:30 - 9:30 PM.

# DO NOT TURN THIS PAGE OVER UNTIL YOU ARE TOLD TO DO SO

**Name:** ____________________

**Recitation Instructor:** ____________________

**TA:** ____________________

| **Question** | Score | Out of |
|---|---|---|
| **1.1** | | 10 |
| **1.2** | | 10 |
| **1.3** | | 10 |
| **1.4** | | 10 |
| **1.5** | | 10 |
| **1.6** | | 10 |
| **1.7** | | 10 |
| **1.8** | | 10 |
| **2.1** | | 10 |
| **2.2** | | 10 |
| **2.3** | | 5 |
| **2.4** | | 5 |
| **Your Grade** | | 110 |

- **For full credit, answers should be algebraic expressions (no integrals), in simplified form. These expressions may involve constants such as $\pi$ or $e$, and need not be evaluated numerically.**
- This quiz has 2 problems, worth a total of 110 points.
- You may tear apart page 3, as per your convenience, **but you must turn them in together with the rest of the booklet.**
- Write your solutions in this quiz booklet, only solutions in this quiz booklet will be graded. Be neat! You will not get credit if we can't read it.
- You are allowed two two-sided, handwritten, 8.5 by 11 formula sheets. Calculators are not allowed.
- You have 120 minutes to complete the quiz.
- Graded quizzes will be returned in recitation on Thursday 11/4.

**Problem 1. (80 points)** In this problem:
(i) $X$ is a (continuous) uniform random variable on $[0, 4]$.
(ii) $Y$ is an exponential random variable, independent from $X$, with parameter $\lambda = 2$.

1. **(10 points)** Find the mean and variance of $X - 3Y$.
2. **(10 points)** Find the probability that $Y \geq X$.
   (Let $c$ be the answer to this question.)
3. **(10 points)** Find the conditional joint PDF of $X$ and $Y$, given that the event $Y \geq X$ has occurred.
   (You may express your answer in terms of the constant $c$ from the previous part.)
4. **(10 points)** Find the PDF of $Z = X + Y$.
5. **(10 points)** Provide a fully labeled sketch of the conditional PDF of $Z$ given that $Y = 3$.
6. **(10 points)** Find $\mathbf{E}[Z \mid Y = y]$ and $\mathbf{E}[Z \mid Y]$.
7. **(10 points)** Find the joint PDF $f_{Z,Y}$ of $Z$ and $Y$.
8. **(10 points)** A random variable $W$ is defined as follows. We toss a fair coin (independent of $Y$). If the result is "heads", we let $W = Y$; if it is tails, we let $W = 2 + Y$. Find the probability of "heads" given that $W = 3$.

**Problem 2. (30 points)** Let $X, X_1, X_2, \ldots$ be independent normal random variables with mean 0 and variance 9. Let $N$ be a positive integer random variable with $\mathbf{E}[N] = 2$ and $\mathbf{E}[N^2] = 5$. We assume that the random variables $N, X, X_1, X_2, \ldots$ are independent. Let $S = \sum_{i=1}^{N} X_i$.

1. **(10 points)** If $\delta$ is a small positive number, we have $\mathbf{P}(1 \leq |X| \leq 1 + \delta) \approx \alpha\delta$, for some constant $\alpha$. Find the value of $\alpha$.
2. **(10 points)** Find the variance of $S$.
3. **(5 points)** Are $N$ and $S$ uncorrelated? Justify your answer.
4. **(5 points)** Are $N$ and $S$ independent? Justify your answer.

Each question is repeated in the following pages. Please write your answer on the appropriate page.

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6.041 / 6.431 Probabilistic Systems Analysis and Applied Probability
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