Tutorial 3
September 30/October 1, 2010

1. Let X and Y be independent random variables. Random variable X has mean μ_X and variance σ_X^2, and random variable Y has mean μ_Y and variance σ_Y^2. Let $Z = 2X - 3Y$. Find the mean and variance of Z in terms of the means and variances of X and Y.

2. Problem 2.40, page 133 in the text.
   A particular professor is known for his arbitrary grading policies. Each paper receives a grade from the set $\{A, A-, B+, B, B-, C+\}$, with equal probability, independently of other papers. How many papers do you expect to hand in before you receive each possible grade at least once?

3. The joint PMF of the random variables X and Y is given by the following table:

$y = 3$	c	c	$2c$
$y = 2$	$2c$	0	$4c$
$y = 1$	$3c$	c	$6c$
	$x = 1$	$x = 2$	$x = 3$

   (a) Find the value of the constant c.

   (b) Find $p_Y(2)$.

   (c) Consider the random variable $Z = YX^2$. Find $\mathbf{E}[Z \mid Y = 2]$.

   (d) Conditioned on the event that $X \neq 2$, are X and Y independent? Give a one-line justification.

   (e) Find the conditional variance of Y given that $X = 2$.

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