MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Department of Electrical Engineering & Computer Science
6.041/6.431: Probabilistic Systems Analysis
(Spring 2006)

Recitation 12
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1. Widgets are packed into cartons which are packed into crates. The weight (in pounds) of a widget, X, is a continuous random variable with PDF

$$f_X(x) = \lambda e^{-\lambda x}, \quad x \geq 0 \qquad .$$

The number of widgets in any carton, K, is a random variable with PMF

$$p_K(k) = \frac{\mu^k e^{-\mu}}{k!}, \quad k = 0, 1, 2, \ldots \qquad .$$

The number of cartons in a crate, N, is a random variable with PMF

$$p_N(n) = p^{n-1}(1-p), \quad n = 1, 2, 3, \ldots \quad .$$

Random variables X, K, and N are mutually independent. Determine

(a) The probability that a randomly selected crate contains exactly one widget.

(b) The expected value and variance of the number of widgets in a crate.

(c) The transform or the PDF for the total weight of the widgets in a crate.

(d) The expected value and variance of the total weight of the widgets in a crate.

2. Using a fair three-sided die (construct one, if you dare), we will decide how many times to spin a fair wheel of fortune. The wheel of fortune is calibrated infinitely finely and has numbers between 0 and 1. The die has the numbers 1,2 and 3 on its faces. Whichever number results from our throw of the die, we will spin the wheel of fortune that many times and add the results to obtain random variable Y.

(a) Determine the expected value of Y.

(b) Determine the variance of Y.