MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Department of Electrical Engineering & Computer Science
6.041/6.431: Probabilistic Systems Analysis
(Spring 2006)

Tutorial 1
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1. In a new version of "Who wants to eat an ice-cream sundae", a student is told that there is a single gift certificate for a sundae in one of 5 envelopes, say envelope 1, 2, 3, 4 and 5. The student picks an envelope, say i, but does not open it. The professor then opens another envelope, say j where $j \neq i$, which the professor knows for sure does *not* contain the gift certificate.

   (a) Assuming that a certificate was equally likely to be in any envelope initially, what is the probability, after the professor showed an empty envelope, that a certificate is in envelope k, where $k = i$ and $k \neq j$ (i.e. the certificate is in the envelope that the student originally picked)?

   (b) Repeat part a), for $k \neq i$ and $k \neq j$ (i.e. the certificate is in envelope k which the student did not pick and the professor did not open.)

   (c) Repeat a) and b), now assuming that there are gift certificates in *two* out of the five envelopes.

   (d) The game in a) is now changed in the following way. The student first picks an envelope and does not open it. Instead of then having the professor show the student an envelope without the certificate in it, the student gets to pick another envelope and open it. If a certificate is in this latter envelope, the student gets to keep that certificate. Otherwise, he gets to open the envelope he/she chose originally. What is the probability that the student finds a certificate by the end of the game?

   (e) Repeat d) when there are originally *two* out of the five envelopes each with a gift certificate in them.

2. A newly formed secret society on campus, with 1% student body membership, decides that it must have its own secret grip, so members may easily tell each other apart from non-members. Unfortunately, the grip they choose is so complicated, that a member has a 12% chance of thinking another member is an impostor, but a 93 % chance of recognizing a non-member to be an impostor. Given that after administering the grip, a member decides someone is also a member, find the probability that that person is in fact a real member.

3. Three persons roll a fair n-sided die once. Let A_{ij} be the event that person i and person j roll the same face. Show that the events A_{12}, A_{13}, and A_{23} are pairwise independent but are not independent.