6.041SC Probabilistic Systems Analysis and Applied Probability, Fall 2013 Transcript – Recitation: Calculating a Cumulative Distribution Function (CDF)

Hi. In this problem, we'll get some practice working with PDFs and also using PDFs to calculate CDFs. So the PDF that we're given in this problem is here. So we have a random variable, z, which is a continuous random variable. And we're told that the PDF of this random variable, z, is given by gamma times 1 plus z squared in the range of z between negative 2 and 1. And outside of this range, it's 0.

All right, so first thing we need to do and the first part of this problem is we need to figure out what gamma is because it's not really a fully specified PDF yet. We need to figure out exactly what the value gamma is. And how do we do that? Well, we've done analogous things before for the discrete case.

So the tool that we use is that the PDF must integrate to 1. So in the discrete case, the analogy was that the PMF had to sum to 1. So what do we know? We know that when you integrate this PDF from negative infinity to infinity, fz of z, it has to equal 1.

All right, so what do we do now? Well, we know what the PDF is-- partially, except for gamma-- so let's plug that in. And the first thing that we'll do is we'll simplify this because we know that the PDF is actually only non-zero in the range negative 2 to 1. So instead of integrating from negative infinity to infinity, we'll just integrate from negative 2 to 1.

And now let's plug in this gamma times 1 plus z squared dc. And now the rest of the problem is just applying calculus and integrating this. So let's just go through that process.

So we get z plus 1/3 z cubed from minus 2 to 1. And now we'll plug in the limits. And we get gamma, and that's 1 plus 1/3 minus minus 2 plus 1/3 times minus 2 cubed.

And then if we add this all up, you get 4/3 plus 2 plus 8/3, which will give you 6. So what we end up with in the end is that 1 is equal to 6 gamma. So what does that tell us? That tells us that, in this case, gamma is 1/6.

OK, so we've actually figured out what this PDF really is. And let's just substitute that in. So we know what gamma is. So it's 1/6.

So from this PDF, we can calculate anything that we want to. This PDF, basically, fully specifies everything that we need to know about this random variable, z. And one of the things that we can calculate from the PDF is the CDF. So the next part of the problem asks us to calculate the CDF.

So remember the CDF, we use capital F. And the definition is that you integrate from negative infinity to this z. And what do you integrate? You integrate the PDF. And all use some dummy variable, y, here in the integration.

So what is it really doing? It's basically just taking the PDF and taking everything to the left of it. So another way to think about this-- this is the probability that the random variable is less than or equal to some little z. It's just accumulating probability as you go from left to right.

So the hardest part about calculating the CDFs, really, is actually just keeping track of the ranges, because unless the PDF is really simple, you'll have cases where the PDF cold be 0 in some ranges and non-zero in other ranges. And then what you really have to keep track of is where those ranges are and where you actually have non-zero probability.

So in this case, we actually break things down into three different ranges because this PDF actually looks something like this. So it's non-zero between negative 2 and 1, and it's 0 everywhere else. So then what that means is that our job is a little simpler because everything to the left of negative 2, the CDF will be 0 because there's no probability density to the left.

And then everything to the right of 1, well we've accumulated all the probability in the PDF because we know that when you integrate from negative 2 to 1, you capture everything. So anything to the right of 1, the CDF will be 1. So the only hard part is calculating what the CDF is in this intermediate range, between negative 2 and 1.

So let's do that case first-- so the case of z is between negative 2 and 1. So what is the CDF in that case? Well, the definition is to integrate from negative infinity to z.

But we know that everything to the left of negative 2, there's no probably density. So we don't need to include that. So we can actually change this lower limit to negative 2. And the upper limit is wherever this z is.

So that becomes our integral. And the inside is still the PDF. So let's just plug that in. We know that it's 1/6 1 plus-- we'll make this y squared-- by. And now it's just calculus again. And in fact, it's more or less the same integral, so what we get is y plus 1/3 y cubed from negative 2 to z.

Notice the only thing that's different here is that we're integrating from negative 2 to z instead of negative 2 to 1. And when we calculate this out, what we get is z plus 1/3 z cubed minus minus 2 plus 1/3 minus 2 cubed, which gives us 1/6 z plus 1/3 z cubed plus plus 2 plus 8/3 gives us 14/3. So that actually is our CDF between the range of negative 2 to 1.

So for full completeness, let's actually write out the entire CDF, because there's two other parts in the CDF. So the first part is that it's 0 if z is less than negative 2. And it's 1 if z is greater than 1.

And in between, it's this thing that we've just calculated. So it's 1/6 z plus 1/3 z cubed plus 14/3 if z is between minus 2 and 1. So that is our final answer.

So the main point of this problem was to drill a little bit more the concepts of PDFs and CDFs. So for the PDF, the important thing to remember is that in order to be a valid PDF, the PDF has to integrate to 1. And you can use that fact to help you calculate any unknown constants in the PDF.

And then to calculate the CDF, it's just integrating the PDF from negative infinity to whatever point that you want to cut off at. And the tricky part, as I said earlier, was really just keeping track of the ranges. In this case, we've broke it down into three ranges. If we had a slightly more complicated PDF, then you would have to keep track of even more ranged. All right, so I hope that was helpful, and we'll see you next time.

6.041SC Probabilistic Systems Analysis and Applied Probability
Fall 2013