Mathematics for Computer Science
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Independent Events

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Definition 1:

Events A and B are independent iff

$$\Pr[A] = \Pr[A \mid B]$$

Definition 2:

Events A and B are independent iff

$$\Pr[A] \cdot \Pr[B] = \Pr[A \cap B]$$

Definitions of Independence

proof of equivalence:

$\Pr[A] = \Pr[A \mid B]$ iff

$\Pr[A] = \dfrac{\Pr[A \cap B]}{\Pr[B]}$ iff

$\Pr[A] \cdot \Pr[B] = \Pr[A \cap B]$

Definitions of Independence

need $\Pr[B] \neq 0$ for Def. 1.

Def. 2 always works:

$$\boxed{\Pr[A\} \cdot \Pr[B] = \Pr[A \cap B]}$$

Independence

$\Pr[A]\cdot\Pr[B] = \Pr[A\cap B]$

symmetric in A and B so,

A independent of B iff

B independent of A

Independence

Corollary: If $\Pr[B]= 0$, then B is independent of every event

Independence

Corollary: If $\Pr[B]= 0$, then B is independent of every event – even itself.

Independence

A independent of B means

Independence

A independent of B
means A is independent of
whether or not B occurs:

Independence

Lemma:

A independent of B iff
A independent of $\overline{B}$.

Independence

Lemma:

A independent of B iff
A independent of $\overline{B}$

Simple proof using:

$\Pr[A-B] = \Pr[A]-\Pr[A\cap B]$