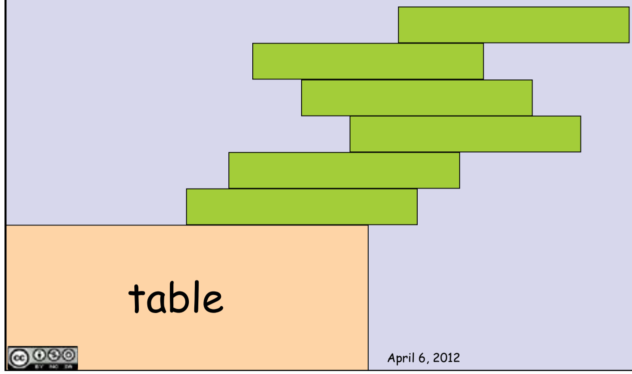
Mathematics for Computer Science
MIT 6.042J/18.062J

Book Stacking

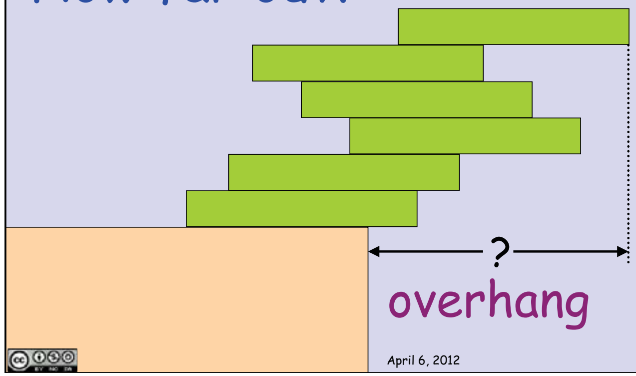
Harmonic Sums

Book Stacking



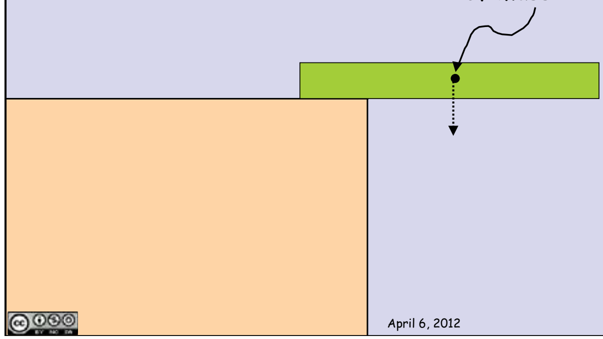
Book Stacking

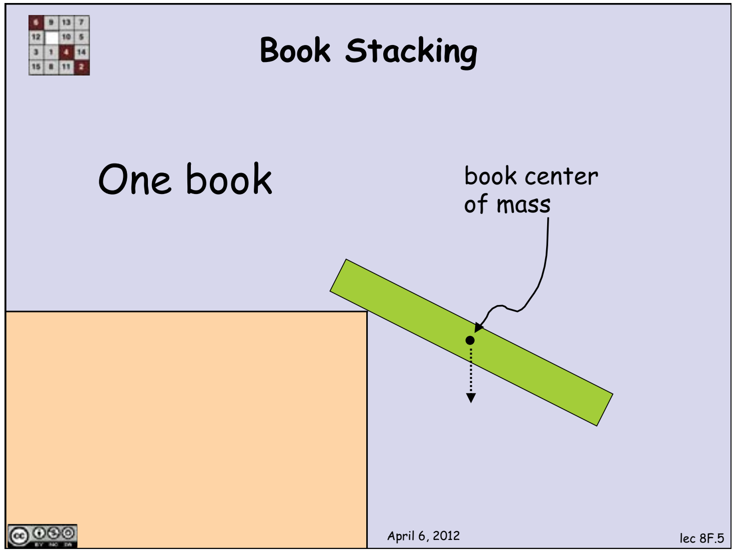
How far out?

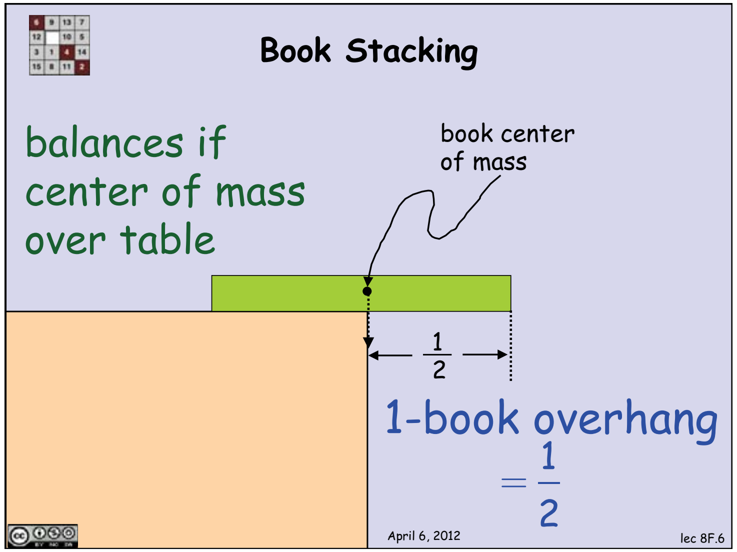


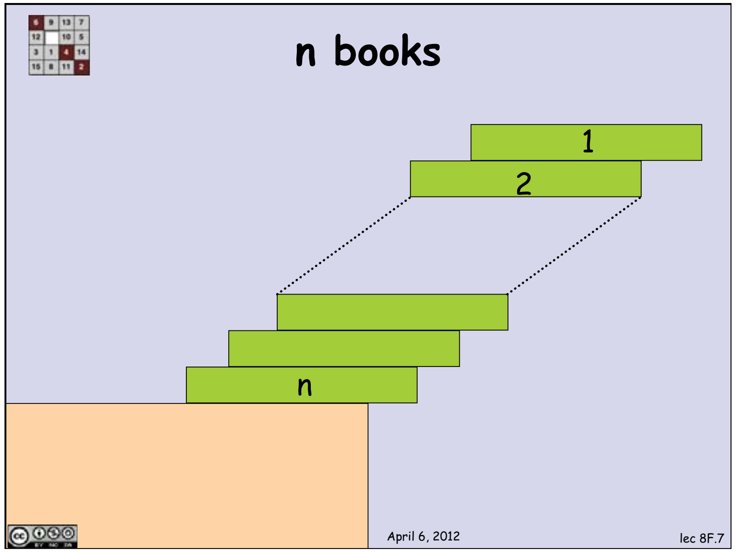
Book Stacking

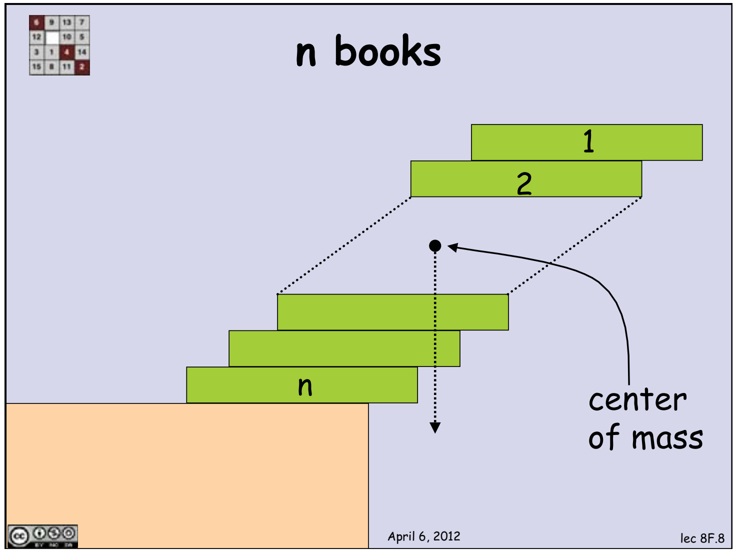
One book



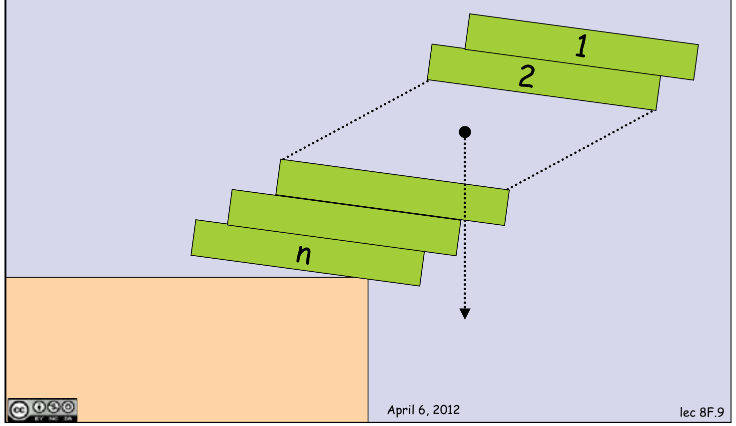

Book Stacking
One book
book center
of mass
April 6, 2012
lec 8F.5


Book Stacking
balances if
center of mass
over table
book center
of mass
1/2
1-book overhang
= 1/2
April 6, 2012
lec 8F.6

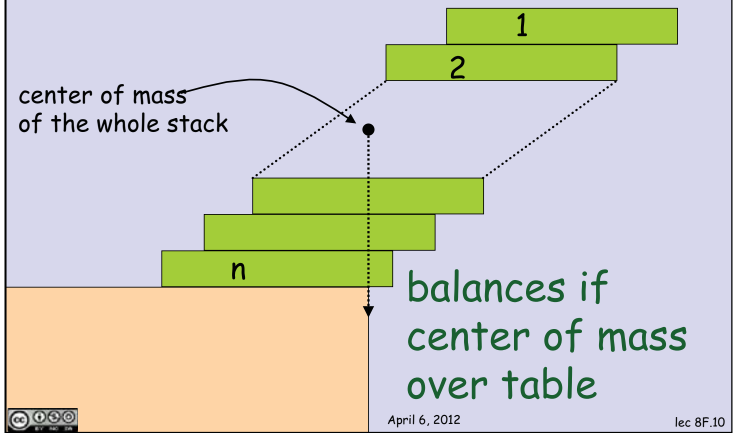

n books
1
2
n
April 6, 2012
lec 8F.7


n books
1
2
n
center
of mass
April 6, 2012
lec 8F.8

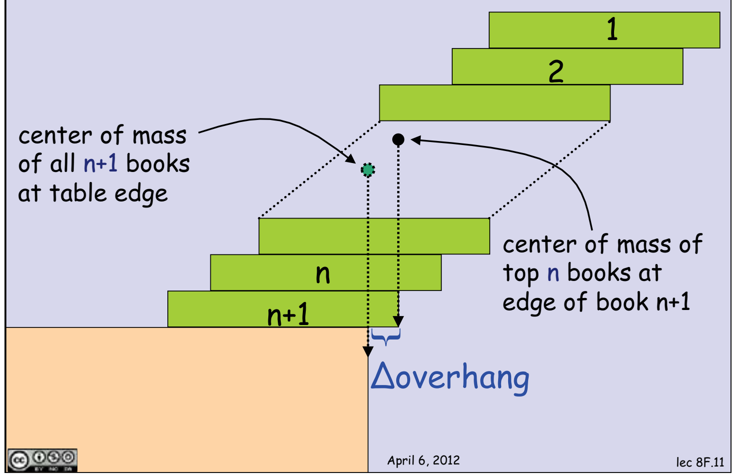
n books



n books

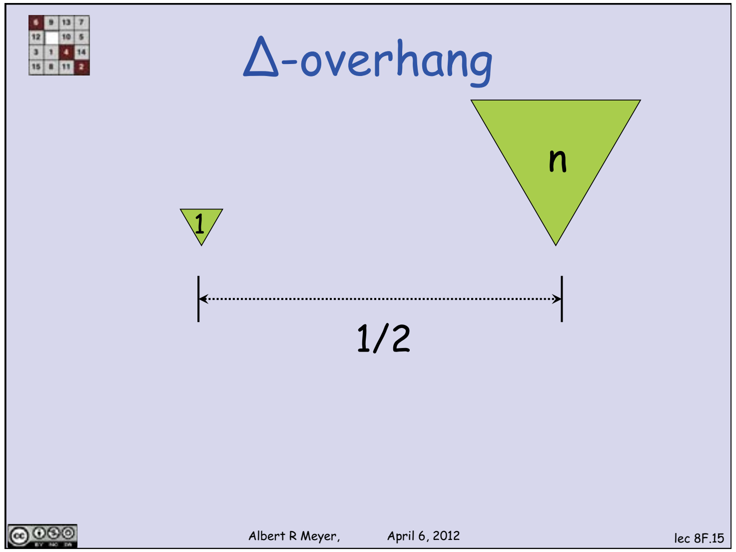


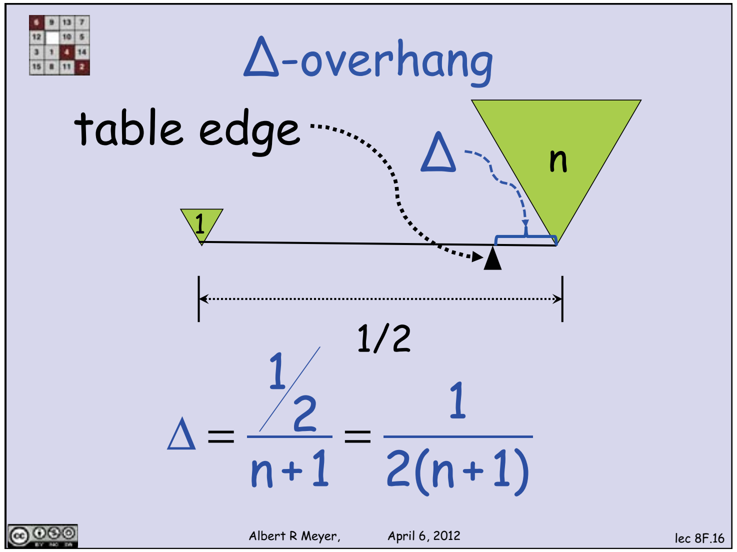
n+1 books

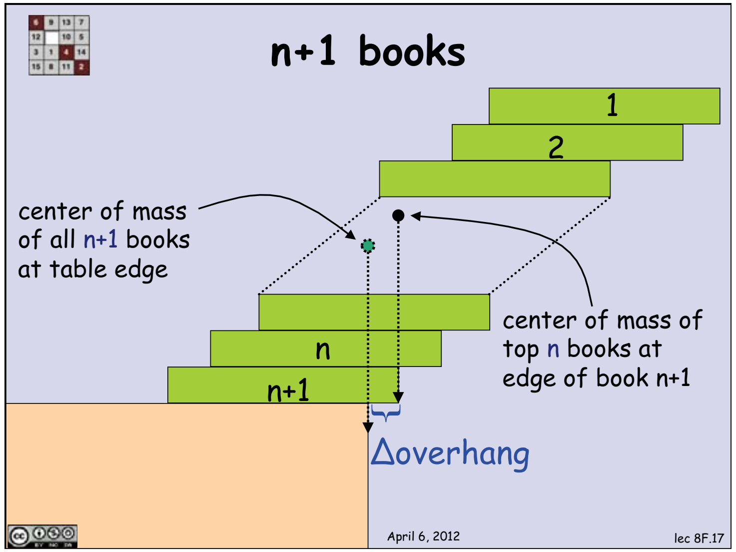


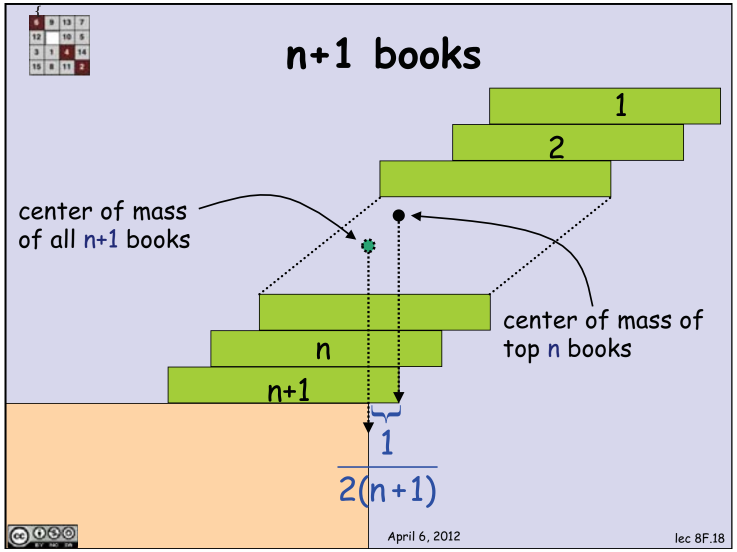
Δ-overhang

Δ-overhang ::=
horizontal distance from
n-book to (n+1)-book
centers of mass


Δ-overhang
n
1
1/2
Albert R Meyer, April 6, 2012
lec 8F.15


Δ-overhang
table edge
Δ
n
1
1/2
$\Delta = \frac{1/2}{n+1} = \frac{1}{2(n+1)}$
Albert R Meyer, April 6, 2012
lec 8F.16


n+1 books
1
2
center of mass of all n+1 books at table edge
center of mass of top n books at edge of book n+1
n
n+1
Δoverhang
April 6, 2012
lec 8F.17


n+1 books
1
2
center of mass of all n+1 books
center of mass of top n books
n
n+1
$\frac{1}{2(n+1)}$
April 6, 2012
lec 8F.18

Book stacking summary

B_n ::= overhang of n books

$B_1 = 1/2$

$$B_{n+1} = B_n + \frac{1}{2(n+1)}$$

$$B_n = \frac{1}{2}\left(1 + \frac{1}{2} + \frac{1}{3} + \cdots + \frac{1}{n}\right)$$

Harmonic Sums

$$H_n ::= 1 + \frac{1}{2} + \frac{1}{3} + \cdots + \frac{1}{n}$$

n^{th} Harmonic number

$$B_n = H_n/2$$