

6	9	13	7
12	10	5	
3	1	4	14
15	8	11	2

*Mathematics for Computer Science*  
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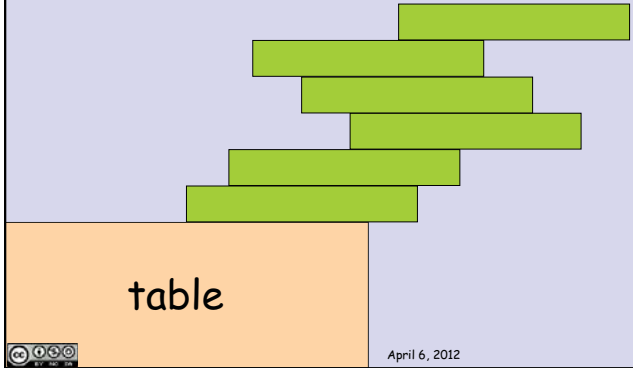
# Book Stacking Harmonic Sums

Albert R Meyer, April 6, 2012

lec 8F.1

6	9	13	7
12	10	5	
3	1	4	14
15	8	11	2

## Book Stacking



table

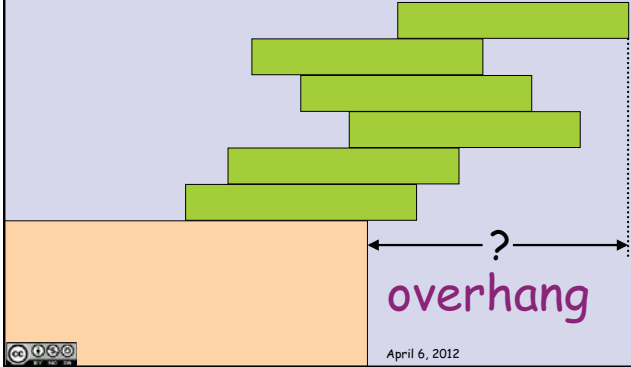
April 6, 2012

lec 8F.2

6	9	13	7
12	10	5	
3	1	4	14
15	8	11	2

## Book Stacking

How far out?



overhang

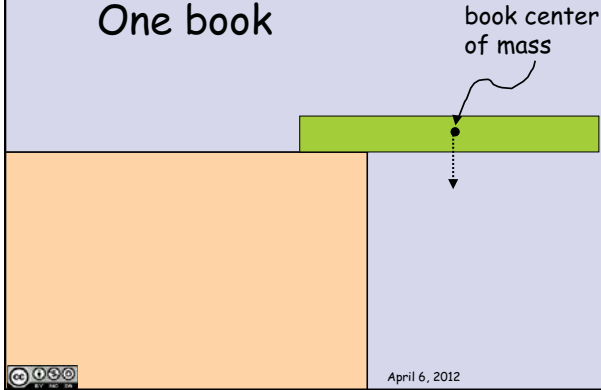
April 6, 2012

lec 8F.3

6	9	13	7
12	10	5	
3	1	4	14
15	8	11	2

## Book Stacking

One book



book center  
of mass

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Book Stacking

One book

book center of mass

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Book Stacking

balances if center of mass over table

book center of mass

$\frac{1}{2}$

1-book overhang =  $\frac{1}{2}$

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n books

1

2

n

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n books

1

2

n

center of mass

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**n books**

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**n books**

center of mass of the whole stack

balances if center of mass over table

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**n+1 books**

center of mass of all  $n+1$  books at table edge

center of mass of top  $n$  books at edge of book  $n+1$

$\Delta$ overhang

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**$\Delta$ -overhang**

$\Delta$ -overhang ::= horizontal distance from  $n$ -book to  $(n+1)$ -book centers of mass

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**Δ-overhang**

1

n

1/2

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**Δ-overhang**

table edge

1

n

1/2

$$\Delta = \frac{1/2}{n+1} = \frac{1}{2(n+1)}$$

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**n+1 books**

center of mass of all n+1 books at table edge

center of mass of top n books at edge of book n+1

n

n+1

1

2

Δoverhang

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**n+1 books**

center of mass of all n+1 books

center of mass of top n books


n

n+1

1

2(n+1)

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## Book stacking summary


$B_n$  ::= overhang of  $n$  books

$B_1 = 1/2$

$$B_{n+1} = B_n + \frac{1}{2(n+1)}$$

$$B_n = \frac{1}{2} \left( 1 + \frac{1}{2} + \frac{1}{3} + \dots + \frac{1}{n} \right)$$



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## Harmonic Sums

$$H_n ::= 1 + \frac{1}{2} + \frac{1}{3} + \dots + \frac{1}{n}$$

$n^{\text{th}}$  Harmonic number

$$B_n = H_n / 2$$


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