Mathematics for Computer Science
MIT 6.042J/18.062J

Finite Cardinality

Mapping Rule (bij)

A bijection from A to B implies

$$|A| = |B|$$

for finite A, B

size of the power set

subsets of a finite set A?

$|pow(A)|$?

for $A = \{a, b, c\}$, $pow(A) =$

$\{ \emptyset, \{a\}, \{b\}, \{c\},$
$\{a,b\}, \{a,c\}, \{b,c\}, \{a,b,c\} \}$

pow(A) bijection to bit-strings

A: $\{a_0, a_1, a_2, a_3, a_4, \ldots, a_{n-1}\}$

subset: $\{a_0, \quad a_2, a_3, \quad \ldots, a_{n-1}\}$

string: 1 0 1 1 0 … 1

this defines a bijection, so

n-bit strings = $|pow(A)|$

pow(A) bijection to bit-strings

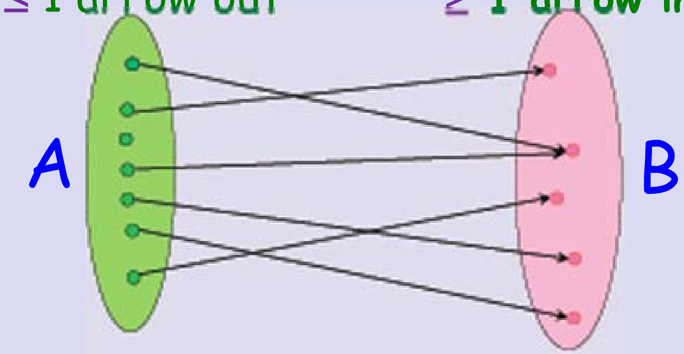
every computer scientist knows #n-bit strings, so

Corollary:

$$|pow(A)| = 2^{|A|}$$

function & surjective

≤ 1 arrow out ≥ 1 arrow in



Mapping Rule (surj)

function: $A \rightarrow B$

Mapping Rule (surj)

[≤ 1 out]: $A \rightarrow B$

IMPLIES $|A| \geq$ #arrows.

surjection: $A \rightarrow B$

Mapping Rule (surj)

$[\leq 1 \text{ out}] : A \to B$

IMPLIES $|A| \geq$ #arrows.

$[\geq 1 \text{ in}] : A \to B$

IMPLIES #arrows $\geq |B|$.

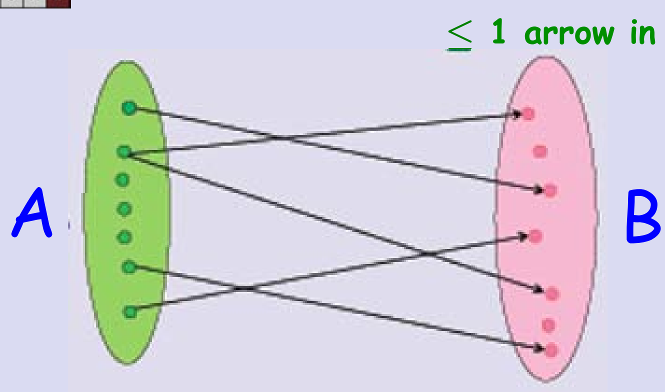
Mapping Rule (surj)

Surjective function from A to B implies

$$|A| \geq |B|$$

for finite A, B

injection archery



Mapping Rule (inj)

total $[\geq 1 \text{ out}]$ IMPLIES

$|A| \leq$ #arrows

injection $[\leq 1 \text{ in}]$ IMPLIES

#arrows $\leq |B|$

Mapping Rule (inj)

Total injective relation from A to B implies

$$|A| \le |B|$$

for finite A, B

"jection" relations

A bij B ::= $\exists$bijection:$A \to B$

A surj B ::= $\exists$surj func:$A \to B$

A inj B ::= $\exists$total inj relation:$A \to B$

Mapping Lemma

A bij B IFF $|A| = |B|$

A surj B IFF $|A| \ge |B|$

A inj B IFF $|A| \le |B|$

for finite A, B

Familiar "size" properties

$|A| = |B| = |C|$ IMPLIES $|A| = |C|$

$|A| \ge |B| \ge |C|$ IMPLIES $|A| \ge |C|$

$|A| \ge |B| \ge |A|$ IMPLIES $|A| = |B|$

for finite A, B, C

Familiar "size" properties

A bij B bij C IMPLIES A bij C

A surj B surj C IMPLIES A surj C

A surj B surj A IMPLIES A bij B

for finite A, B, C

by the Mapping Lemma

Familiar "size" properties

A bij B bij C IMPLIES A bij C

A surj B surj C IMPLIES A surj C

A surj B surj A IMPLIES A bij B

for infinite A, B, C, also

1st two implications: easy

3rd is tricky