

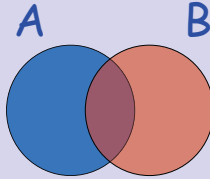

Mathematics for Computer Science
 MIT 6.042J/18.062J


Inclusion-Exclusion


2 set proof

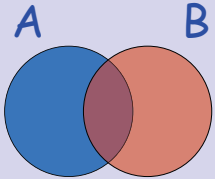

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Inclusion-Exclusion

$$|A \cup B| = |A| + |B| - |A \cap B|$$




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Inc-Exc from Sum Rule




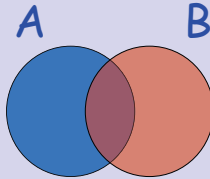
proof:

$$A \cup B = A \cup (B - A)$$


 disjoint


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Inc-Exc from Sum Rule



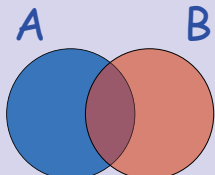
proof:

$$|A \cup B| = |A| + |B - A|$$

by Sum Rule


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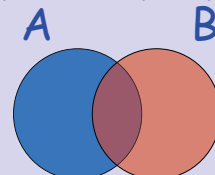
Inc-Exc from Sum Rule



$$|A \cup B| = |A| + \underbrace{|B - A|}_{|B| - |A \cap B|}$$

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Lemma: $|B - A| = |B| - |A \cap B|$



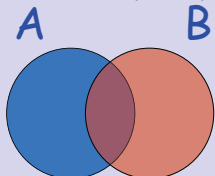
proof:

$$B = (B \cap A) \cup (B - A)$$

↙ ↘
disjoint

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Lemma: $|B - A| = |B| - |A \cap B|$



QED

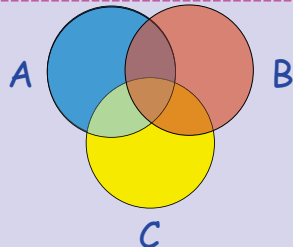
proof:

$$|B| = |B \cap A| + |B - A|$$


by Sum Rule

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Inclusion-Exclusion (3 Sets)


$$|A \cup B \cup C| = |A| + |B| + |C| - |A \cap B| - |A \cap C| - |B \cap C| + |A \cap B \cap C|$$


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Incl-Excl (n sets)

$$|A_1 \cup A_2 \cup \dots \cup A_n| =$$

$$\sum_{\emptyset \neq S \subseteq \{1, 2, \dots, n\}} (-1)^{|S|+1} \left| \bigcap_{i \in S} A_i \right|$$


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Incl-Excl Formula: Proofs

by induction on n
 --uninformative

by binomial counting
 --next

by distributivity
 --also



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