Mathematics for Computer Science
MIT 6.042J/18.062J

Proof by Cases: Friends & Strangers

Friends & Strangers

Six people. Every two are either friends or strangers.

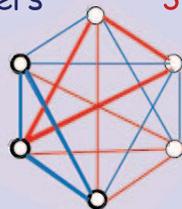Claim: there is a set of

3 mutual friends or

3 mutual strangers

Friends & Strangers

People are circles

3 mutual strangers   3 mutual friends

red line shows friends
blue line shows strangers

Friends & Strangers

Take 3 minutes to find a counter-example

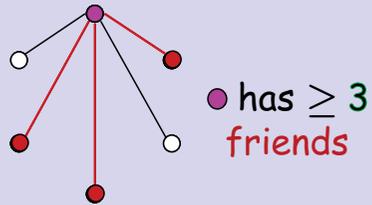--or convince yourself there isn't any counterexample, that is, the Claim is true.

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A Proof of the Claim

⇒ Person ● has a line to each of the **5** other people.
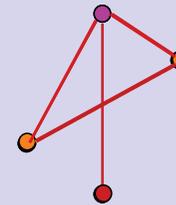- lines are red or blue, so at least **3** must be the same color.

● has ≥ **3** friends

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A Proof of the Claim

Case 1: some pair of these friends are friends of each other, then we have 3 mutual friends:

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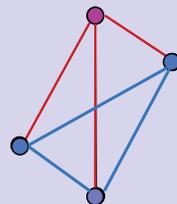
A Proof of the Claim

Case 2: no pair of these friends are friends of each other, so we have 3 mutual strangers:

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A Proof of the Claim

Since the Claim is true in either case, and one of these cases always holds, the Claim is always true.

QED

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Ramsey's Theorem

For any k, every *large enough* group of people will include either

k mutual friends, or

k mutual strangers.

Ramsey's Theorem

For any k, every *large enough* group of people will include either

size-k red *clique*, or

size-k blue *clique*.

Let R(k) be the *large enough* size.

So we've proved that R(3) = 6.

Ramsey's Numbers

Turns out that R(4) = 18 (not easy!)
R(5) is unknown!
Paul Erdös considered finding R(6) a hopeless challenge!
So in our second class, we have reached a research frontier!

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