Mathematics for Computer Science
MIT 6.042J/18.062J

The Well Ordering Principle

Well Ordering principle

Every nonempty set of
nonnegative integers
has a
least element.

Familiar? Now you mention it, Yes.
Obvious? Yes.
Trivial? Yes. But watch out:

Well Ordering principle

Every nonempty set of
nonnegative rationals
has a
least element.

NO!

Well Ordering principle

Every nonempty set of
~~*nonnegative*~~ *integers*
has a
least element.

NO!

What is the

- *youngest* age of MIT graduate?
- *smallest* # neurons in any animal?
- *smallest* #coins = \$1.17?

$\mathbb{N}$ ::= nonnegative integers

For rest of this talk, "number" means nonnegative integer

$\sqrt{2}$ proof used Well Ordering

Proof: …suppose $\sqrt{2} = \frac{m}{n}$

…can always find such m, n>0 without common factors…

why always ?

Proof using Well Ordering

Find smallest number m s.t. $\sqrt{2} = \frac{m}{n}$. If m, n had a common factor, c>1, then $\sqrt{2} = \frac{(m/c)}{(n/c)}$ and $m/c < m$

Proof using Well Ordering

Find smallest number m s.t. $\sqrt{2} = \frac{m}{n}$.

This contradiction implies m, n have no common factors.

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