6.080 / 6.089 Great Ideas in Theoretical Computer Science
Spring 2008

6.080/6.089 Problem Set 3

Assigned: Thursday, March 13, 2008 / Due: Thursday, March 20, 2008

1. Consider the following problem:

   - *Given a positive integer M, as well as a list of positive integers $x_1, \ldots, x_n$, find the closest you can get to M by adding a subset of x_i's without exceeding M. In other words, find the maximum of $\sum_{i \in S} x_i$ over all subsets $S \subseteq \{1, \ldots, n\}$ such that $\sum_{i \in S} x_i \leq M$.*

   In the general case—where M could be much larger than n—it is known that the above problem is NP-complete. On the other hand, describe an algorithm to solve this problem whose running time is a polynomial function of M and n. [*Hint:* Use dynamic programming, the same basic technique we used in class to solve the Longest Increasing Subsequence problem. In other words, show how a solution to the whole problem can be built recursively out of solutions to a reasonable number of subproblems.]

2. **"The Equivalence of Search and Decision Problems."** Suppose there's a polynomial-time algorithm to decide whether a given Boolean formula $\varphi(x_1, \ldots, x_n)$ has a satisfying truth assignment. (In other words, suppose P = NP.) Show that this implies that we can actually *find* a satisfying assignment for any Boolean formula φ in polynomial time, whenever one exists. [*Hint:* Give an algorithm that constructs a satisfying assignment for φ, one variable at a time, repeatedly calling the decision algorithm as an oracle]

3. Suppose problem X is proved NP-complete, by a polynomial-time reduction that maps size-n instances of SAT to size-n^3 instances of problem X. And suppose that someday, some genius manages to prove that SAT requires $\Omega(c^n)$ time, for some constant $c > 1$. Then what can you conclude about the time complexity of problem X?

4. Let $EXACT4SAT$ be the following problem:

   - *Given a Boolean formula φ, consisting of an AND of clauses involving exactly 4 distinct literals each (such as $(x_2 \vee \neg x_3 \vee \neg x_5 \vee x_6)$), decide whether φ is satisfiable.*

   Show that $EXACT4SAT$ is NP-complete. You can use the fact, which we proved in class, that $3SAT$ is NP-complete.