Massachusetts Institute of Technology

Department of Electrical Engineering and Computer Science

6.245: MULTIVARIABLE CONTROL SYSTEMS

by A. Megretski

Problem Set 6 (due March 17, 2004) [1]

Problem 6.1

A standard feedback control design setup is defined by the differential equations

$$\dot{x}_1 = -ax_1 + u, \quad \dot{x}_2 = -x_2 + w,$$

and by

$$y = w + bx_2, \;\; z = \begin{bmatrix} cu \\ x_1 \end{bmatrix},$$

where a, b, c are real parameters.

(a) Find matrices $A, B_1, B_2, C_1, C_2, D_{11}, D_{12}, D_{21}, D_{22}$ for this setup.

(b) Find all values of a, b, c for which the setup is singular, inficating frequency, multiplicity, and type (control/sensor) of the singularity.

Problem 6.2

Find H-Infinity and H2 norms of $G(s) = G_a(s)$ as a function of real parameter a:

(a) $G(s) = 1/(s + a) - 2/(s + 2a)$;

(b) $G(s) = (1 - \exp(-a^2 s))/s$.

[1] Version of March 10, 2004

Problem 6.3

Find L2 gains of systems described below (input f, output g, defined for $t \geq 0$). You are not required to *prove* correctness of your answer.

(a) $y(t) = e^{-at} f(t)$.

(b) $y(t) = f(t)/(1 + a^2 f(t)^2)$.

(c) $y(t) = f(t + a)$.

Problem 6.4

A standard H2 optimization setup is defined by the following transfer functions:

$$P_{wz}(s) = \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \; P_{wy}(s) = 1, \; P_{uz}(s) = \begin{bmatrix} 1/(s+a) \\ 1 \end{bmatrix}, \; P_{uy}(s) = -1/(s+a),$$

where $a \in \mathbf{R}$ is a parameter. For all values of a for which the setup is non-singular, find the H2 optimal controller, together with the associated Hamiltonian matrices, solutions of Riccati equations, and controller/observer gains.

Problem 6.5

Open-loop plant $P(s)$ is strictly proper, has a double pole at $s = 0$, and a zero at $s = 0.1$. Controller $C = C(s)$ stabilizes the system on Figure 6.1, and ensures good tracking

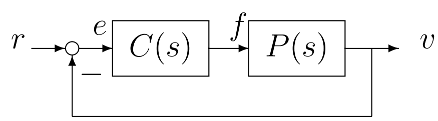

Figure 6.1: A SISO Feedback Setup

(transfer function from r to e has gain less than 0.1) for frequencies up to 1 rad/sec. Find a good lower bound on the maximal gain from r to e.

Problem 6.6

Find the square of the H2 norm of

$$G(s) = \frac{s}{s^2 + a_1 s + a_0}$$

as a rational function of real parameters a_1, a_0.