Problem 1 *(Supermodular Games)* Are the two games below supermodular?

P1 \ P2	A	B	C
A	$0,0$	$0,-1$	$-4,-2$
B	$-1,0$	$1,1$	$-1,-1$
C	$-2,-4$	$-1,-1$	$2,2$

P1 \ P2	A	B	C
A	$0,0$	$0,3$	$1,1$
B	$-1,-4$	$2,2$	$2,-1$
C	$0,0$	$-4,-1$	$-1,0$

Problem 2 *(Supermodular Games)* A supermodular game has *positive spillovers* if each player's payoff is increasing in the actions of others, so for each i, $u_i(s_i, s_{-i})$ is increasing in s_j, $j \neq i$.

Define the socially efficienct profile s^E as the solution to

$$\max_{s_1,\ldots,s_I} \sum_{i=1}^{I} u_i(s_1,\ldots,s_I).$$

Assume that this problem has a unique local optimum. Show that if s^N is a pure strategy NE, then $s_i^N \leq s_i^E$ for all i.

Problem 3 *(Potential games)*

- (a) Which of the following games are potential or ordinal potential? Justify your answer.

P1 \ P2	H	T
H	$1,-1$	$-1,1$
T	$-1,1$	$1,-1$

P1 \ P2	B	S
B	$2,1$	$0,0$
S	$0,0$	$1,2$

- (b) Is there a game with a unique pure strategy Nash Equilibrium, which does not have an ordinal potential?

Problem 4 *(The Stag Hunt Game - A Game of Social Cooperation)* The stag hunt is a game which describes a conflict between safety and social cooperation. Other names for it or its variants include "assurance game", "coordination game", and "trust dilemma". Inspired by the philosopher Jean-Jacques Rousseau, the game involves two individuals that go out on a hunt. Each can individually choose to hunt a stag or hunt a hare. If an individual hunts a stag, he must have the cooperation of his partner in order to succeed. An individual can get a hare by himself, but a hare is worth less than a stag. The game is succinctly described by the payoff matrix below:

	stag	hare
stag	a,a	$0,b$
hare	$b,0$	$b/2,b/2$

In particular, if they both cooperate and hunt a stag, they succeed and get a. Alternatively, one goes for hare, succeeds and get a lower payoff b, whereas the other that went for stag gets 0, since stag hunting needs cooperation. Finally, if both go for hare, then they both obtain $b/2$. The main assumption is that $a > b > 0$.

(i) Compute all Nash Equilibria of the stag hare game, both in pure and mixed strategies.

(ii) Show that the pure strategy Nash Equilibria are evolutionary stable. How about the mixed strategy equilibrium?

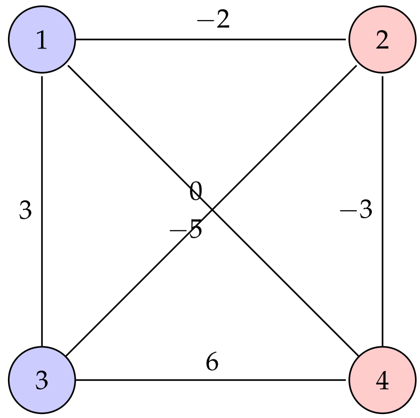

Figure 1: Same color indicates the nodes belong to the same cut set.

(iii) Consider the continuous time replicator dynamics for the stag hare game. Write down their expression and show that the pure strategy Nash equilibria are asymptotically stable.

Problem 5 *(Graph Cut as a Potential Game)* Consider a weighted undirected graph $G = (V, E)$, where V denotes the set of vertices, and E denotes the set of edges. Let w_{ij} denote the weight on the edge between the vertices i and j. The goal is to partition the vertices set V into two distinct subsets V_1, V_2, where $V_1 \bigcup V_2 = V$. We formulate this problem as a game. Let each vertex i be a player, with strategy space $s_i \in \{-1, 1\}$, where $s_i = 1$ means $i \in V_1$ and $s_i = -1$ means $i \in V_2$. The weight on each edge denotes how much the corresponding vertices 'want' to be on the same set. Thus, define the payoff function of player i as $u_i(s_i, s_{-i}) = \sum_{j \neq i} w_{ij} s_i s_j$.

For example, in the cut given in Figure 1, where $s_1 = s_3 = 1$ and $s_2 = s_4 = -1$. It can be seen that player $1, 2, 3$ has no incentive to unilaterally deviate, while player 4 can do better by deviating to $s_4 = 1$ and receive a positive payoff of 3.

Show that this game is a potential game by writing down explicitly the associated exact potential function.

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