6.334 Power Electronics
Spring 2007

Power Electronics Notes - D. Perreault

★★ Intro. Magnetics

Ampere's Law: $\oint H \cdot dl = \int J \cdot dA + \cancel{\frac{d}{dt}\int \epsilon E \cdot dA}^{\approx 0}$

Faraday's Law $\oint E \cdot dl = -\frac{d}{dt}\int B \cdot dA$

Flux Continuity $\oint B \cdot dA = 0$ $(\nabla \cdot B = 0)$

	units	
H magnetic field intensity	A/m	
B magnetic flux density	T	$\left(1T = 1\frac{\text{Joule}}{A \cdot m^2} = 10^4 \text{ gauss}\right)$

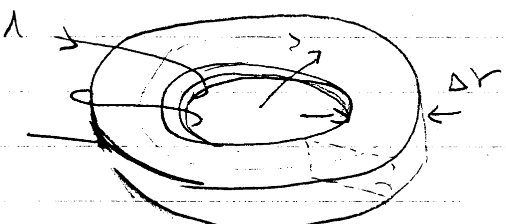
$B = \mu H$, μ = permeability (material property) (H/m)
μ_0 = permeability of free space $(4\pi \times 10^{-7}$ H/m)

(note: in general $B = f(H)$ → nonlinear relation)

★ Let's consider the design of an inductor

Assume $r_c \gg \Delta r$, $\mu_c \gg \mu_0$

$l_c = 2\pi r_c$



To find inductance
1. find H
2. find Φ
3. find λ → $\lambda = L i$

1. Ampere's Law $\oint H \cdot dl = \int J \cdot dA$

$$H \cdot l_c = N i$$

2. $B = \mu H$, $\Phi = BA_c \rightarrow \Phi = \mu_c A_c H$

$$\Phi_B = \frac{\mu A_c N}{l_c} i$$

3. λ flux linkage $\lambda = N\Phi_B = \underbrace{\frac{N^2 \mu A_c}{l_c}} i$

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$$\lambda = L i \Rightarrow L = \frac{N^2 \mu A_c}{l_c}$$ ← depends on geometry, mat'l properties, # turns

By Faraday's Law $\frac{d\lambda}{dt} = v = \frac{d}{dt}(Li) = L\frac{di}{dt}$

Notes

1. $L \propto N^2$ (not N!!)

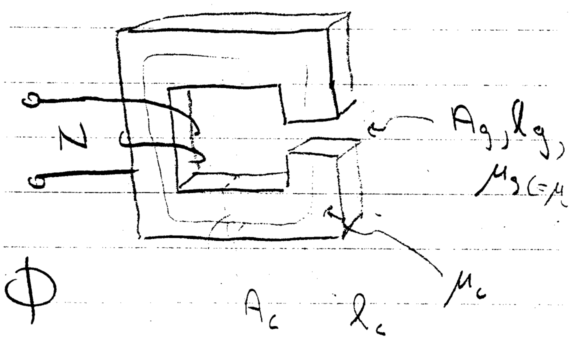
   Some manufacturers will characterize cores with an A_L value, where A_L = # nH for 1 turn (or # mH @ 1000 turns)

   $\therefore L = A_L * N^2$ (nH)

2. $L \propto \mu_c$ (material property)

   because μ can vary with H, Temp, time, etc., so can L! For very accurate & stable values of L we need to do additional work → one method: Add an air gap! (see below)

Consider a core with a gap in it



$$H_c l_c + H_g l_g = N i$$

by flux continuity $B_c A_c = B_g A_g = \Phi$

$H_c = \frac{B_c}{\mu_c}$, $H_g = \frac{B_g}{\mu_g}$

$$\frac{\Phi}{\mu_c A_c} l_c + \frac{\Phi l_g}{\mu_g A_g} = N i$$

start here for $\mathcal{R}$

or $$\Phi\left[\frac{l_c}{\mu_c A_c} + \frac{l_g}{\mu_g A_g}\right] = N i \quad (*)$$

$$\therefore \lambda = N\Phi = \frac{N^2 i}{\left[\frac{l_c}{\mu_c A_c} + \frac{l_g}{\mu_g A_g}\right]}$$

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$$\therefore\ L = \frac{\lambda}{i} = \frac{N^2}{\left[\frac{l_c}{\mu_c A_c} + \frac{l_g}{\mu_g A_g}\right]}$$

for $\mu_g = \mu_o$, $A_c = A_g \Rightarrow$

$\Rightarrow$ IF $\frac{l_g}{\mu_o} >> \frac{l_c}{\mu_c}$ then inductance doesn't depend much on μ_c !!!

$\rightarrow$ in this case, most energy stored in gap $W_m = \frac{1}{2}\iiint_{Vol} B \cdot H \, dv$

★ Let's step back and look at flux computation : Reluctance Models

$$\Phi = \frac{(Ni)}{\left(\frac{l_c}{\mu_c A_c} + \frac{l_g}{\mu_g A_g}\right)}$$

Similar in form to "$i = \frac{V}{R_1 + R_2}$"

Define Ni = magnetomotive force MMF

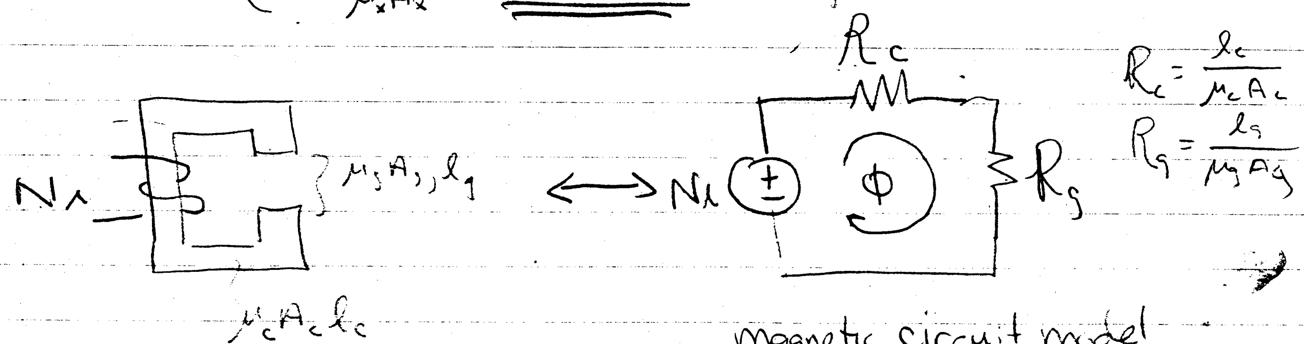
Φ = flux in magnetic circuit

$\mathcal{R} = \frac{l_x}{\mu_x A_x}$ = Reluctance of magnetic circuit element

resistance to flux !!

$\mathcal{R}_c = \frac{l_c}{\mu_c A_c}$

$\mathcal{R}_g = \frac{l_g}{\mu_g A_g}$



magnetic circuit model

$\Rightarrow$ also works for multiple windings (MMF's) and parallel branches

$\Rightarrow$ Rapid computation of magnetic flux in more complicated structures

Note Resistance $R = \frac{l}{\sigma A}$ Reluctance $= \frac{l}{\mu A}$

$\therefore$ permeability is like "magnetic conductivity" (Reluctance is resistance to flux)

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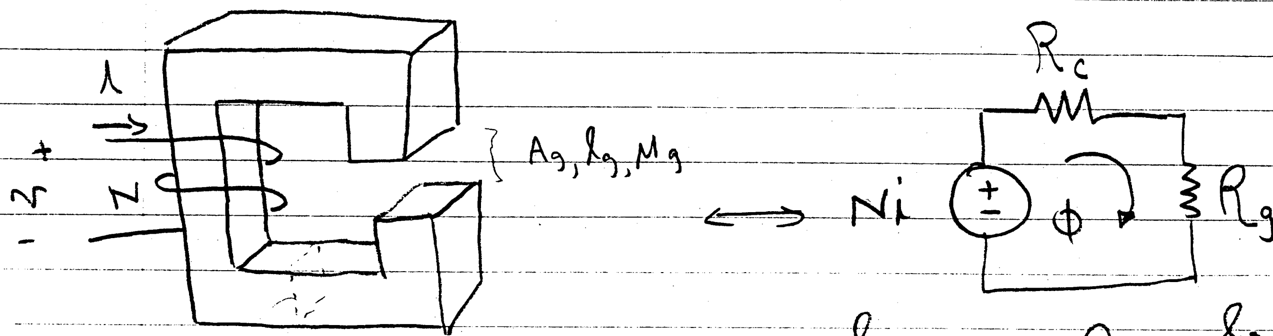
★ Summary: Reluctance Model

Analogue of Electric Circuit Model for magnetics

Electric		Magnetic	
Voltage (EMF)	$\longleftrightarrow$	Magnetomotive Force (MMF)	
Current	$\longleftrightarrow$	flux	$i \rightarrow$
Resistance $\frac{\ell}{\sigma A}$	$\longleftrightarrow$	Reluctance $\frac{\ell}{\mu A}$	



$A_c \ell_c \mu_c$

$$R_c = \frac{\ell_c}{\mu_c A_c} \qquad R_g = \frac{\ell_g}{\mu_g A_g}$$

$$\phi = \frac{Ni}{R_c + R_g} = \frac{Ni}{\frac{\ell_c}{\mu_c A_c} + \frac{\ell_g}{\mu_g A_g}}$$

To find terminal voltage: Voltage of each turn is $V_{Turn} = \frac{d\phi}{dt}$, Voltage of whole coil is N times this. Hence the definition of flux linkage:

$$\lambda = N\phi \qquad \text{(flux linkage)}$$

$$v = \frac{d\lambda}{dt} = \underbrace{\left[\frac{N^2}{\frac{\ell_c}{\mu_c A_c} + \frac{\ell_g}{\mu_g A_g}}\right]}_{L} \frac{di}{dt}$$

$(\lambda = L\,i)$

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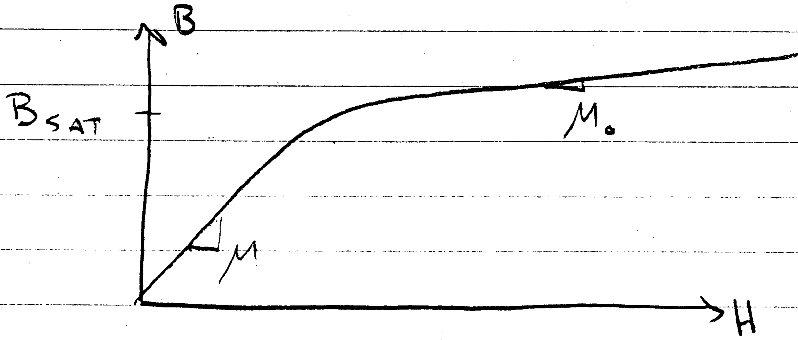
Notes: ① $L \propto N^2$

$(L = N^2 A_L)$ where A_L is usually def. as nH @ 1 turn.

② for accuracy, we need $\mu_c \gg \mu_o$, $l_c \gg \sqrt{A_c}$, $l_g \ll \sqrt{A_c}$

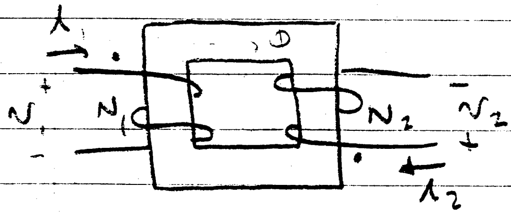
③ These relations rely on material property $B = \mu H$.
In general $B = f(H)$

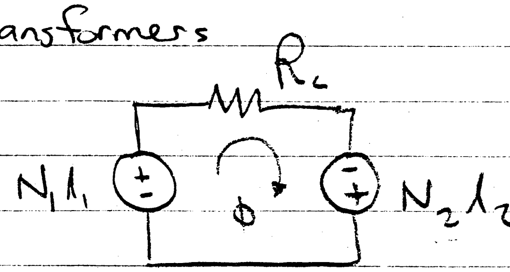


above some flux density B_{sat}, material saturates $\frac{\partial B}{\partial H} \to \mu_o$

- we need to operate @ $B < B_{SAT}$ (Iron: $\mu \to 10^4 \mu_o$, $B_{sat} \sim 2T$)

★ Now, lets consider transformers





"DOT CONVENTION: All currents into dots throw flux the same direction"

$$v_1 = \frac{d\lambda_1}{dt} = N_1 \frac{d\phi}{dt}$$
$$v_2 = \frac{d\lambda_2}{dt} = N_2 \frac{d\phi}{dt}$$

$$\boxed{v_2 = \frac{N_2}{N_1} v_1}$$

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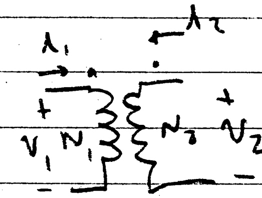
currents: $\Phi = \dfrac{N_1 i_1 + N_2 i_2}{R_c}$

If $\mu_c \to \infty$ $\therefore$ $R_c \to 0$ so for finite Φ we need

$$\boxed{N_1 i_1 = -N_2 i_2}$$

$\Rightarrow$ These are the ideal transformer relations!



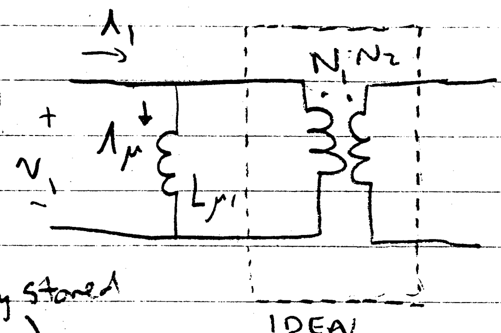
Consider nonidealities : Magnetizing

Since $\mu_c < \infty$, $R_c > 0$ $\therefore$ $N_1 i_1 + N_2 i_2 = \Phi R_c \neq 0$
(there is an error in the current relation.

For $i_2 = 0$ (open ckt on secondary)

$$\Phi = \frac{N_1 i_1}{R_c} \longrightarrow v_1 = \frac{N_1^2}{R_c}\, i_1 = L_{\mu 1}\frac{d i_1}{dt}$$

Similar results for other winding



- magnetizing inductance L_μ
  reflects the fact that there is energy stored in the core (in the process of induction).

- Could place magnetizing inductance on either side of xformer; with appropriate scaling for turns ratio. (N^2)

- Magnetization of the core is the reason that transformers don't work at dc

$$V_1 = N_1 \frac{d\Phi}{dt} \quad \therefore \quad \Phi = \frac{1}{N_1}\int V_1\, dt$$

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$$\therefore B_c = \frac{\Phi}{A_c} = \frac{1}{N_1 A_c}\int V_1\,dt$$

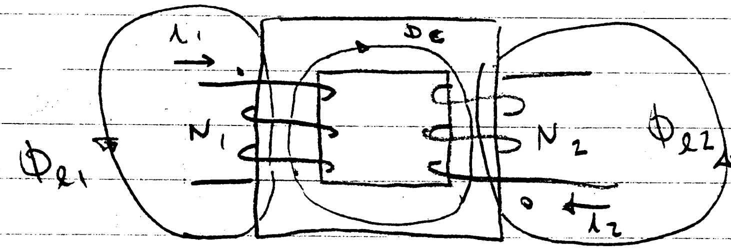
Since we need $B_c < B_{sat}$ to avoid core saturation

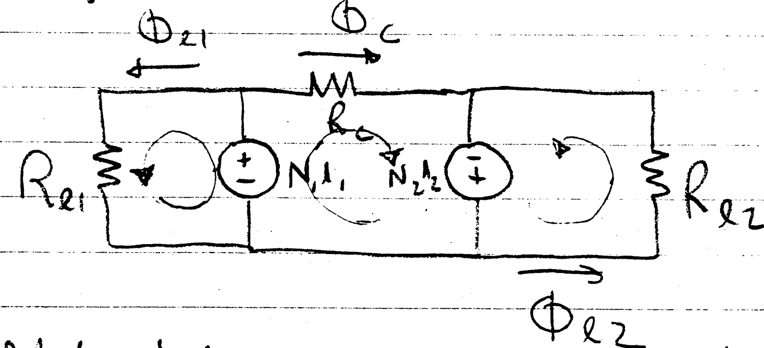
$$\therefore \boxed{\int V_1\,dt \leq N_1 B_{SAT} A_c}$$

Volt-sec limit on the transformer!

Leakage Inductance

In reality, all flux does not follow core path and some flux from each winding does not link the other:





$$\Phi_{\ell 1} = \frac{N_1 i_1}{R_{\ell 1}} \qquad \Phi_c = \frac{N_1 i_1 + N_2 i_2}{R_c} \qquad \Phi_{\ell 2} = \frac{N_2 i_2}{R_{\ell 2}}$$

$$\lambda_1 = N_1(\Phi_c + \Phi_{\ell 1}) = \frac{N_1^2}{R_c} i_1 + \frac{N_1^2}{R_{\ell 1}} i_1 + \frac{N_1 N_2}{R_c} i_2$$

$$\lambda_2 = N_2(\Phi_c + \Phi_{\ell 2}) = \frac{N_2^2}{R_c} i_2 + \frac{N_2^2}{R_{\ell 2}} i_2 + \frac{N_1 N_2}{R_c} i_1$$

$$\begin{bmatrix}\lambda_1 \\ \lambda_2\end{bmatrix} = \begin{bmatrix}\frac{N_1^2}{R_c} + \frac{N_1^2}{R_{\ell 1}} & \frac{N_1 N_2}{R_c} \\ \frac{N_1 N_2}{R_c} & \frac{N_2^2}{R_c} + \frac{N_2^2}{R_{\ell 2}}\end{bmatrix}\begin{bmatrix} i_1 \\ i_2\end{bmatrix} = \begin{bmatrix} L_{11} & L_m \\ L_m & L_{22}\end{bmatrix}\begin{bmatrix} i_1 \\ i_2\end{bmatrix}$$

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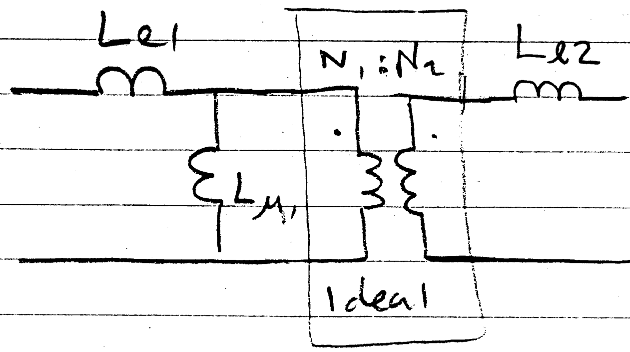
$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} L_{11} & L_M \\ L_M & L_{22} \end{bmatrix} \begin{bmatrix} \dot{\lambda}_1 \\ \lambda_2 \end{bmatrix}$$

inductance matrix description

It is easy to show that this is equivalent to the circuit model:



where $L_{e_1} = \dfrac{N_1^2}{R_{e1}}$ $L_{\mu_1} = \dfrac{N_1^2}{R_c}$ $L_{e2} = \dfrac{N_2^2}{R_{e2}}$

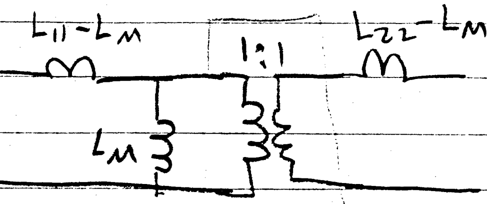
and $L_{11} = L_{e1} + L_{\mu 1}$, $L_m = \left(\dfrac{N_2}{N_1}\right) L_{\mu 1}$, $L_{22} = L_{e2} + \left(\dfrac{N_2}{N_1}\right)^2 L_{\mu 1}$

- So a real transformer has both magnetizing inductance and leakage inductances. These are important for practical applications

- OTHER Model parameters are possible:
To characterize a two-winding transformer, we need three inductances L_{11}, L_M, L_{22}. The circuit model actually has 4 parameters: $L_{e1}, L_{\mu 1}, L_{e2}, N_2/N_1$. If we are willing to use NONPHYSICAL values (e.g. for N_2/N_1), we can come up with an infinite array of circuit models with the SAME terminal characteristics

e.g. make $\left(\dfrac{N_2}{N_1}\right) = 1$

Just in MODEL, not physical value.



Also works to describe behavior!

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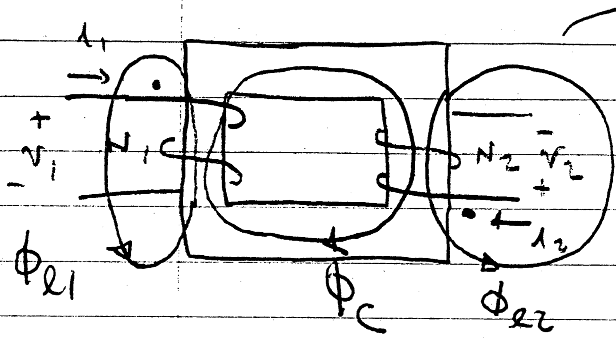
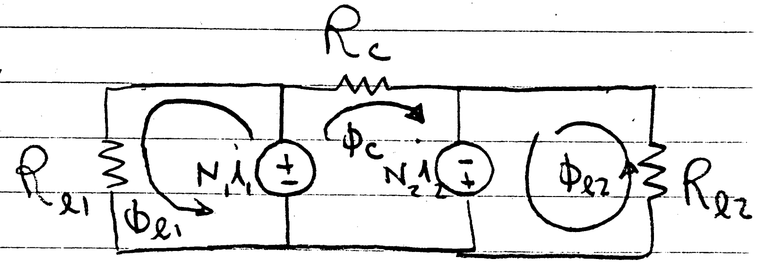
★ Summary of Transformers

GIVEN A TWO-WINDING MAGNETIC STRUCTURE



we get a magnetic circuit model



Notes: ① The directions of the MMF sources (+fluxes) are determined by the orientation of the windings

② The electrical behavior at the winding terminals is determined by flux linkage. Appropriate polarities from Lenz's Law (λ is $N\Phi$ from + side of MMF sourse)

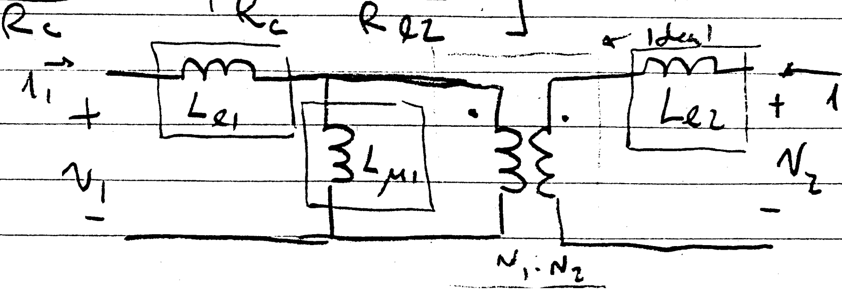
$$\int V_1\,dt = \lambda_1 = N_1(\Phi_{l1} + \Phi_c)$$

$$\int V_2\,dt = \lambda_2 = N_2(\Phi_{l2} + \phi_c)$$

working out the math relating flux linkage (λ) to flux (Φ) to current (i)

$$\begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} \frac{N_1^2}{R_c} + \frac{N_1^2}{R_{l1}} & \frac{N_1 N_2}{R_c} \\ \frac{N_1 N_2}{R_c} & \frac{N_2^2}{R_c} + \frac{N_2^2}{R_{l2}} \end{bmatrix} \begin{bmatrix} \dot{\imath}_1 \\ \dot{\imath}_2 \end{bmatrix} = \begin{bmatrix} L_{11} & L_M \\ L_M & L_{22} \end{bmatrix} \begin{bmatrix} \dot{\imath}_1 \\ \dot{\imath}_2 \end{bmatrix}$$

A CIRCUIT MODEL



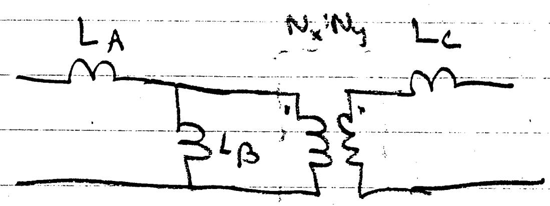
where $L_{\mu 1} = \frac{N_1^2}{R_c}$ $L_{\ell 1} = \frac{N_1^2}{R_{\ell 1}}$ $L_{\ell 2} = \frac{N_2^2}{R_{\ell 2}}$

Note: our inductance matrix description of the system has 3 parameters: L_{11}, L_{22}, L_M.

Our circuit model has 4 parameters: $L_{\ell 1}, L_{\mu}, L_{\ell 2}, \left(\frac{N_2}{N_1}\right)$

Using the physical turns ratio, we then get unique relations for $L_{\ell 1}, L_{\mu}, L_{\ell 2}$. However, that is not necessary! There are an infinite # of circuit models that match the terminal behavior of the device (specified by the inductance matrix

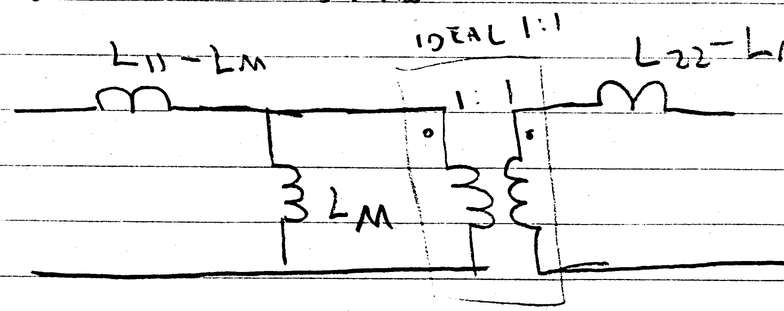
$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} L_{11} & L_M \\ L_M & L_{22} \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} \longleftrightarrow$$



3 indep params

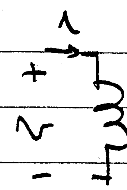
4 indep params. Choose any one + match other 3!

e.g. if we arbitrarily select a MODEL with $(N_y/N_x) = 1$ (regardless of "real" turns ratio



This also gives us the exact same terminal behavior! (Though it does not reflect the physical system, it is a good terminal model.)

Note: for a real magnetic system, there are physical limits on the parameter values:



in a 1-port system $V = L\frac{d\lambda}{dt}$, we must have $L > 0$
(cons. of energy)

Similarly in a 2-port magnetic circuit (inverting the matrix)

$$\frac{d}{dt}\begin{bmatrix}\lambda_1 \\ \lambda_2\end{bmatrix} = L^{-1}\begin{bmatrix}V_1 \\ V_2\end{bmatrix} = \frac{1}{L_{11}L_{22}-L_M^2}\begin{bmatrix}L_{22} & -L_M \\ -L_M & L_{11}\end{bmatrix}\begin{bmatrix}V_1 \\ V_2\end{bmatrix}$$

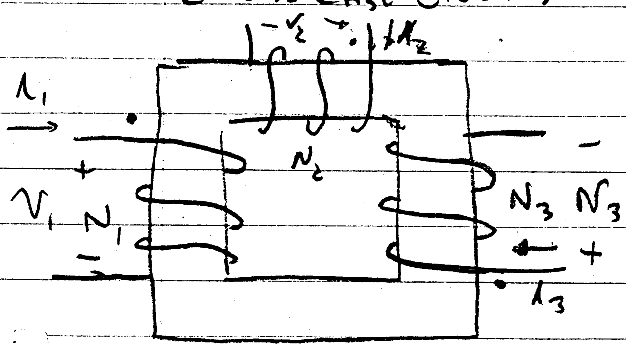
For a real system, we must have $|L_M| < \sqrt{L_{11}L_{22}}$.

Otherwise if $V_2 = 0$ (short) + we apply $+V_1$, $\frac{d\lambda_1}{dt} < 0$ and we would get energy out of the system forever!

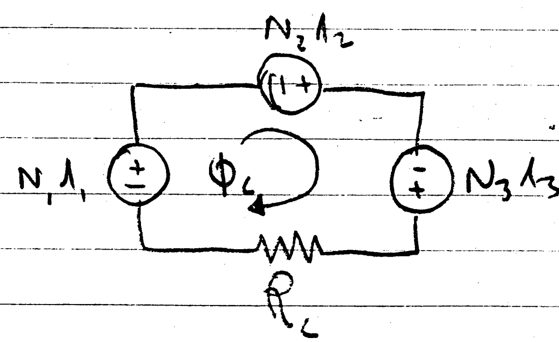
In practice, $L_M = \sqrt{L_{11}L_{22}}$ is perfect coupling (no leakage)

Coupling coeff. $K \triangleq \frac{L_M}{\sqrt{L_{11}L_{22}}}$ $\quad -1 < k < 1$

★ CONSIDER TRANSFORMERS WITH MORE THAN 2 WINDINGS:
(IDEAL CASE ONLY)



No LEAKAGE
$\longleftrightarrow$



SERIES CASE

If $R_c \to 0$, no leakage
$(\mu \to \infty)$

$$\boxed{N_1\lambda_1 + N_2\lambda_2 + N_3\lambda_3 = \phi R_c = 0}$$

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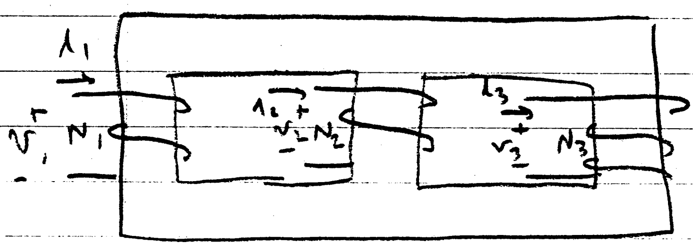
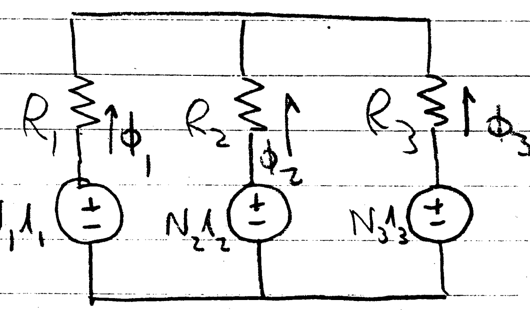
Also
$$v_1 = \frac{d\lambda_1}{dt} = N_1 \frac{d\Phi}{dt}$$
$$v_2 = \frac{d\lambda_2}{dt} = N_2 \frac{d\Phi}{dt}$$
$$v_3 = \frac{d\lambda_3}{dt} = N_3 \frac{d\Phi}{dt}$$
$$\boxed{\frac{v_1}{N_1} = \frac{v_2}{N_2} = \frac{v_3}{N_3}}$$

Note: as there is only one magnetic path, the dot convention is clear!

WE CAN ALSO HAVE A PARALLEL STRUCTURE



$\longleftrightarrow$



If $\mu_c \to \infty$ $R_1, R_2, R_3 \to 0$

$\therefore$ from model, for $\phi_1 - \phi_3 < \infty$

$$\boxed{N_1\lambda_1 = N_2\lambda_2 = N_3\lambda_3}$$

Also $\Phi_1 + \Phi_2 + \Phi_3 = 0$

$$\frac{\lambda_1}{N_1} + \frac{\lambda_2}{N_2} + \frac{\lambda_3}{N_3} = 0$$

$$\therefore \boxed{\frac{v_1}{N_1} + \frac{v_2}{N_2} + \frac{v_3}{N_3} = 0}$$

1. WE GET DIFFERENT RELATIONS IN SERIES + PARALLEL CASES
2. PARALLEL CASE: "DOT" CONVENTION IS NO LONGER A SUFFICIENT DESCRIPTION
3. If we included nonidealities, we would get a 3×3 inductance matrix description.

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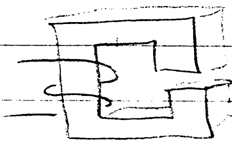
★ Inductor Design Tips

So far, we have mostly considered analysis. Design is more challenging, and is partly an art.

Some tips for designing inductors: (we will focus on identifying the limits that a design must meet. There are many paths to designs that meet the limits)

1. Given a core geometry and gap, we get inductance proportional to the number of turns squared:

$$L = \frac{N^2}{(R_c + R_g)}$$



This is sometimes expressed in A_L $\quad L = N^2 A_L$ (nH)

2. WE MUST NOT exceed the allowed flux density of the material

$$B_{MAX} = \frac{\Phi}{A_c} = \frac{L i_{max}}{N A_c} \leq B_{SAT}$$

If we choose to store all energy in the gap

$$W_M = \frac{1}{2} L i^2 = \frac{1}{2} \iiint_V B \cdot H \, dv = \frac{1}{2} \frac{B_{MAX}^2}{\mu_0} \cdot A_g l_g$$

→ for selecting A_g, l_g: $\quad l_g \ll \sqrt{A_c} \ll l_c$

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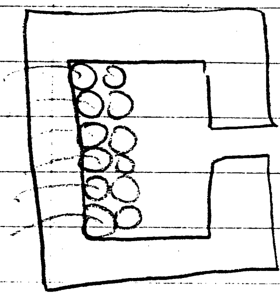
3 Wire must be thick enough to carry current w/o overheating.

Typically, a current density limit is imposed

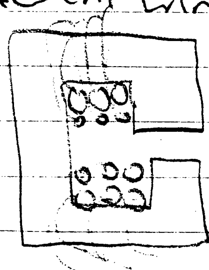
$$J_{MAX} \leq 500 \ A/cm^2 \quad \text{or} \quad 3000 \ A/in^2$$

4 WIRE MUST FIT IN THE WINDOW AREA

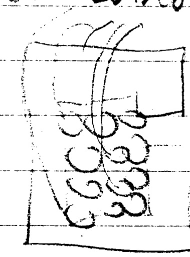


The packing factor K_w tells us the fraction of actual window area that can be filled with copper (typ $K_w \sim 0.5$, but depends on wire size, winding method.)

Different winding methods yield different winding lengths



2 leg windings may be better than one



5 losses + Temp rise must be acceptable (core + copper loss) ⇒ This is often the main limiting factor!

→

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★ **Loss mechanisms in magnetics:**

We care about loss both for efficiency and because the size of a magnetic component is often determined by loss.

Generally, we can break down losses into 2 components: Those associated with the winding and those associated with the core. Each has special effects associated with them

1. Winding Loss

@ low frequency (dc) winding loss is just due to the dc resistance in the winding + is easy to calculate

$$R_{wire} = \frac{\rho_{cu} l_{wire}}{A_{wire}} \qquad P_{diss} = I_{rms}^2 R_{wire} = I_{dc}^2 R$$

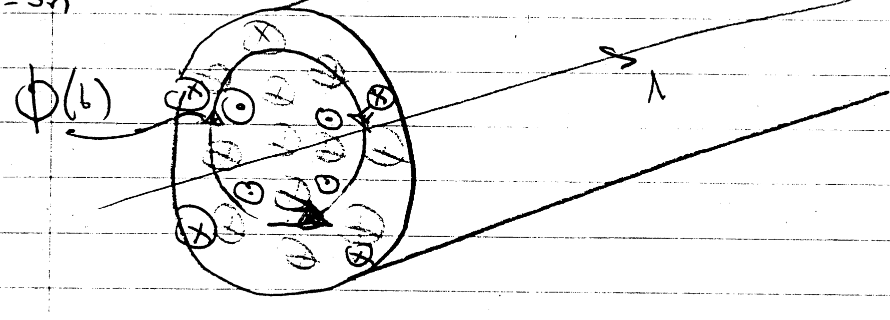
for dc current.

At higher frequencies, there are additional effect we must worry about: skin effect and proximity effect

Skin effect is the "self shielding" effect of conductors: Due to eddy currents generated by changes in magnetic field of an ac current, the fields + currents may not penetrate inside a conductor @ high frequency

simplified model

end-on



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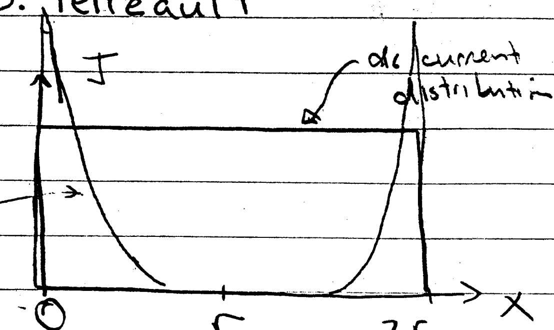
@ low frequency $\left|\frac{d\Phi}{dt}\right|$ small

@ high freq. $\left|\frac{d\Phi}{dt}\right|$, eddy currents large



The "self shielding" due to eddy currents may be expressed as a magnetic diffusion problem: magnetic fields diffuse into the conductor.

$$J \sim J_o e^{-x/\delta} \quad \text{where} \quad \delta = \sqrt{\frac{\rho_{cu}}{\pi \mu_{cu} f}} = \frac{K}{\sqrt{f}}$$

@ 60 Hz $\delta_{cu} \sim 0.75$ cm

So, if we need to carry high frequency current, a wire of radius $> \delta$ is not useful, since the current will be carried only on the surface of the wire

Approximate only!

$$R_{AC} \cong R_{dc}\left(1 + \sqrt{f/f_x}\right)$$ f_x is freq @ which $2r = \delta$

$$\cong R_{dc}\left(1 + \frac{W}{\delta}\right)$$ W is effective wire thickness ~ 2r

neglects porosity, etc.

So if we want to carry large, high-frequency current, we should parallel isolated wires of thickness $\leq 1\delta$. Sometimes use Litz wire, where strands are transposed so each spends same time in inner + outer positions.
(show sample)

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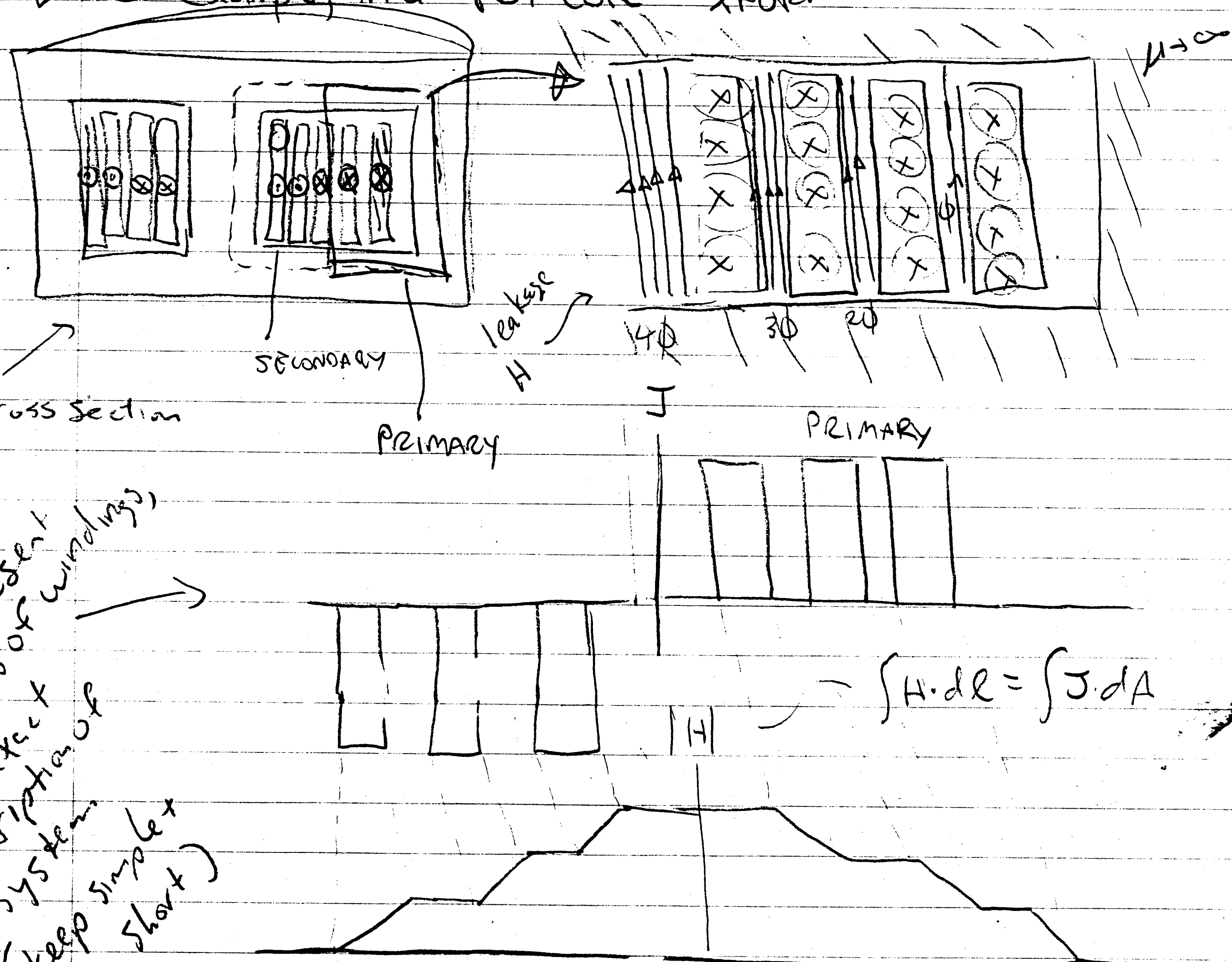
Proximity Effect

PROXIMITY EFFECT IS MOST IMPORTANT IN XFORMERS (WHERE BIG AC CURRENTS EXIST) BUT ALSO APPLY TO AC COMPONENTS IN INDUCTORS.

Skin Effect Arises because of eddy currents due to changing H fields due to the conductors own current.

However, even stronger eddy currents can be generated due to the fields from currents in neighboring conductors, especially in multi layer windings.

For example, in a "pot core" XFORMER



$$\int H \cdot dl = \int J \cdot dA$$

Just present as layers of windings, without exact description of system (keep simple + short)

So AMONG LAYERS OF WINDINGS, $|H| \uparrow$, $\therefore \frac{dB}{dt} \uparrow$

$\therefore$ Eddy currents + losses $\uparrow$

So: If we have many layers of conductor, ac losses may be even larger than suggested by skin effect, since the ac fields from other conductors may induce losses in a conductor.

⇒ This is esp. important in transformer design (where ac currents predominate)

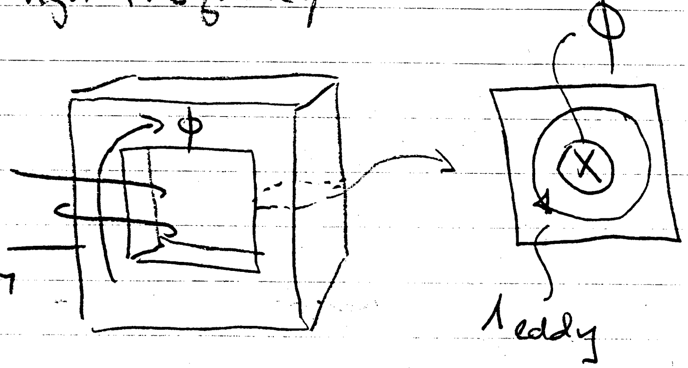
⇒ we may avoid high #'s of layers, interleave layers to keep peak field magnitudes down,

⇒ Usually, min loss is for wire < δ, and smaller as # of layers ↑.

★ Core losses: Up to now, we have considered core material to be "ideal" high μ material. However, there are mechanisms that cause loss in the core, and may cause problems @ high frequency

Eddy currents

Many core materials have significant conductivity



$\frac{dB}{dt}$ through core generates voltage which drive eddy currents around core. These eddy currents oppose changing core flux!

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2 effects:

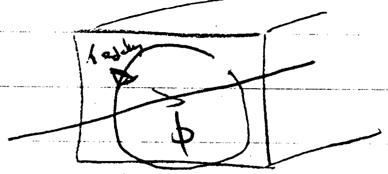
① we get losses! $i_{eddy} \propto \frac{dB}{dt} \propto f \Rightarrow i^2_{EDDY} R_{core}$ loss

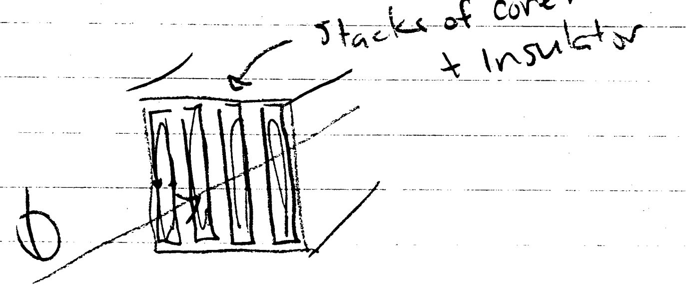
② once $\delta_{core} \lesssim$ core thickness, the eddy currents will cause the flux to be rejected from the core!

$\Rightarrow$ It will not act to guide flux + not be useful

$\Rightarrow$ This is why iron is good for low frequency ~ 60Hz but not useful @ high frequency ~ 100kHz. We must use high resistivity materials @ high f.

$\Rightarrow$ lamination Helps

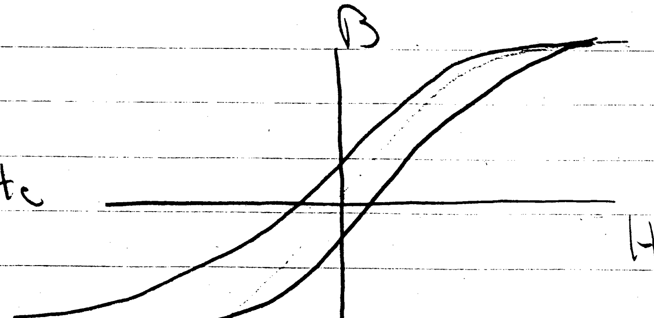




Effective core resistivity ↑ with laminations if lamination thickness $\ll \delta$ in appropriate dimension.

Hysteresis loss:

We actually dissipate energy moving the magnetic domains around. This appears as a hysteresis in the B-H loop.



Ideally we lose energy each time we go around loop
$P_{HYS} \propto f$, (but this is not true in reality)

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To model core loss, we often use the empirical Steinmetz Model (named for Charles Proteus Steinmetz):

$$\boxed{\text{loss per unit volume (of core)} \quad P_v = C_m f^{\alpha} \hat{B}_{AC}^{\beta}}$$

where parameters C_m, α, β are determined from manufacturer's data

e.g. for 3F3 Ferrite $\quad C_m \doteq 9.6 \times 10^{-13}\ \mathrm{W/cm^3}$

$\alpha = 1.231 \quad (f \text{ in Hz})$

$\beta = 2.793 \quad (B \text{ in gauss})$

These are usually only accurate for sinusoidal ac waves, but do account for both hysteresis + eddy current effects.

$\Rightarrow$ total loss $\quad P_{TOT} = P_{cu} + P_{core}$.