MASSACHUSETTS INSTITUTE OF TECHNOLOGY

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THE POISSON PROCESS CONTINUED

Readings:
Posted excerpt from Chapter 6 of [BT, 2nd edition], as well as starred exercises therein.

1 MEMORYLESSNESS IN THE POISSON PROCESS

The Poisson process inherits the memorylessness properties of the Bernoulli process. In particular, if we start watching a Poisson process at some time t^*, or at some causally determined random time S, the process we will see is a Poisson process in its own right, and is independent from the past. This is an intuitive consequence of the close relation of the Poisson and Bernoulli processes. We will give a more precise statement below, but we will not provide a formal proof. Nevertheless, we will use these properties freely in the sequel.

We first introduce a suitable definition of the notion of a stopping time. A nonnegative random variable S is called a stopping time if for every $s \geq 0$, the occurrence or not of the event $\{S \leq s\}$ can be determined from knowledge of the values of the random variables $N(t)$, $t \leq s$.[1]

Example: The first arrival time T_1 is a stopping time. To see this, note that the event $\{T_1 \leq s\}$ can be written as $\{N(s) \geq 1\}$; the occurrence of the latter event can be determined from knowledge of the value of the random variable $N(s)$.

The arrival process $\{M(t)\}$ seen by an observer who starts watching at a stopping time S is defined by $M(t) = N(S+t) - N(S)$. If S is a stopping time, this new process is a Poisson process (with the same parameter λ). Furthermore, the collection of random variables $\{M(t) \mid t \geq 0\}$ (the "future" after S) is independent from the collection of random variables $\big\{N(\min\{t, S\}) \mid t \geq 0\big\}$ (the "past" until S).

[1] In a more formal definition, we first define the σ-field $\mathcal{F}_s$ generated by all events of the form $\{N(t) = k\}$, as t ranges over $[0, s]$ and k ranges over the integers. We then require that $\{S \leq s\} \in \mathcal{F}_s$, for all $s \geq 0$.

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