6.450 Principles of Digital Communication   Monday Dec 14, 2009
MIT, Fall 2009

Final Exam

- You have 180 minutes to complete the quiz.

- This is an open-book quiz. You may use your book and six pages of notes. Calculators are allowed, but probably won't be useful.

- The problems are not necessarily in order of difficulty. Do the problems in whichever order you find most natural.

- A correct answer does not guarantee full credit and a wrong answer does not guarantee loss of credit. You should concisely indicate your reasoning and show all relevant work. The grade on each problem is based on our judgement of your level of understanding as reflected by what you have written.

- If we can't read it, we can't grade it.

- If you don't understand a problem, please ask.

Problem Q1:

True or False. Explain your answer.

(a) (5 points) Suppose two Gaussian random variables are uncorrelated. Thus, they are independent.

(b) (5 points) Let X and Y be iid zero mean Gaussian random variables. Let $Z = UX$ where U is $+1$ with probability $1/2$, and -1 with probability $1/2$. U is independent of X and Y. Then, Z and Y are not necessarily jointly Gaussian.

(c) (5 points) Let $Z \sim N(0,1)$ be a single random variable and let $N(t)$ be the random process denoted by $N(t) = Z$ for each t. Then $N(t)$ is a stationary Gaussian random process.

(d) (5 points) The coherence time τ_c for a narrowband communication system over a multipath wireless channel is proportional to the reciprocal of the carrier frequency.

Problem Q2 :

A single binary digit is sent over the following channel:

$$Y[t] = a_0 X[t] + a_1 X[t-1] + N[t]$$

where $N[t]$ is a normal random process.

(a) (10 points) Suppose $X[t]$ is whether $[1, 0]$ or $[0, 1]$ with equal probability. Compute LLR and design an optimal decoder for this scheme.

(b) (10 points) In this part, we want to capture the effect of multipath channel using a proper colored noise model. Specially, suppose our channel model is as follows:

$$Y'[t] = aX[t] + N'[t].$$

Find the probability density function of $N'(t)$ which leads to the same LLR as part (a).

Problem Q3: Suppose $X_c(t) = A\cos(w_0 t + \phi_k)$ is a modulated QPSK signal such that $\phi_k \in \{0, \pi/2, \pi, 3\pi/2\}$. We use the following demodulator:
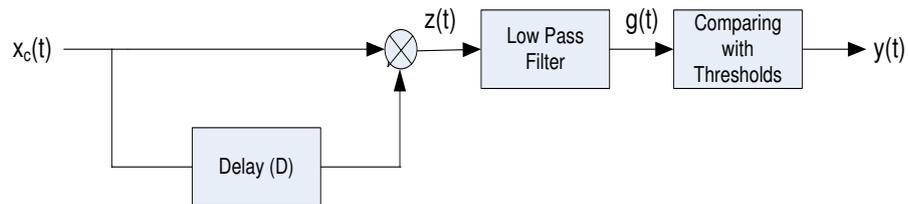
$w_0 D = \pi + 2m\pi$, where m is an integer and $1/D$ is the symbol transmission rate.

(a) (10 points) Compute $z(t)$ and $g(t)$ depicted in the figure. What is the bandwidth of the lowpass filter?

(b) (5 points) Explain how this demodulator works.

(c) (5 points) What is the disadvantage of this mod/demod scheme?

Problem Q4:

Assume that $\{h(\tau, t) : \mathbb{R}^2 \to \mathbb{C}\}$ is $\mathcal{L}_1$ and $\hat{h}(f, t)$ be its Fourier transform. Let ϵ be any given positive number.

(a) (10 points) Show that for sufficiently large T, $\int_{|\tau|>T} |h(\tau, t)e^{-2i\pi f\tau} - h(\tau, t)e^{-2i\pi(f-\delta)\tau}| < \epsilon/2$ for all f and $\delta > 0$.

(b) (10 points) For the ϵ and T selected above, show that $\int_{|\tau|\leq T} |h(\tau, t)e^{-2i\pi f\tau} - h(\tau, t)e^{-2i\pi(f-\delta)\tau}| < \epsilon/2$ for all f and sufficiently small $\delta > 0$. This shows that $\hat{h}(f, t)$ is continuous.

Problem Q5:

Consider the following constellation points.
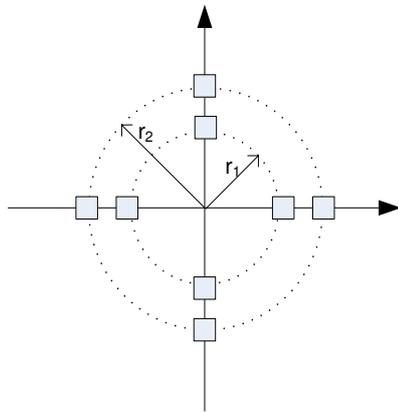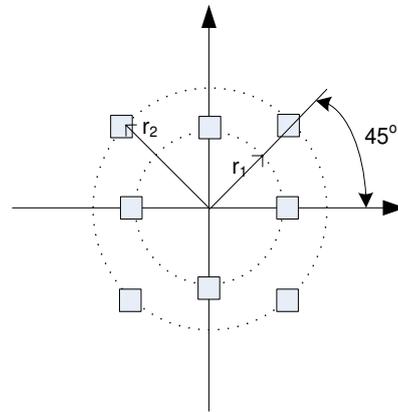
(i)   (ii)

suppose $r_1 = 1$ and $r_2 = 3/2$.

(a) (5 points) Compute the expected power of a transmitted signal for schemes (i) and (ii), assuming that basis are orthonormal and all points have the same probability in each scheme.

(b) (15 points) Compute an approximation for the probability of error of each scheme in the presence of a normal noise. In terms of the probability of error, which constellation works better?

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